

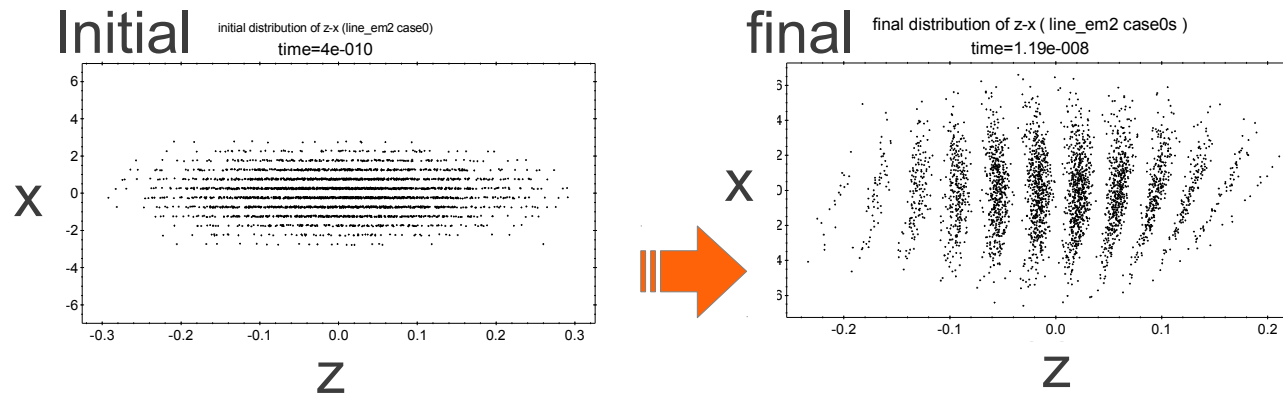
シミュレーションによるエミッタンス交換
を用いた
マイクロバンチ生成の検討

広島大学理学部物理科学科4年
ビーム物理研究室
大槻祥平

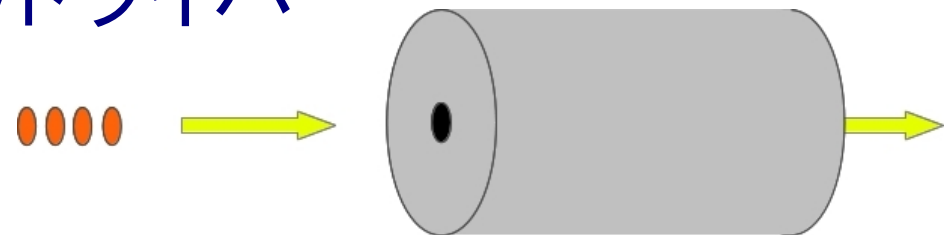
担当教員 栗木雅夫

目的・背景

- バンチ内部の明確な離散的粒子群の生成



- 方法: ビームの位相空間内の回転
(エミッタンス交換)の応用
- 応用: 誘電体加速のドライバ



検討内容

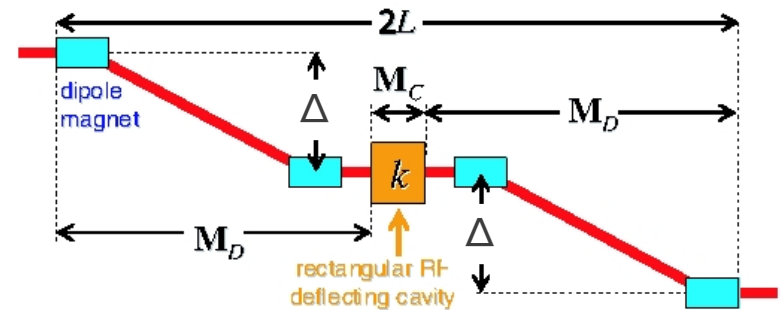
- エミッタンス交換を用いたマイクロバンチ生成が可能か
- 輸送行列の特性
- より好まれる条件
- 誘電体加速への応用
 - 加速電場・同期

※ 先行研究

GENERATION OF FEMTOSECOND BUNCH TRAINS USING A
LONGITUDINAL-TO-TRANSVERSE PHASE SPACE EXCHANGE
TECHNIQUE, Yin-e Sun, Philippe Pot

エミッタンス交換とは

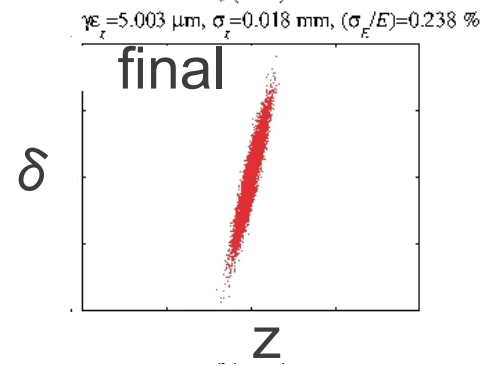
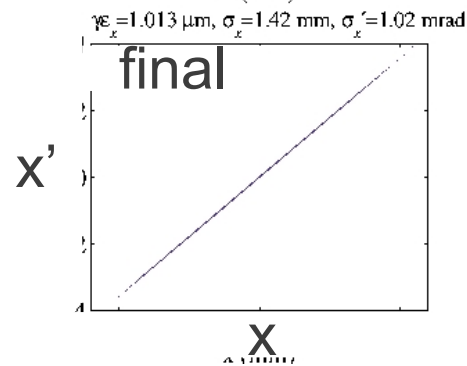
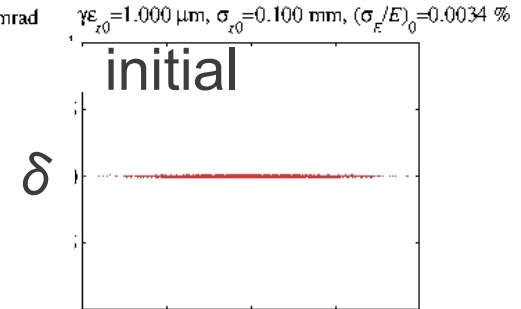
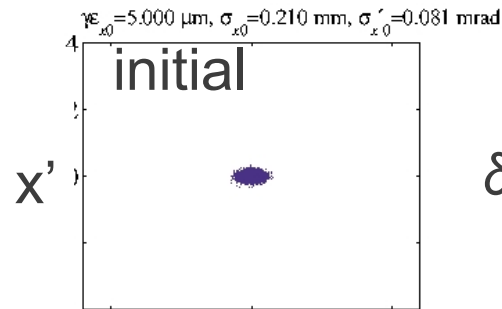
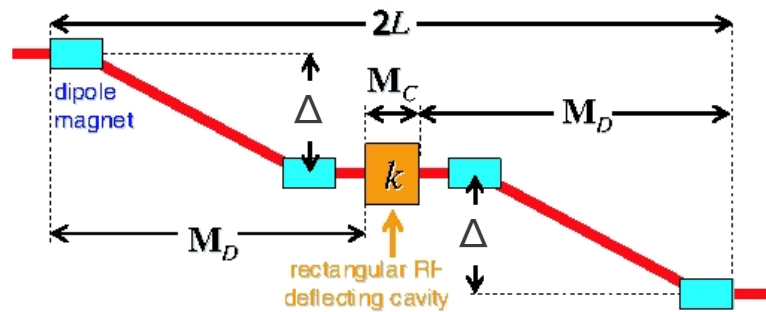
- 構成: ドッグレッグ ×2
TM210 加速空洞
- 考案の背景: X-FEL



x-z 方向のエミッタンスを交換
各方向のエミッタンスを最適化

$$\begin{cases} \gamma \varepsilon_{x0} = 5 \text{ } [\mu\text{m}] \\ \gamma \varepsilon_{z0} = 1 \text{ } [\mu\text{m}] \end{cases} \Rightarrow \begin{cases} \gamma \varepsilon_x \simeq 1 \text{ } [\mu\text{m}] \\ \gamma \varepsilon_z \simeq 5 \text{ } [\mu\text{m}] \end{cases}$$

エミッタンス交換とは



$$\begin{cases} \gamma\varepsilon_{x0} = 5 \text{ } [\mu\text{m}] \\ \gamma\varepsilon_{z0} = 1 \text{ } [\mu\text{m}] \end{cases} \Rightarrow \begin{cases} \gamma\varepsilon_x \simeq 1 \text{ } [\mu\text{m}] \\ \gamma\varepsilon_z \simeq 5 \text{ } [\mu\text{m}] \end{cases}$$

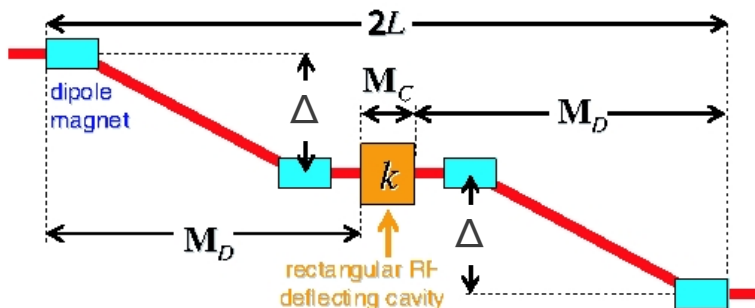
M. Cornacchia and P. Emma, Phys. Rev. - ACCELERATOR AND BEAMS, 5, 084001 (2002)

P. Emma and Z.Huang, Phys. Rev. - ACCELERATOR AND BEAMS, 9, 100702 (2006)

Rec.TM210 deflecting cavity

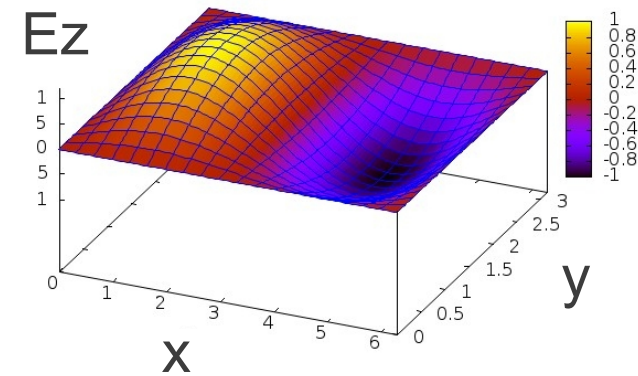
$$\mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ E_0 \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b} \end{bmatrix} \sin(\omega t + \phi)$$

$$\mathbf{B} = B_0 \begin{bmatrix} \frac{\pi}{b} \sin\left(\frac{2\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right) \\ \frac{\pi}{a} \cos\left(\frac{2\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \\ 0 \end{bmatrix} \cos(\omega t + \phi)$$



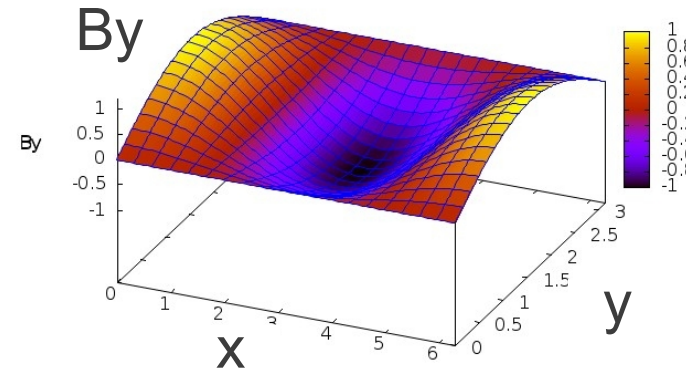
schematic Ez-field of TM210 (Dipole-mode) cavity

$\sin(x)*\sin(y)$ —



schematic Ez-field of TM210 (Dipole-mode) cavity

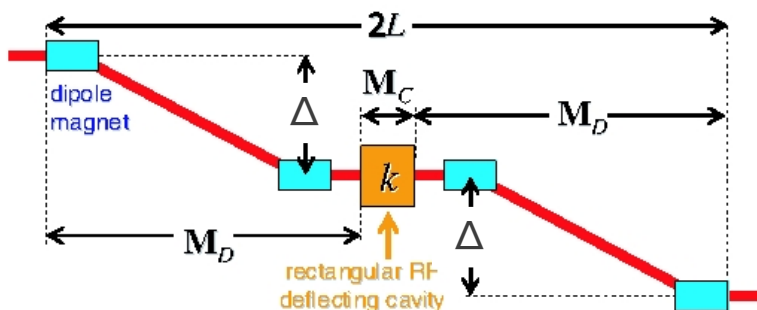
$\cos(x)*\sin(y)$ —



Rec.TM210 deflecting cavity

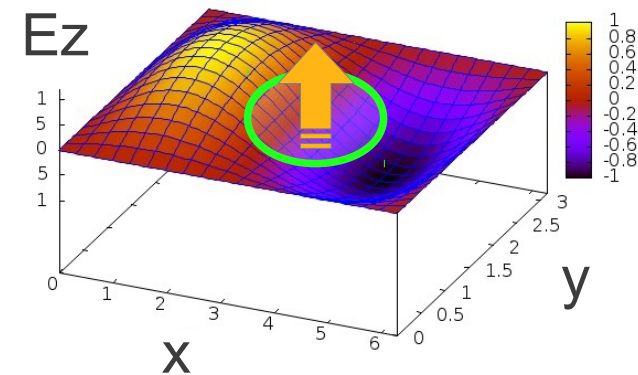
$$\mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ E_0 \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b} \end{bmatrix} \sin(\omega t + \phi)$$

$$\mathbf{B} = B_0 \begin{bmatrix} \frac{\pi}{b} \sin\left(\frac{2\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right) \\ \frac{\pi}{a} \cos\left(\frac{2\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \\ 0 \end{bmatrix} \cos(\omega t + \phi)$$



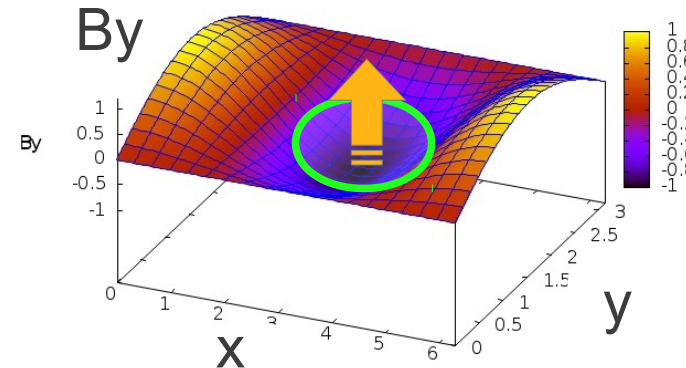
schematic Ez-field of TM210 (Dipole-mode) cavity

$\sin(x) \cdot \sin(y)$ —



schematic Ez-field of TM210 (Dipole-mode) cavity

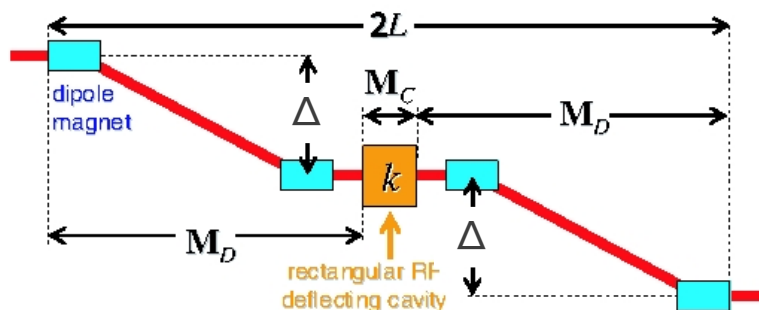
$\cos(x) \cdot \sin(y)$ —



Rec.TM210 deflecting cavity

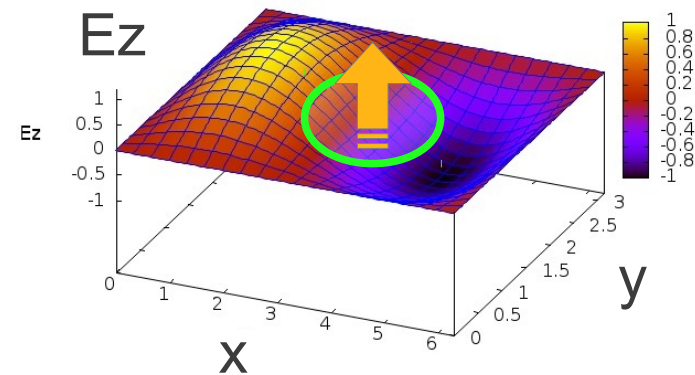
$$\mathbf{E} \simeq \begin{bmatrix} 0 \\ 0 \\ E_0 \frac{2\pi x}{a} \sin(\omega t + \phi) \end{bmatrix}$$

$$\mathbf{B} \simeq \begin{bmatrix} 0 \\ B_0 \frac{\pi}{a} \cos(\omega t + \phi) \\ 0 \end{bmatrix}$$



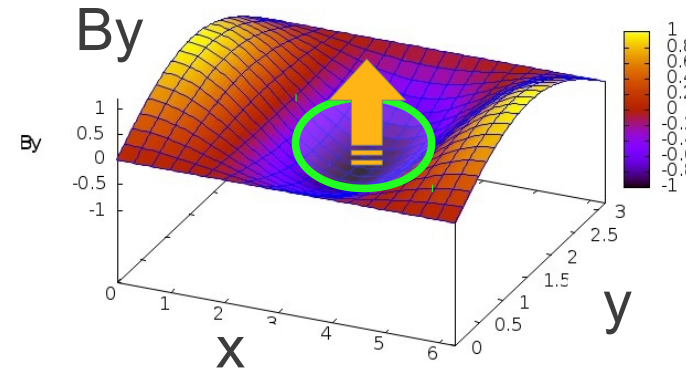
schematic Ez-field of TM210 (Dipole-mode) cavity

$\sin(x) \cdot \sin(y)$

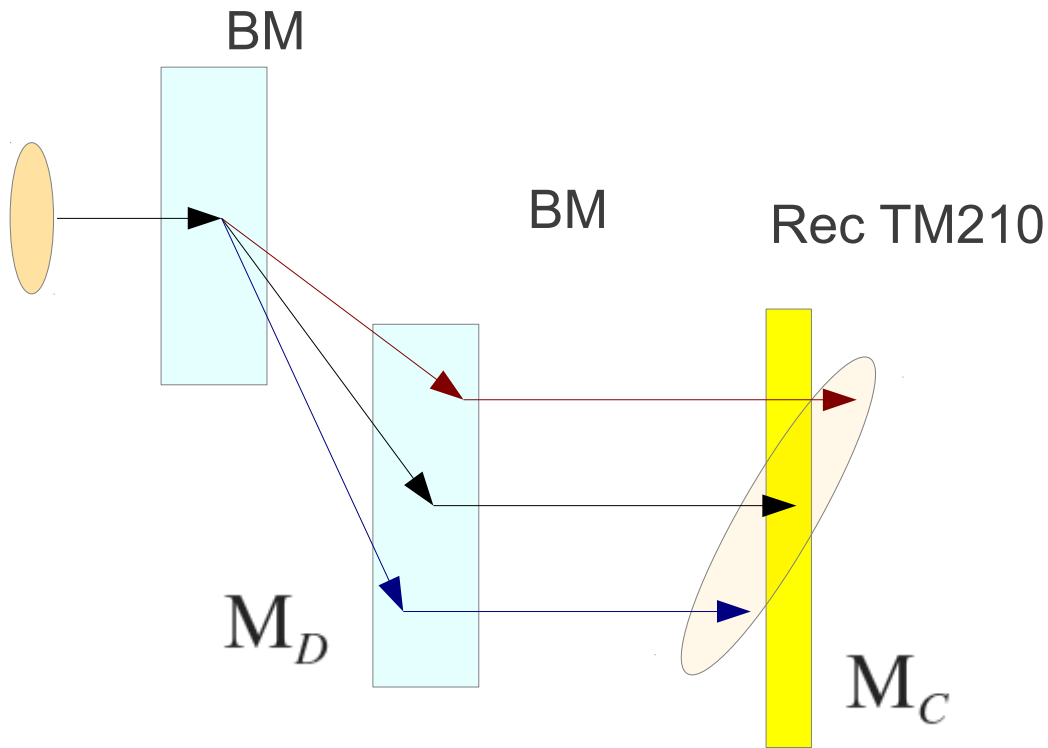


schematic Ez-field of TM210 (Dipole-mode) cavity

$\cos(x) \cdot \sin(y)$



TM210 cavity とドッグレッグ



$$s = \begin{bmatrix} x \\ x' \\ z \\ \delta \end{bmatrix}$$

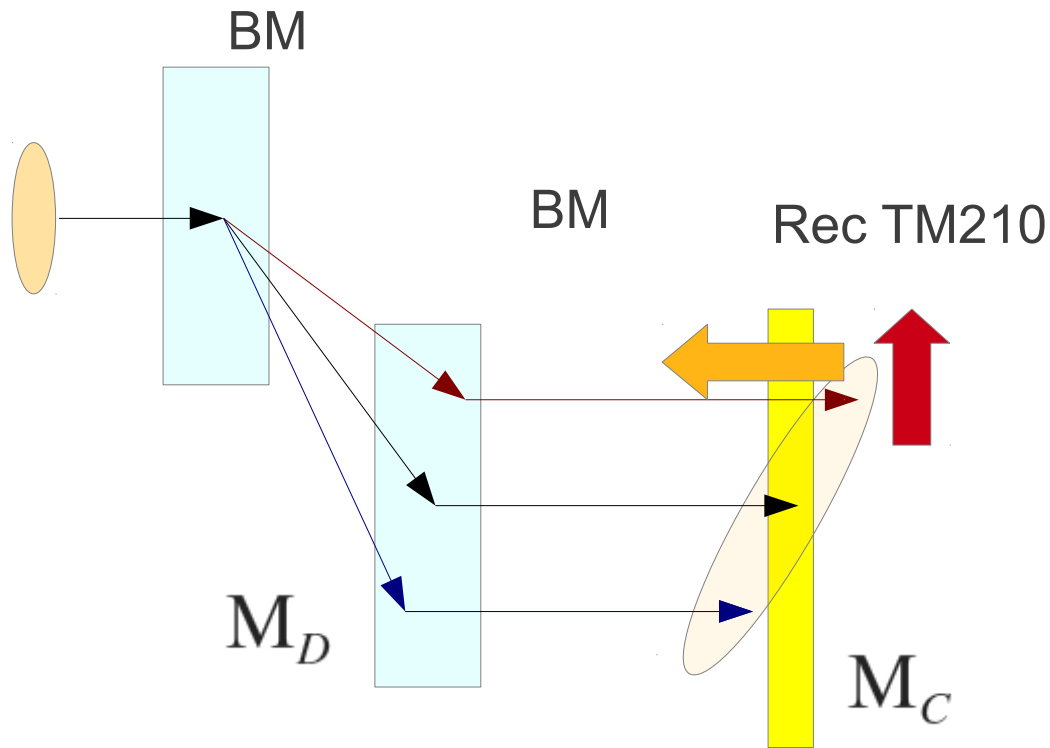
$$M_C(k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{bmatrix}$$

電磁場の一次近似より

$$\begin{cases} \Delta\delta \simeq kx \\ \Delta x' \simeq kz \end{cases}$$

$$k \equiv \frac{eV_0}{aE}$$

TM210 cavity とドッグレッグ



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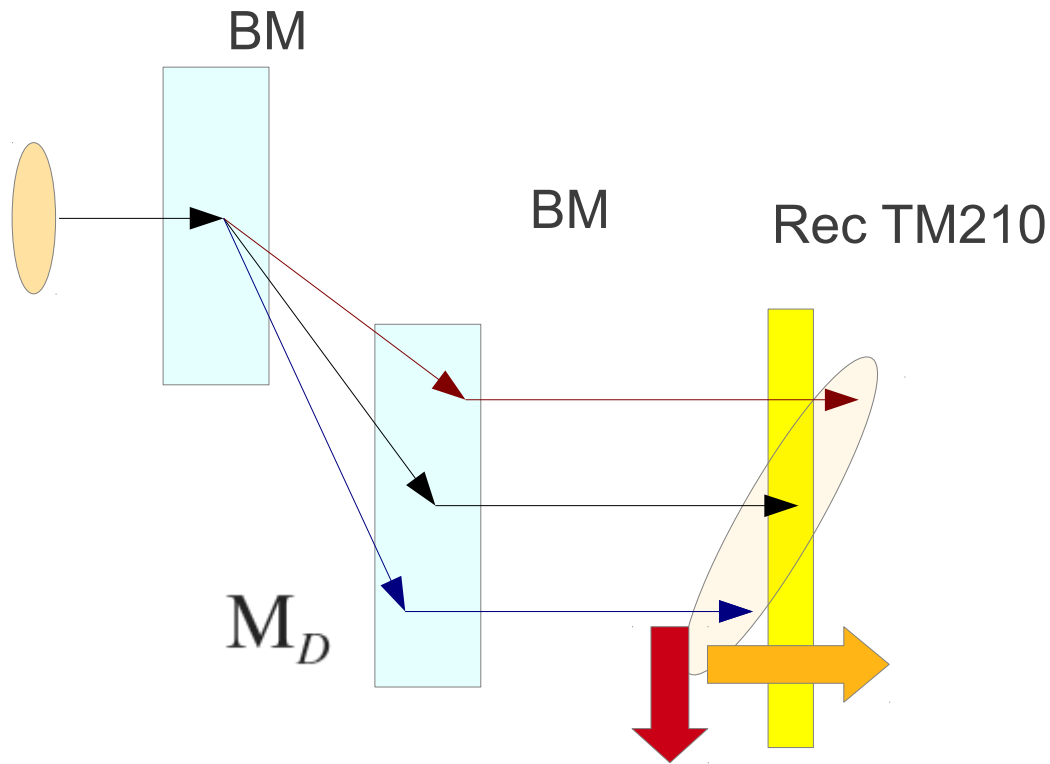
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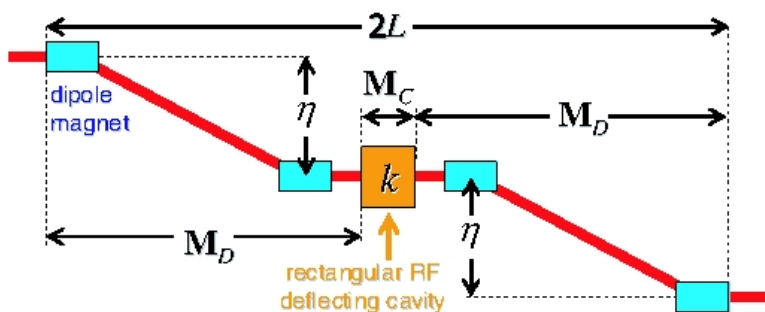
$$k \equiv \frac{eV_0}{aE}$$

エミッタンス交換の輸送行列

$$M = M_D M_C M_D$$

$$M_C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{bmatrix}$$

$$M_D = \begin{bmatrix} 1 & 0 & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \xi \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



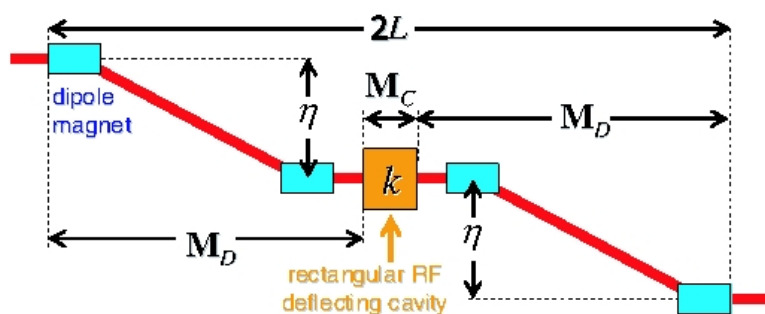
η : dispersion-function $\left(\Delta x = \eta \frac{\Delta P}{P} \right)$

ξ : momentum-compaction $\left(\Delta z = \xi \frac{\Delta P}{P} \right)$

エミッタンス交換の輸送行列

$$M = M_D M_C M_D$$

$$= \begin{bmatrix} 1 + \eta k & 2L(1 + \eta k) & kL & \eta(1 + \eta k) + \eta + kL\xi \\ 0 & 1 + \eta k & k & k\xi \\ \xi k & \eta + L\xi k + \eta(1 + \eta k) & 1 + \eta k & 2\xi(1 + \eta k) \\ k & kL & 0 & 1 + \eta k \end{bmatrix}$$



η : dispersion-function $\left(\Delta x = \eta \frac{\Delta P}{P} \right)$

ξ : momentum-compaction $\left(\Delta z = \xi \frac{\Delta P}{P} \right)$

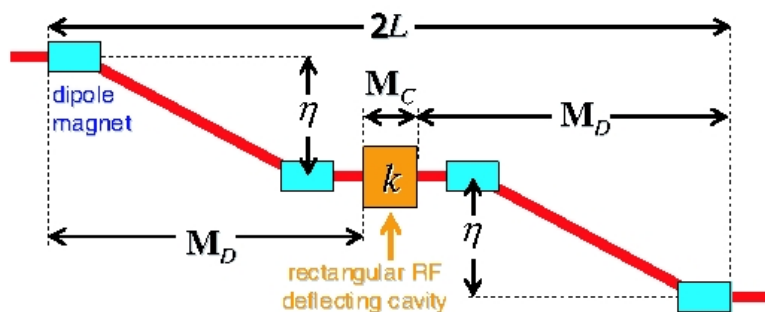
エミッタンス交換の輸送行列

$1 + k\eta = 0$ の条件が満たされるとき

$$s = \begin{bmatrix} x \\ x' \\ z \\ \delta \end{bmatrix}$$

$$M_{EX} = M_D M_C M_D$$

$$= \begin{bmatrix} 0 & 0 & kL & \eta + kL\xi \\ 0 & 0 & k & k\xi \\ k\xi & \eta + kL\xi & 0 & 0 \\ k & kL & 0 & 0 \end{bmatrix}$$



η : dispersion-function $\left(\Delta x = \eta \frac{\Delta P}{P} \right)$

ξ : momentum-compaction $\left(\Delta z = \xi \frac{\Delta P}{P} \right)$

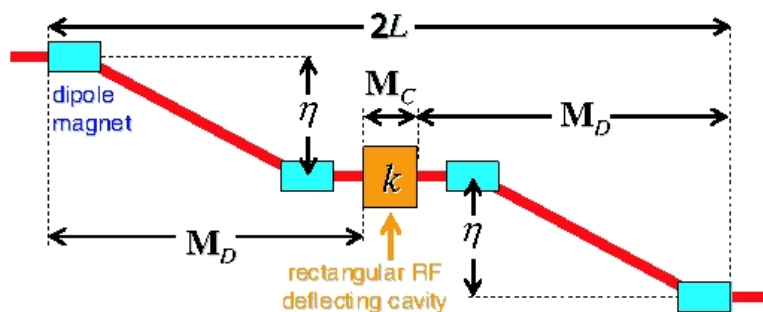
エミッタンス交換の輸送行列

$1 + k\eta = 0$ の条件が満たされるとき

極めて寄与が小さい

$$M_{EX} = M_D M_C M_D$$

$$= \begin{bmatrix} 0 & 0 & kL & \eta + kL\xi \\ 0 & 0 & k & k\xi \\ k\xi & \eta + kL\xi & 0 & 0 \\ k & kL & 0 & 0 \end{bmatrix}$$

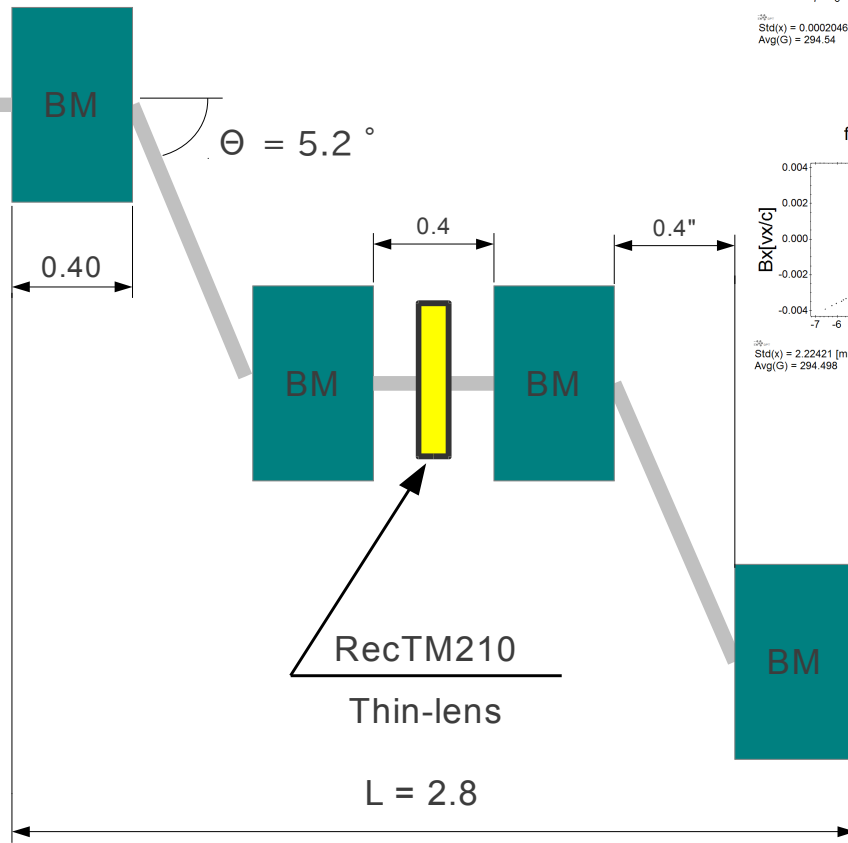


η : dispersion-function $\left(\Delta x = \eta \frac{\Delta P}{P} \right)$

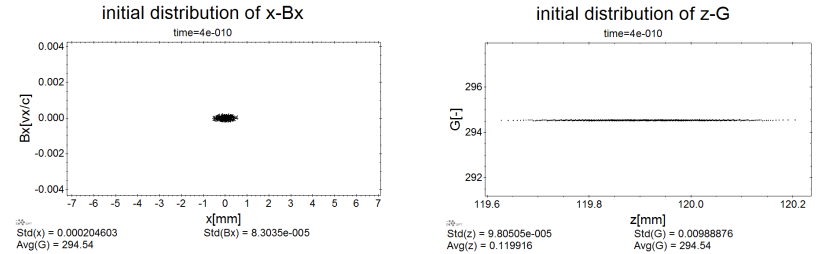
ξ : momentum-compaction $\left(\Delta z = \xi \frac{\Delta P}{P} \right)$

Simulation : 仮定するビームライン

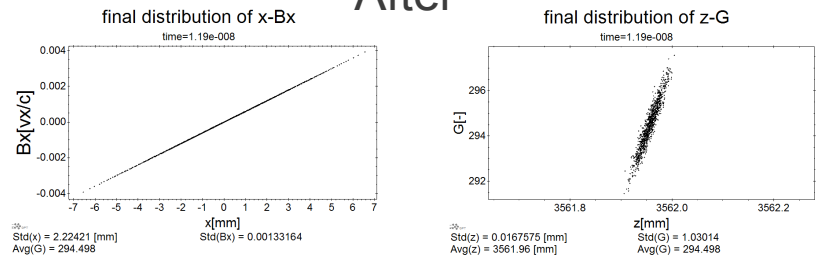
入射



Before



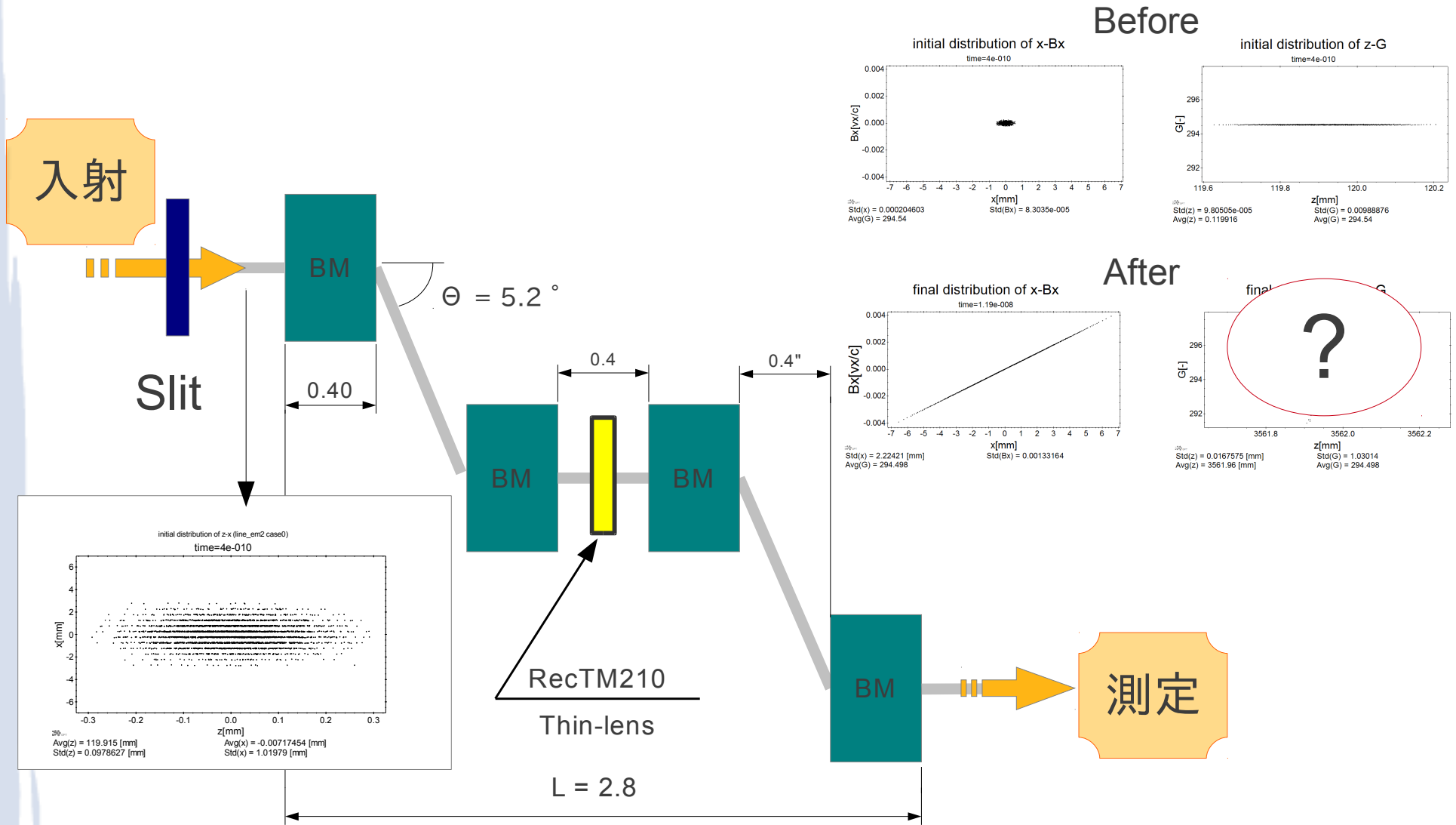
After



そのままでは
エミッタンス交換がおこるだけ

測定

Simulation : 仮定するビームライン



マイクロバンチ生成の試行

- 初期ビーム

$$E_0 = 150 \text{ MeV}$$

$$\gamma\varepsilon_x = 25 \text{ } \mu\text{m}$$

$$\gamma\varepsilon_z = 1 \text{ } \mu\text{m}$$

$$\sigma_r = 1.0 \text{ mm}$$

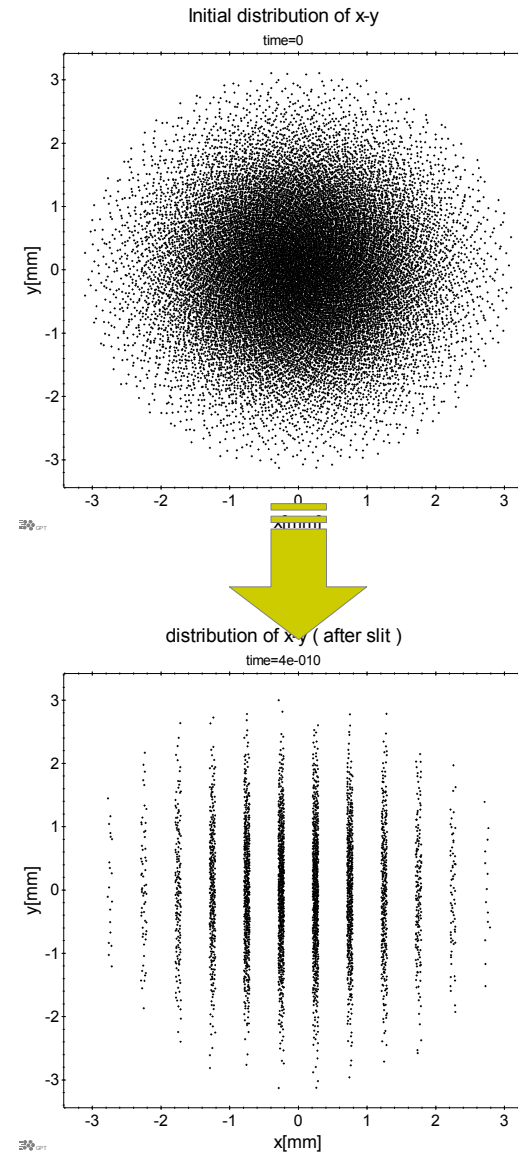
$$\sigma_z = 0.1 \text{ mm}$$

$$Q = 100 \text{ pC}$$

- スリット

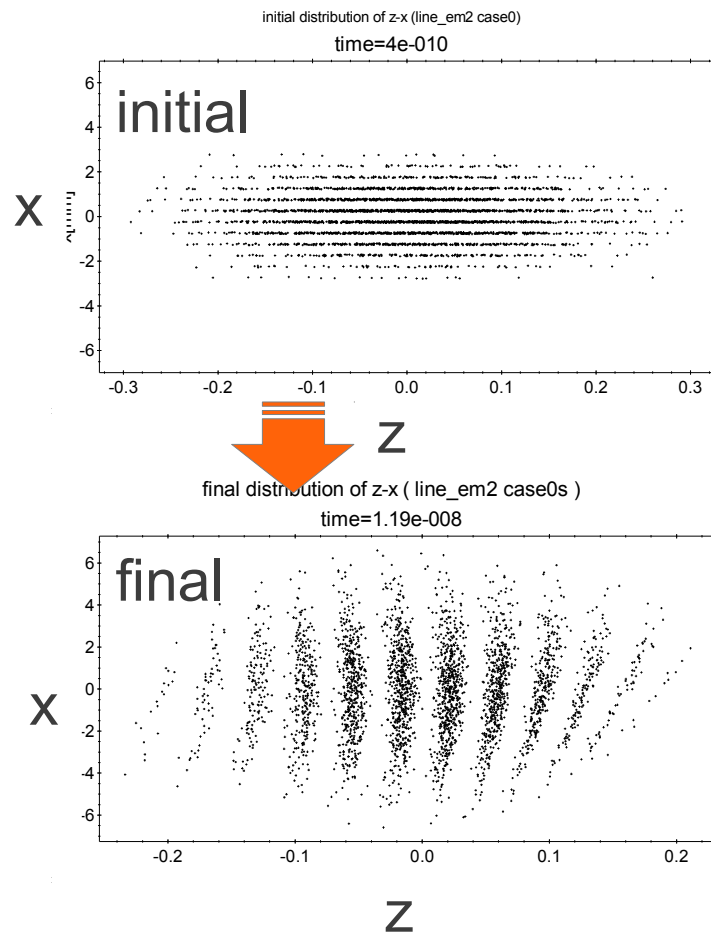
$$\Delta_{\text{opening}} = 80 \text{ } \mu\text{m}$$

$$\Delta_{\text{interval}} = 500 \text{ } \mu\text{m}$$

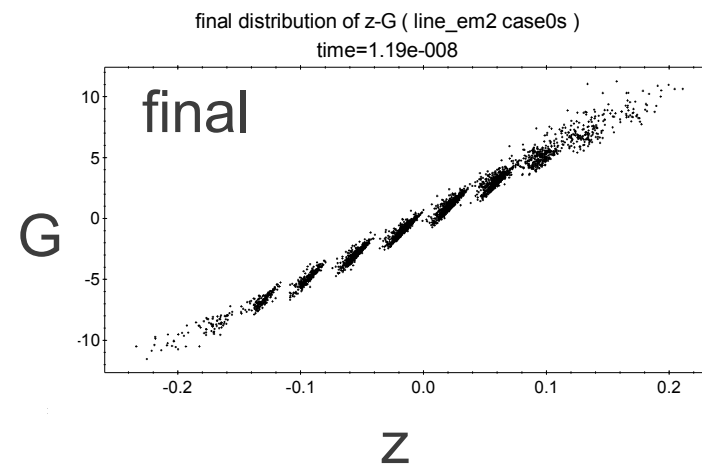


試行1

● 実空間 z-x 分布



● 相空間 z- δ 間分布



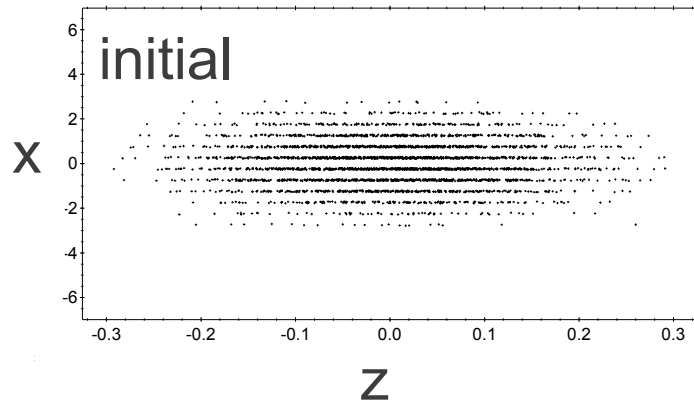
- ・ 周波数: $f = 10$ [THz]
- ・ マイクロバンチの長さ: $\sigma \sim 30$ [fs]

Z位相空間回転でバンチ圧縮・最適化
(アーク or セクタ型マグネット)

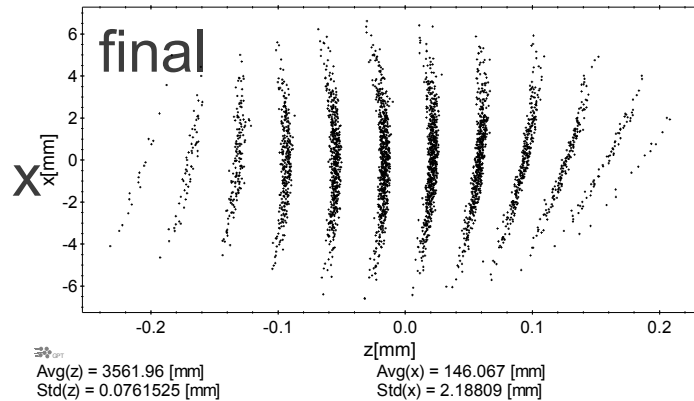
試行2: $\gamma \varepsilon_x \rightarrow 1/5$ 倍

● 実空間 z-x 分布

initial distribution of z-x (line_em2 case0)
time=4e-010

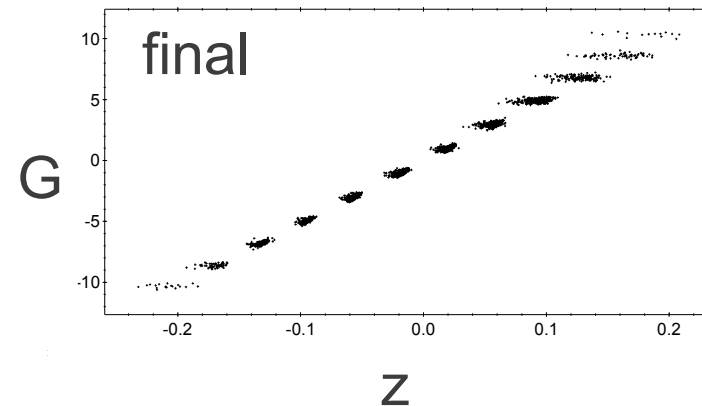


final distribution of z-x (line_em2 case0s)
time=1.19e-008



● 相空間 z-δ 間分布

final distribution of z-G (line_em2 case0s)
time=1.19e-008

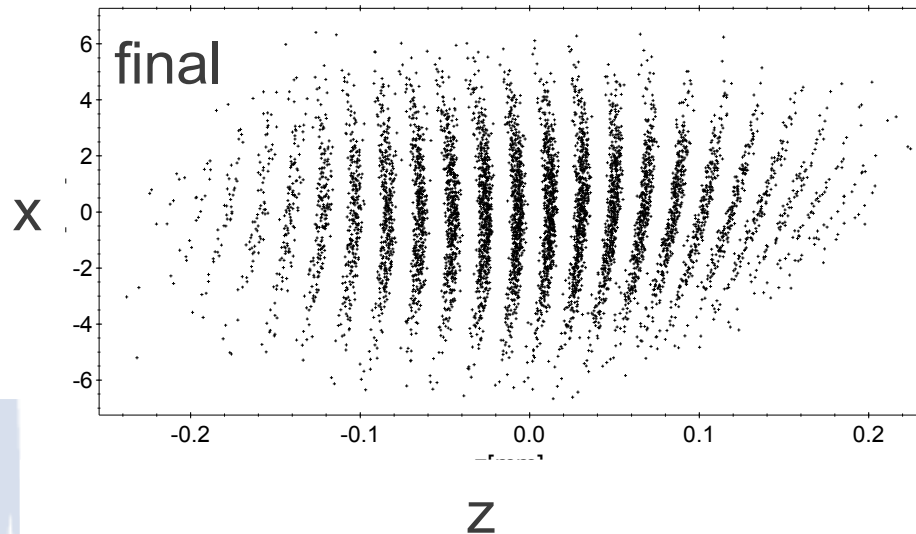


- ・ 周波数: $f = 10$ [THz]
- ・ マイクロバンチの長さ: $\sigma \sim 7$ [fs]

試行3: スリット間隔 1/2

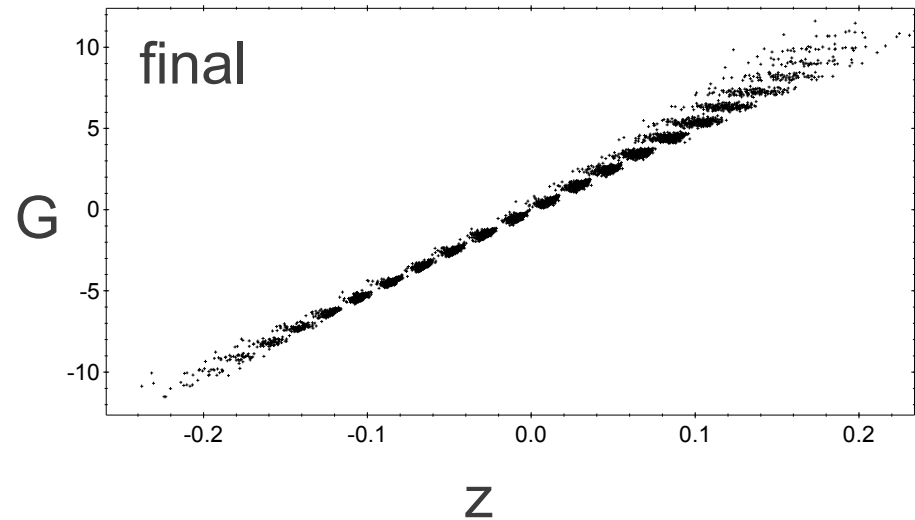
- 実空間 z - x 分布

final distribution of z - x (line_em2 case2s)
time=1.19e-008



- 相空間 z - δ 間分布

final distribution of z - G (line_em2 case2s)
time=1.19e-008



・ 周波数: $f = 20$ [THz]

・ マイクロバッチの長さ: $\sigma \sim 7$ [fs]

バンチ間隔を狭めるために (位相空間制御)

$$s = \begin{bmatrix} x \\ x' \\ z \\ \delta \end{bmatrix}$$

- 1. $k\xi \rightarrow$ 小

→ - 曲げ角 $\theta \rightarrow$ 小

- $k\xi \sim 0.05$ が可

$$M_{EX} = M_D M_C M_D$$

$$= \begin{bmatrix} 0 & 0 & kL & \eta + kL\xi \\ 0 & 0 & k & k\xi \\ k\xi & \eta + kL\xi & 0 & 0 \\ k & kL & 0 & 0 \end{bmatrix}$$

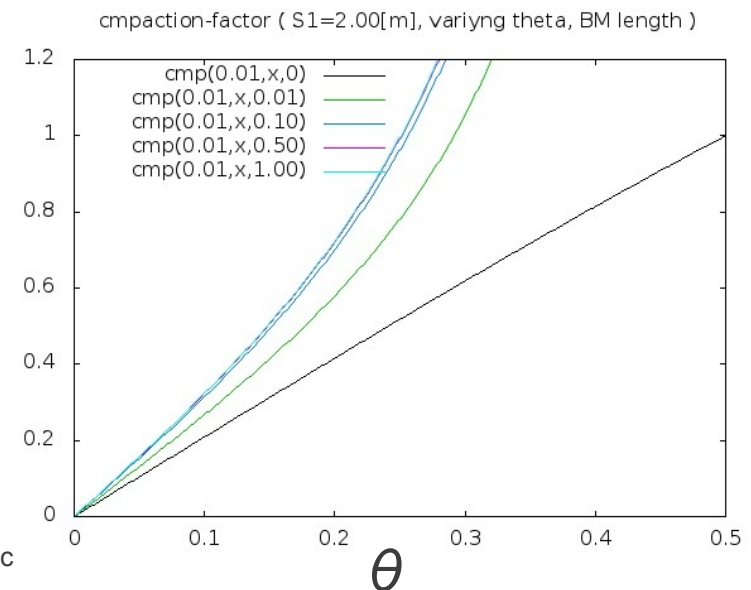
- 3. z相空間回転

- Δz を $\sim 1/3$

→ $f \sim 100$ [THz]

(cf. 近赤外線, 光ファイバー)

$k\xi$

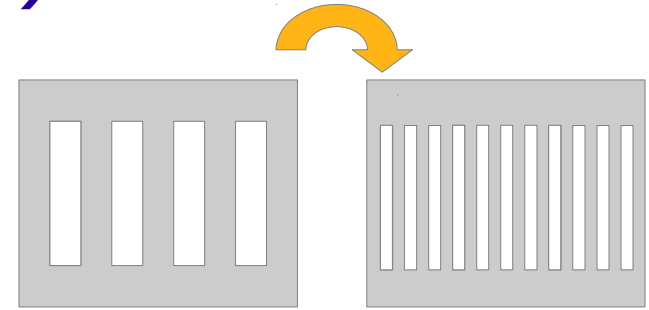


バンチ間隔を狭めるために (初期 x 分布)

- 1. 各バンチの交わらない

スリット幅 : スリット間隔

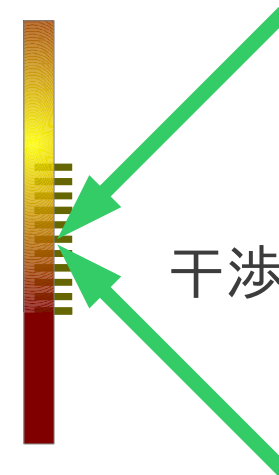
を保てば, 原理上どこまでも周波数が上がる



- 光の干渉の応用

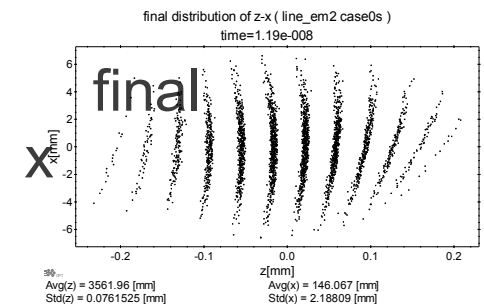
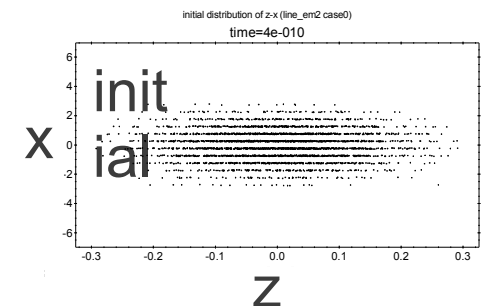
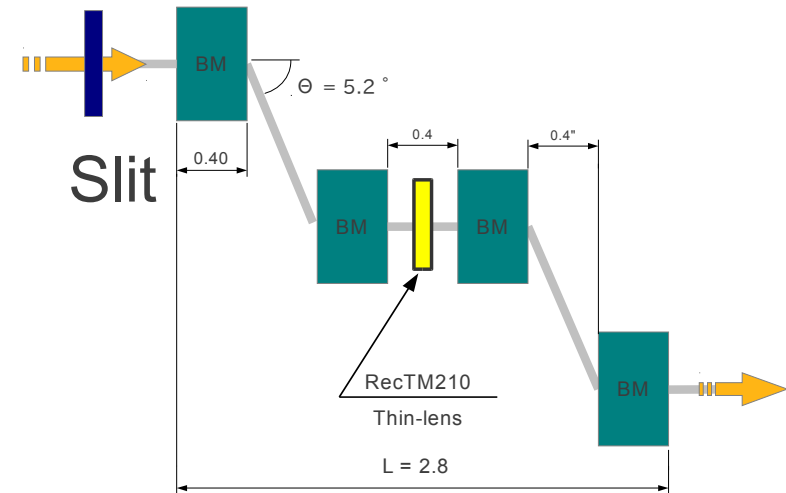
$$\rightarrow f = 1 \text{ [PHz]} \\ (= 1000 \text{ [THz]})$$

$$\Delta t = 100 \text{ [as]}$$



マイクロバンチ生成検討の結論

- EEXの応用でマイクロバンチ生成は可能
- バンチ幅: ~ 10 [fs]
- 周波数 : ~ 40 [THz]
- Z位相空間回転でバンチを圧縮・最適化

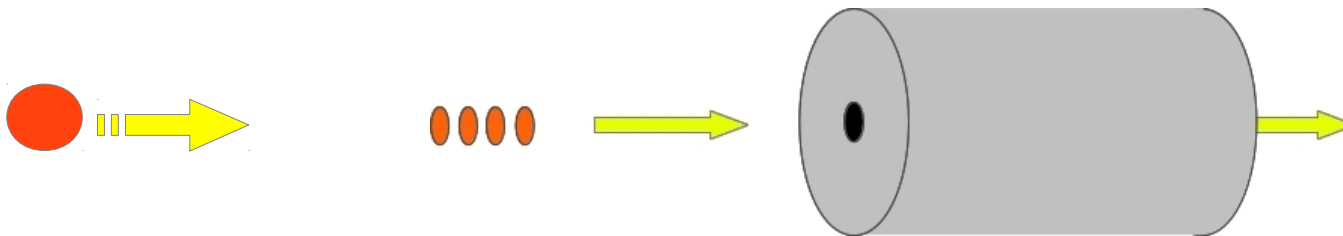


※ 先行研究

GENERATION OF FEMTOSECOND BUNCH TRAINS USING A LONGITUDINAL-TO-TRANSVERSE PHASE SPACE EXCHANGE TECHNIQUE, Yin-e Sun, Philippe Pot

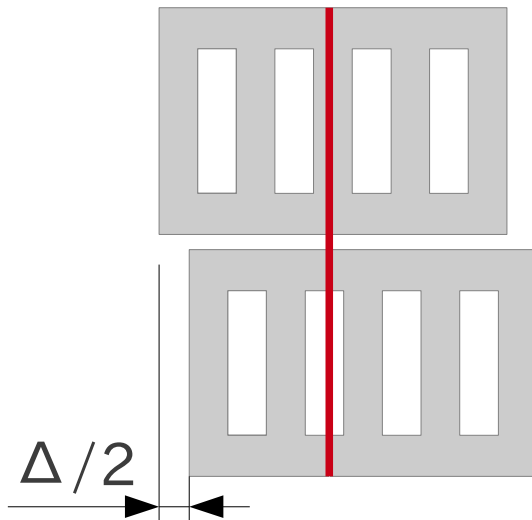
誘電体加速への応用

- マイクロバンチを誘電体の加速空洞(～数cm)に通過, その後に通るバンチを加速
 - バンチの誘起する強力な電場
 - シャントインピーダンスが大 ($Z_{sh} \propto \sqrt{\omega}$)
 - 周波数 : 10 ~ 100[THz]
 - 原理的に同期が可能



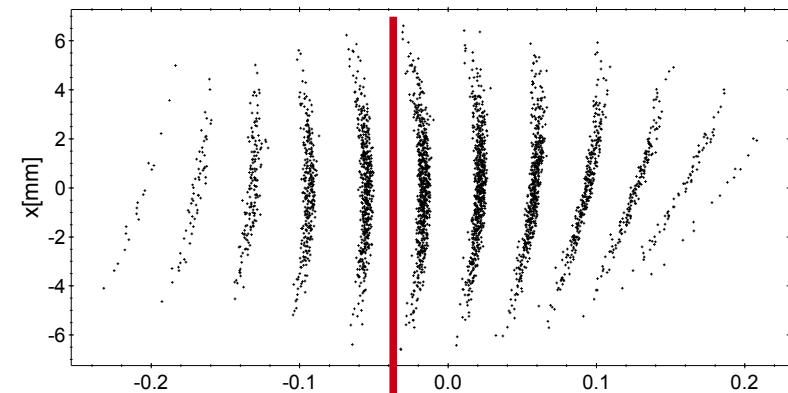
同期への自由度

- スリット位置と位相が対応
- 同期に自由度
- 機械精度で位相制御

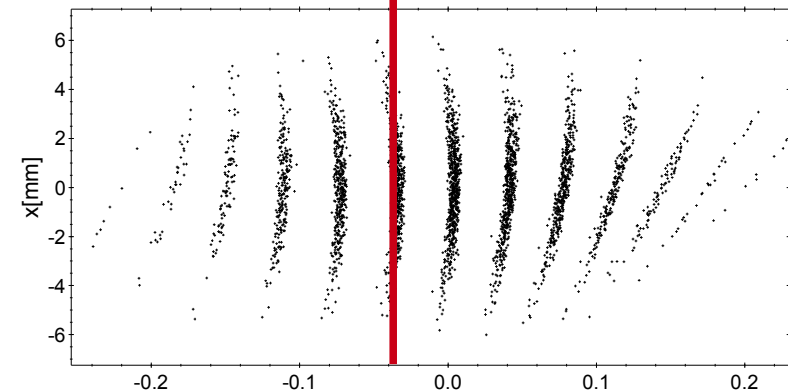


Final z-x dist

final distribution of z-x (line_em2 case0s)
time=1.19e-008



final distribution of z-x (line_em2 case3s)
time=1.19e-008
Avg(z) = 3561.96 [mm]
Std(z) = 0.0761525 [mm]
Avg(x) = 146.067 [mm]
Std(x) = 146.068 [mm]



Avg(z) = 3412.06 [mm]
Std(z) = 0.0766116 [mm]
Avg(x) = 146.098 [mm]
Std(x) = 1.97713 [mm]

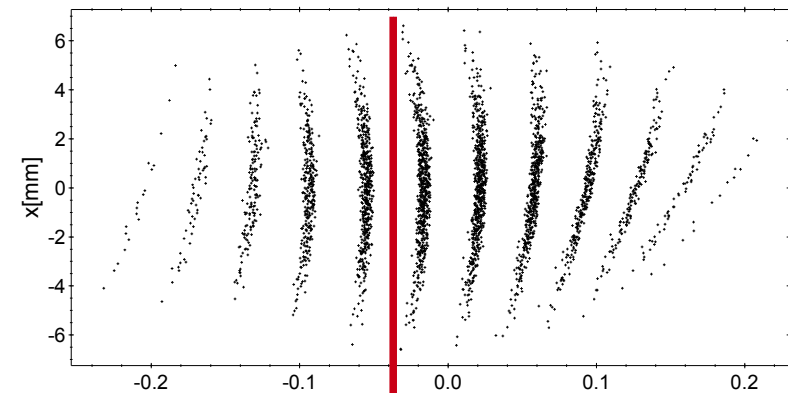
同期への自由度

- スリット位置と位相が対応
- 同期に自由度
- 機械精度で位相制御

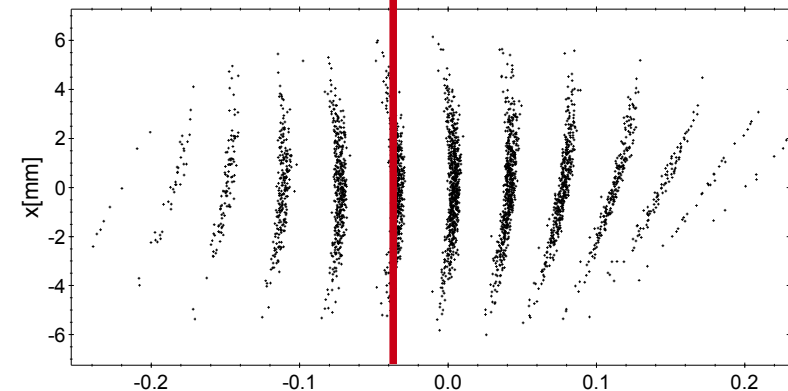
$$\Delta x = 7 [\mu\text{m}]$$
$$\rightarrow \Delta\theta = 5 [^\circ]$$

Final z-x dist

final distribution of z-x (line_em2 case0s)
time=1.19e-008



final distribution of z-x (line_em2 case3s)
time=1.19e-008
Avg(z) = 3561.96 [mm]
Std(z) = 0.0761525 [mm]
Avg(x) = 146.067 [mm]
Std(x) = 1.18809 [mm]



Avg(z) = 3412.06 [mm]
Std(z) = 0.0766116 [mm]
Avg(x) = 146.098 [mm]
Std(x) = 1.97713 [mm]

まとめ

- EEXの応用でマイクロバンチ生成は可能
- バンチ幅: ~ 7 [ps]
- 周波数 : ~ 10 [THz]
- Z位相空間回転でバンチを圧縮・最適化
- 誘電体加速のドライバの応用
- 同期に新たな自由度による位相制御
↑ 機械精度

期待される加速勾配の概算

$$k = \frac{\omega}{4} \frac{R}{Q}$$

$$G = 967[\Omega]$$

$$E_{max} = 2kq \cos\omega t$$

$$= 4\pi \frac{cG}{\lambda^2}$$

$$\rightarrow 1 \sim 100 \text{ [GV/m]}$$

$$T = \frac{\sin \frac{\pi L}{\lambda}}{\frac{\pi L}{\lambda}}$$

以下 補助資料

TM210 cavity の近似: 電場

$$\mathbf{E} = \begin{bmatrix} 0 \\ 0 \\ E_0 \sin \frac{2\pi x}{a} \sin \frac{\pi y}{b} \end{bmatrix} \sin(\omega t + \phi)$$

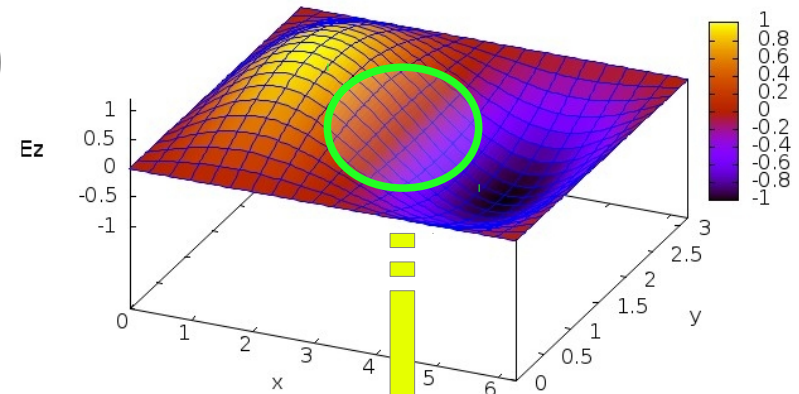


$$E_x = E_y = 0$$

$$E_z = 2\pi E_0 \frac{x}{a} \sin(\omega t + \phi),$$

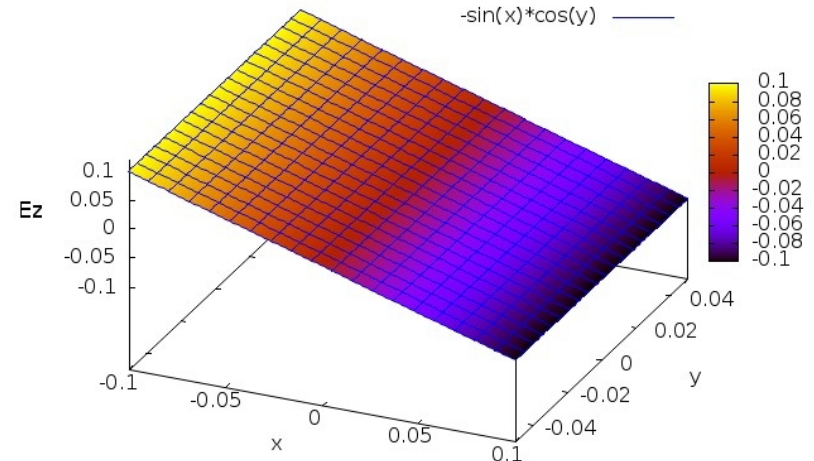
schematic Ez-field of TM210 (Dipole-mode) cavity

$\sin(x) \cdot \sin(y)$ —



schematic Ez-field of TM210 (Dipole-mode) cavity

$-\sin(x) \cdot \cos(y)$ —



TM210 cavity の近似: 磁場

$$\mathbf{B} = B_0 \begin{bmatrix} \frac{\pi}{b} \sin\left(\frac{2\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right) \\ \frac{\pi}{a} \cos\left(\frac{2\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \\ 0 \end{bmatrix} \cos(\omega t + \phi)$$

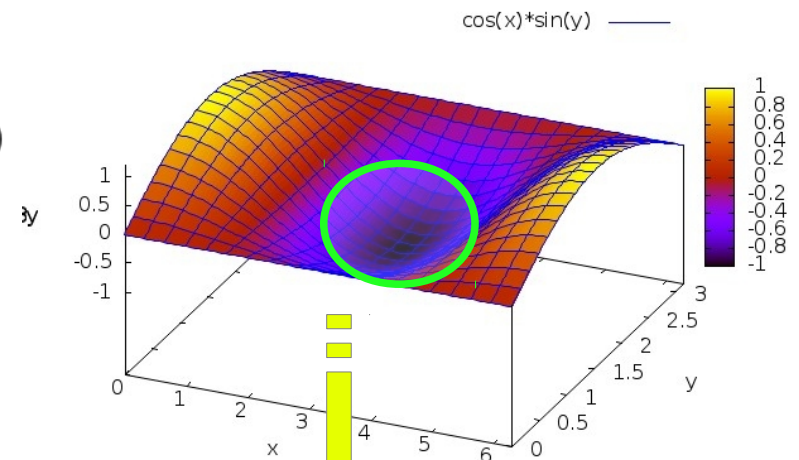


$$B_z = B_x = 0,$$

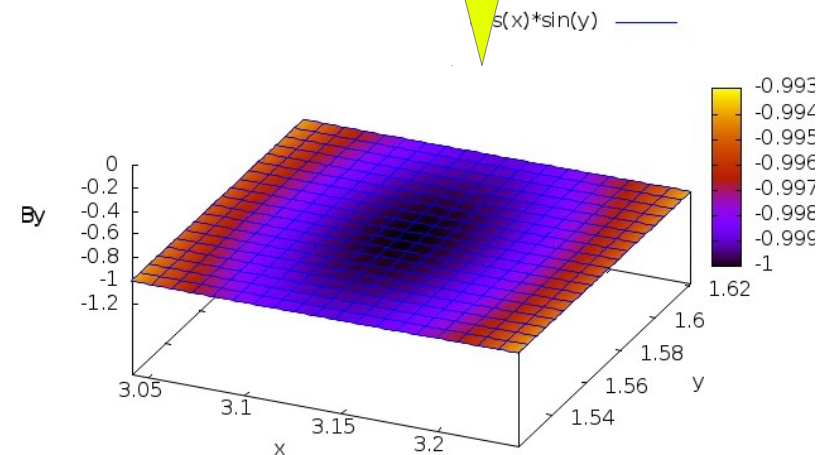
$$B_y = \frac{2\pi B_0}{a} \cos(\omega t + \phi)$$

定数

schematic Ez-field of TM210 (Dipole-mode) cavity



schematic Ez-field of TM210 (Dipole-mode) cavity



TM210 cavity の近似: 磁場

$$\mathbf{B} = B_0 \begin{bmatrix} \frac{\pi}{b} \sin\left(\frac{2\pi x}{a}\right) \cos\left(\frac{\pi y}{b}\right) \\ \frac{\pi}{a} \cos\left(\frac{2\pi x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \\ 0 \end{bmatrix} \cos(\omega t + \phi)$$

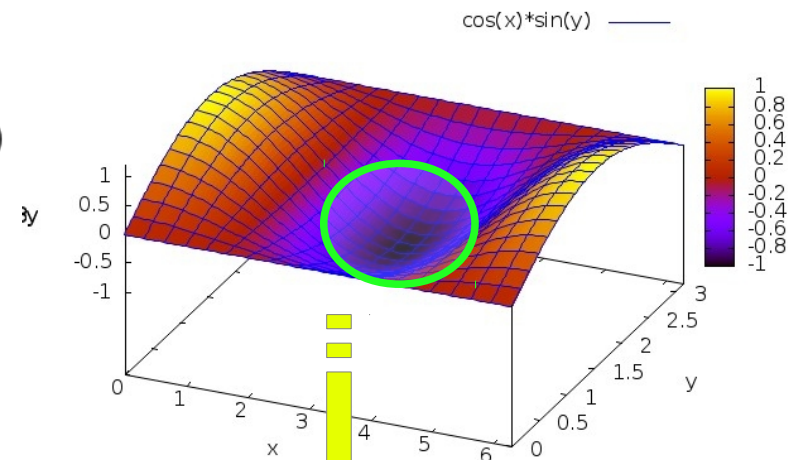


$$B_z = B_x = 0,$$

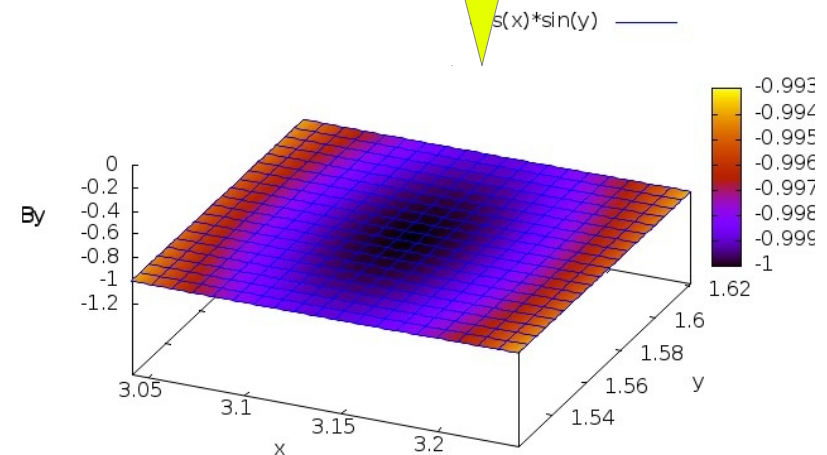
$$B_y = \frac{2\pi B_0}{a} \cos(\omega t + \phi)$$

定数

schematic Ez-field of TM210 (Dipole-mode) cavity



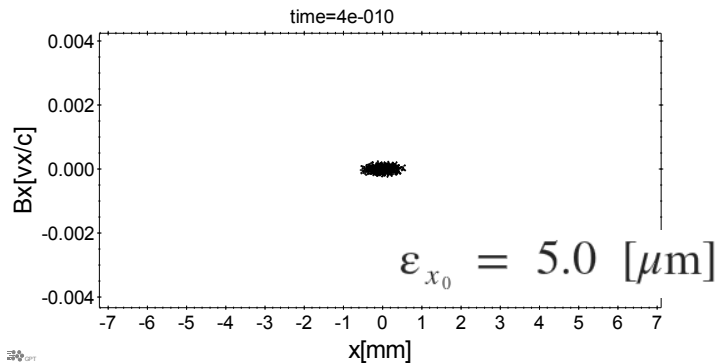
schematic Ez-field of TM210 (Dipole-mode) cavity



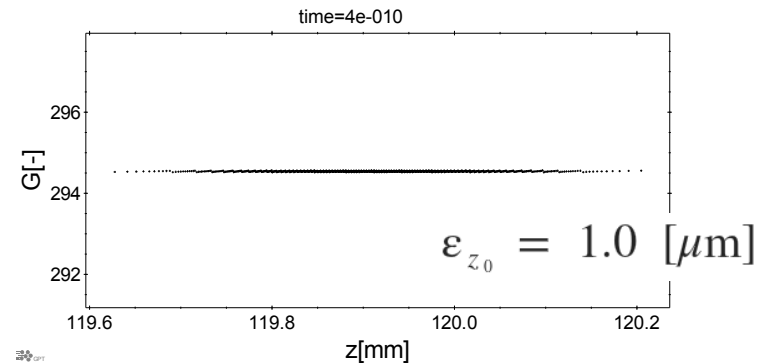
エミッタンス交換の試行

$$\sqrt{\gamma \epsilon_z \cdot \gamma \epsilon_x} \sim 1.1 \sqrt{\gamma \epsilon_{z_0} \cdot \gamma \epsilon_{x_0}}$$

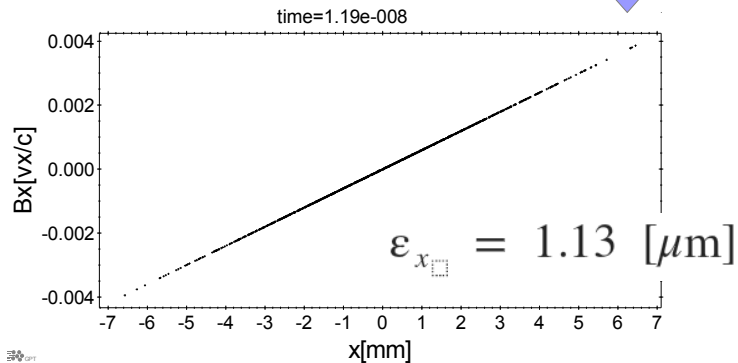
initial distribution of x-Bx



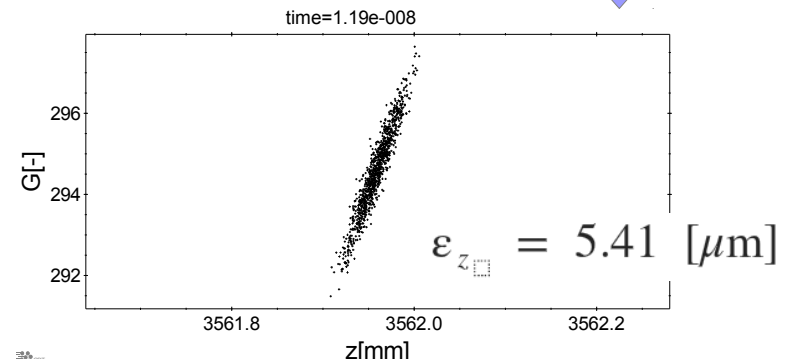
initial distribution of z-G



final distribution of x-Bx



final distribution of z-G

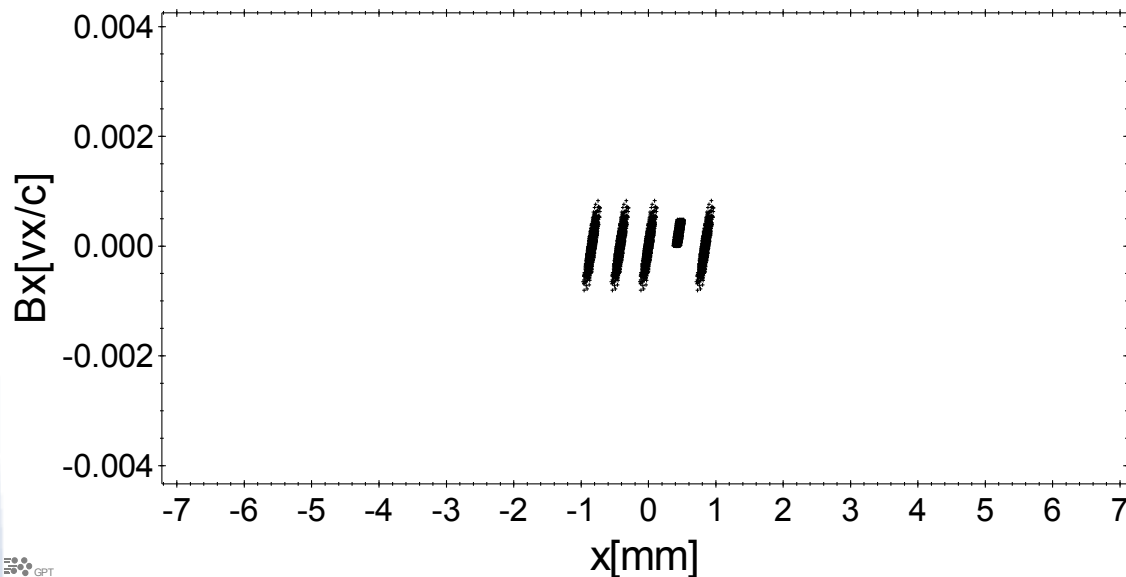


マイクロバンチ生成の試行 1

初期ビーム

initial distribution of x-Bx

time=4e-010



Std(x) = 0.00060086
Avg(G) = 294.54

Std(Bx) = 0.000275292

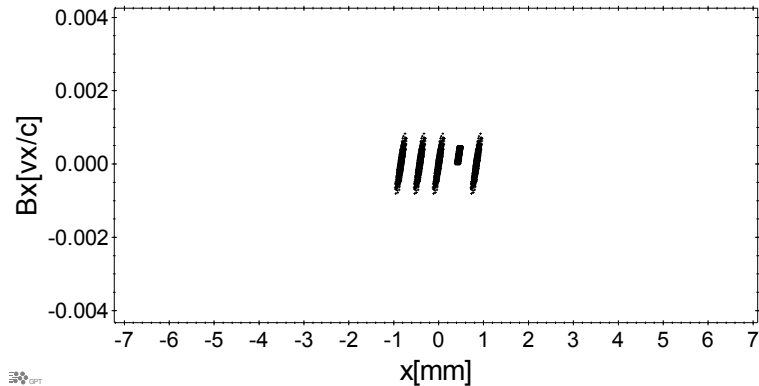
$$\sigma_{x_0} \sim 0.21 \text{ [mm]}$$

$$\Delta = \sigma_{x_0}$$

マイクロバンチ生成の試行 1

initial distribution of x-Bx

time=4e-010

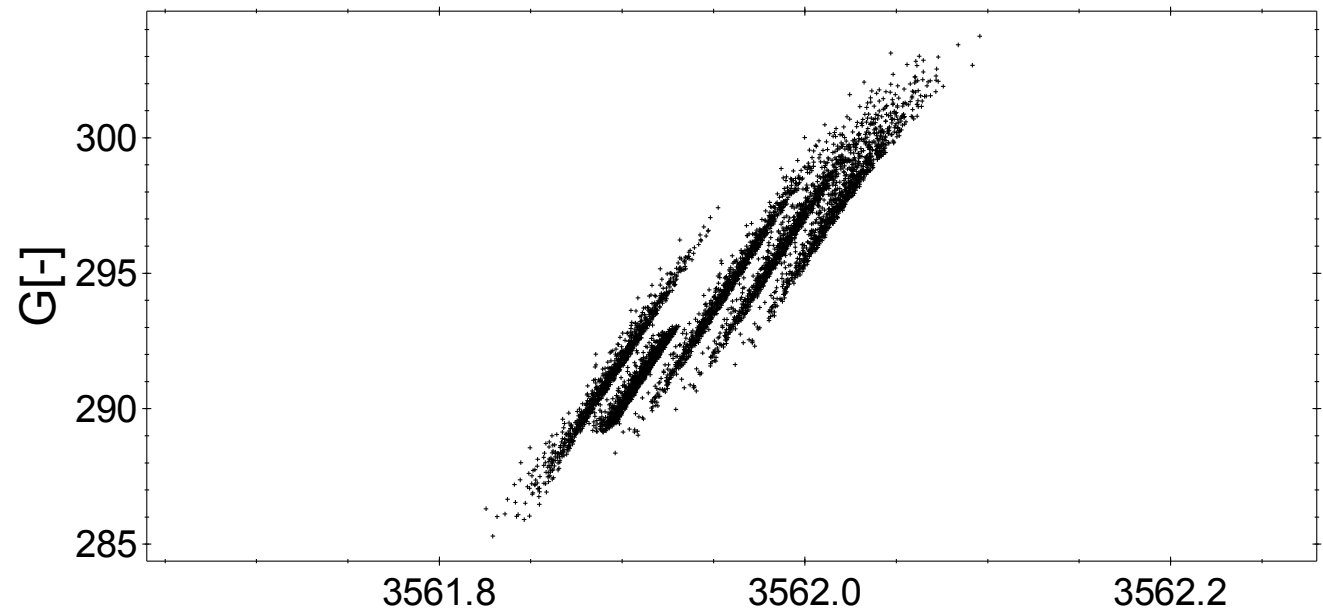


Std(x) = 0.00060086
Avg(G) = 294.54

Std(Bx) = 0.000275292

final distribution of z-G

time=1.19e-008



Std(z) = 5.16398e-005

Avg(z) = 3.56195

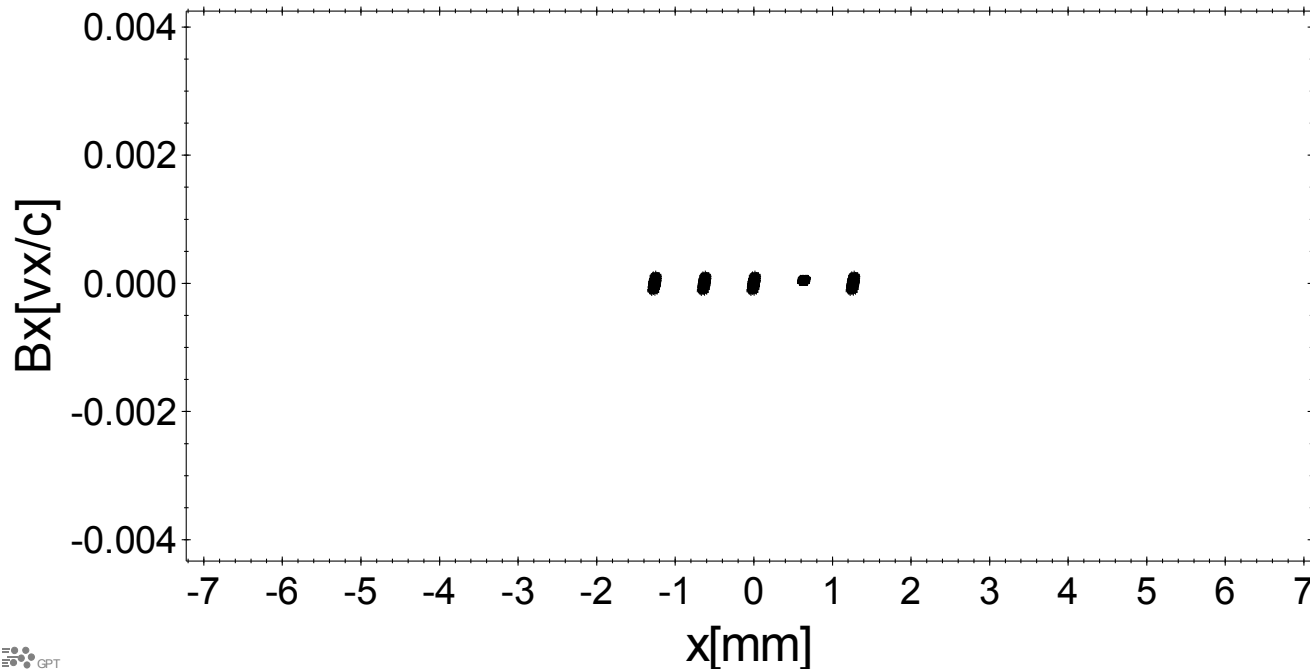
Std(G) = 3.33138

Avg(G) = 294.214

マイクロバンチ生成の試行 2

initial distribution of x-Bx

time=4e-010



Std(x) = 0.00089401

Avg(G) = 294.54

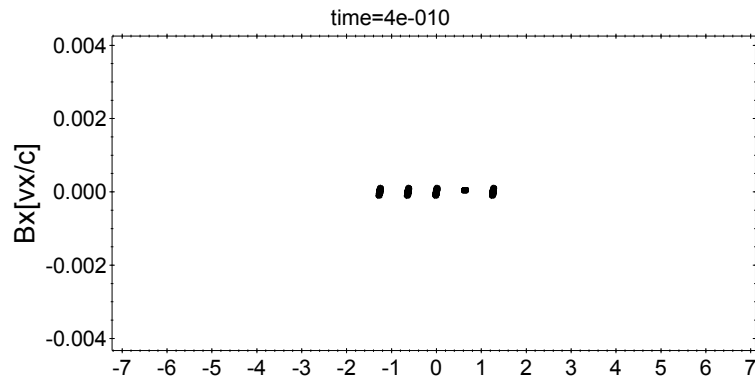
Std(Bx) = 5.50584e-005

$\varepsilon_{x0} = 1.0 [\mu\text{m}] \leftarrow (5.0 [\mu\text{m}]),$

setting the value of x_0^{\square} and y_0^{\square} 1/5

マイクロバンチ生成の試行 2

initial distribution of x-Bx

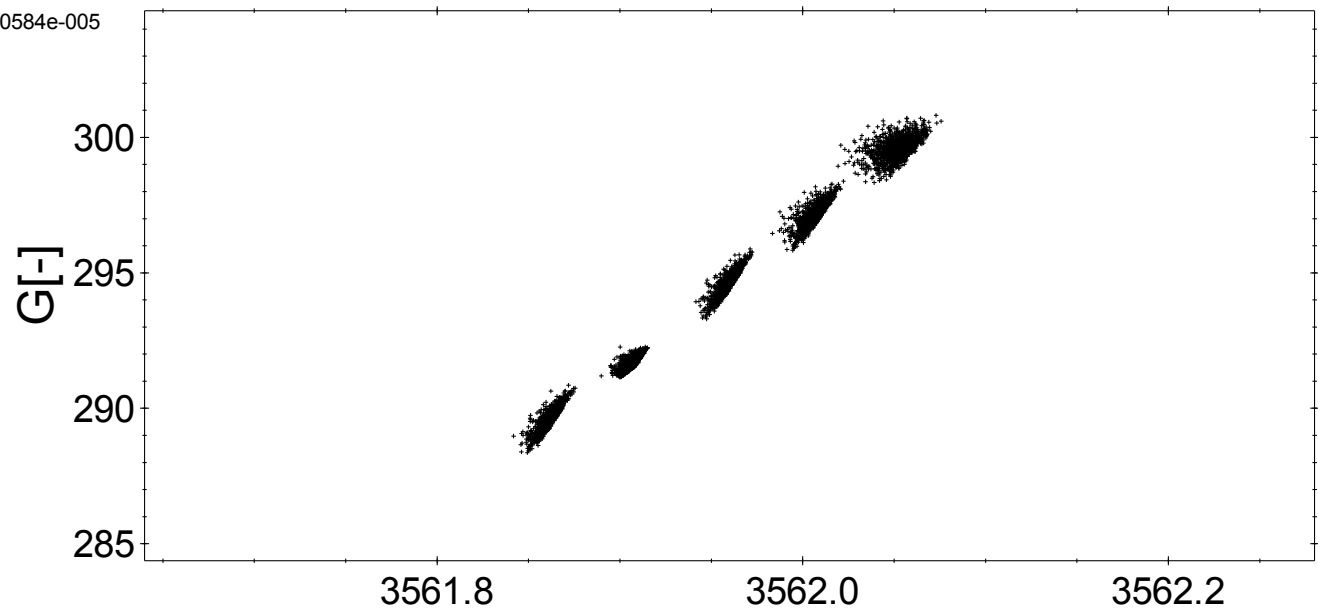


Std(x) = 0.00089401
Avg(G) = 294.54

Std(Bx) = 5.50584e-005

final distribution of z-G

time=1.19e-008



Std(z) = 6.8067e-005
Avg(z) = 3.56196

Std(G) = 3.61521
Avg(G) = 294.509

マイクロバンチ生成の試行 2

エネルギーの