

# 陰極近傍での空間電荷効果について

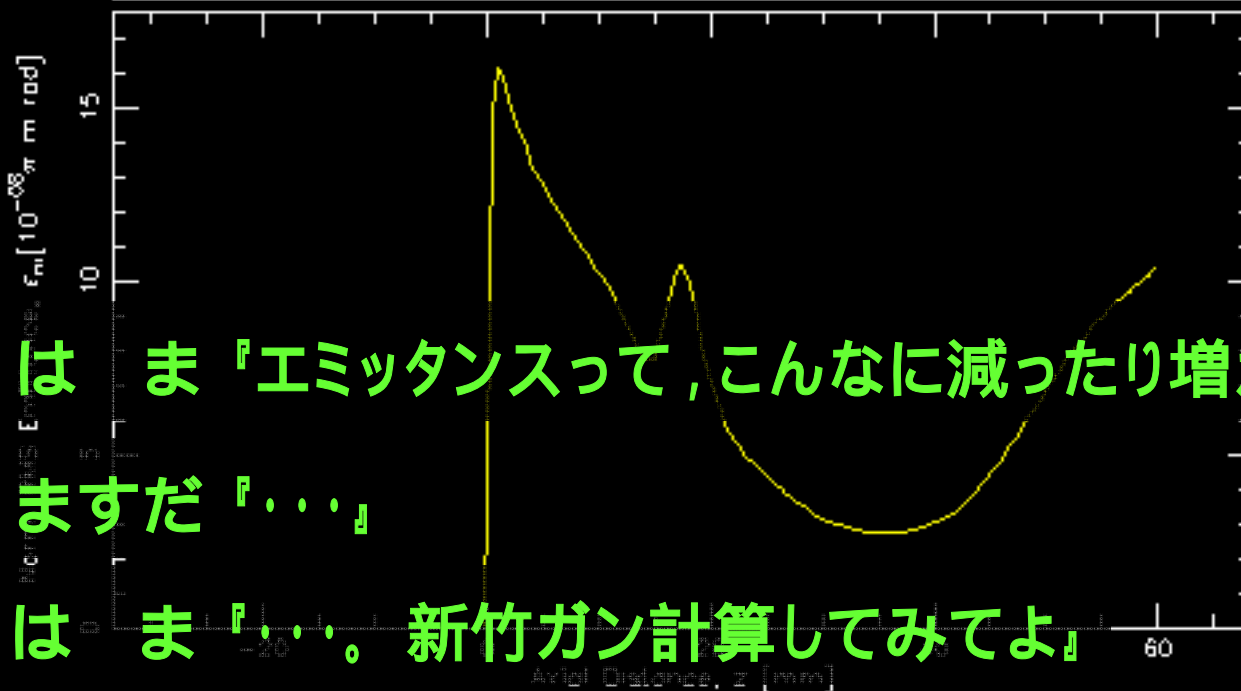
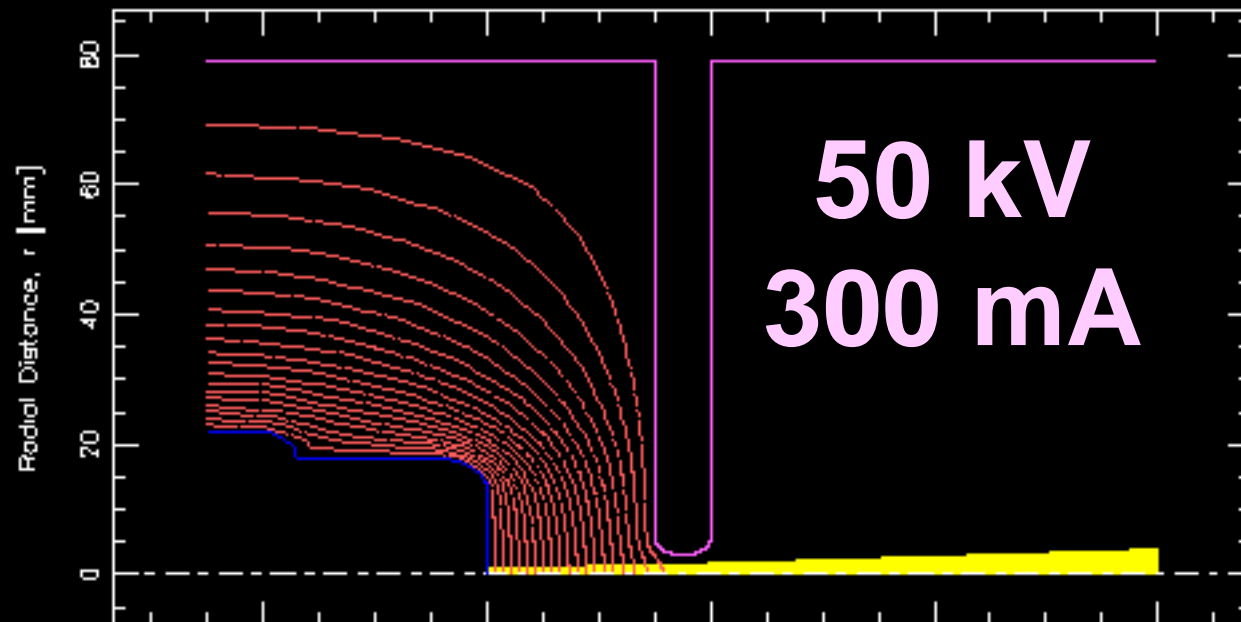
高輝度電子銃シミュレーション研究会

2006年12月7日

京大宇治キャンパス

京都大学エネルギー理工学研究所

増田 開



KUAD2 v2.20

*dartplot*

Sat Jun 10 14:03:24 2006  
driffileID, gun0001

B<sup>n</sup> iteration

Error

$\phi_b$  - 0.37[%]  
 $H_{b,\theta}$  - 0.65[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $i_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi$

Classification by Color

— electron

Injection Current

300.000[mA]

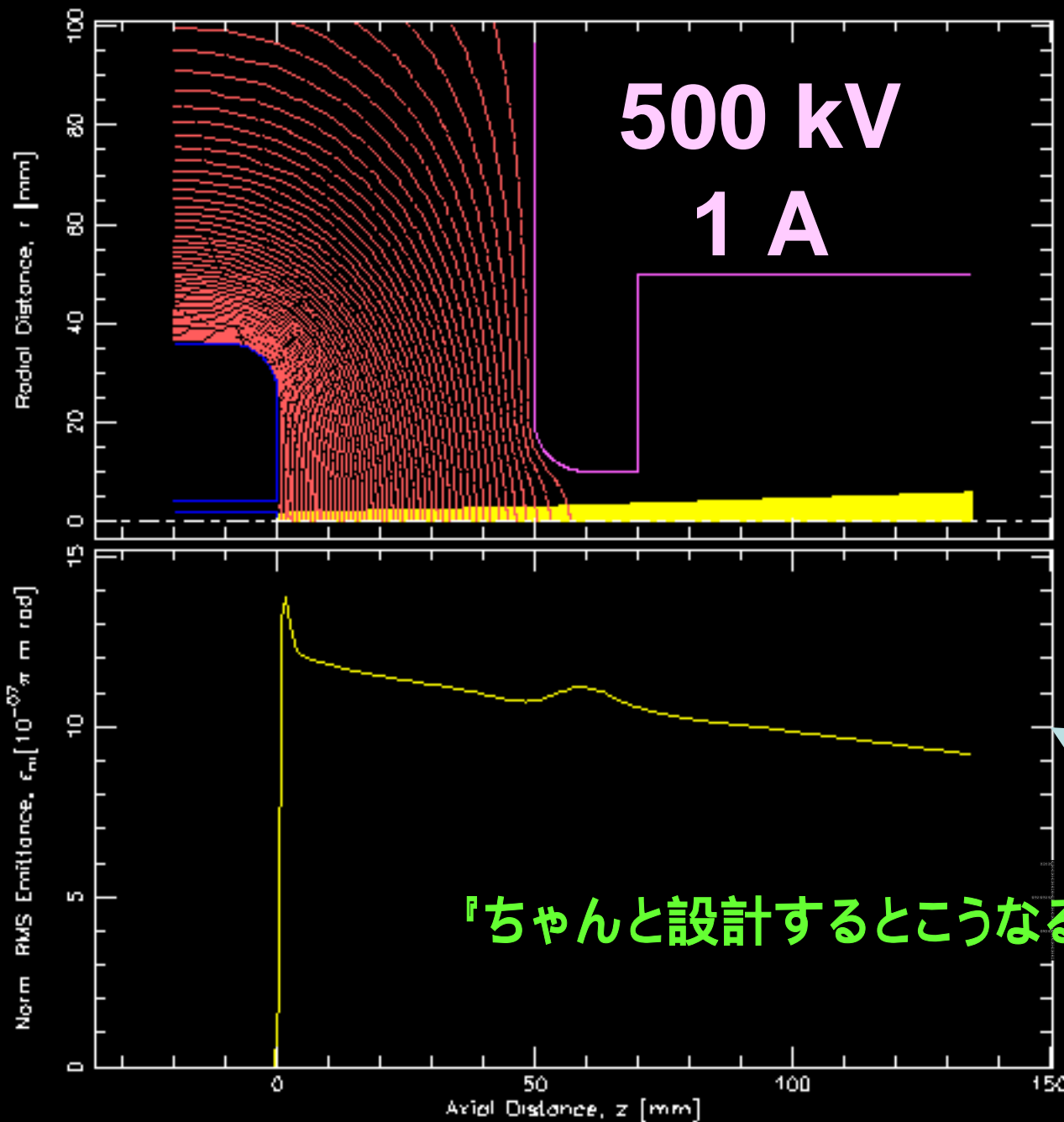
Electrodes

1 2

は ま「エミッタンスって、こんなに減ったり増えたりするの？」

ますだ「・・・」

は ま「・・・。新竹ガン計算してみてよ」



KUAD2 v2.20  
dartplot

Sun Jun 18 19:36:30 2006  
driffileID, gun0101

$B^n$  iteration  
Error  
 $\phi_b$  - 0.50[%]  
 $H_{b,\theta}$  - 0.36[%]  
 $A_{\theta,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1  
Contour of  $\phi$   
Classification by Color  
— electron  
Injection Current  
1000.000[mA]  
Electrodes  
1 2

ここが1

『ちゃんと設計するとこうなるのかぁ・・・』

# エミッタンスの定義

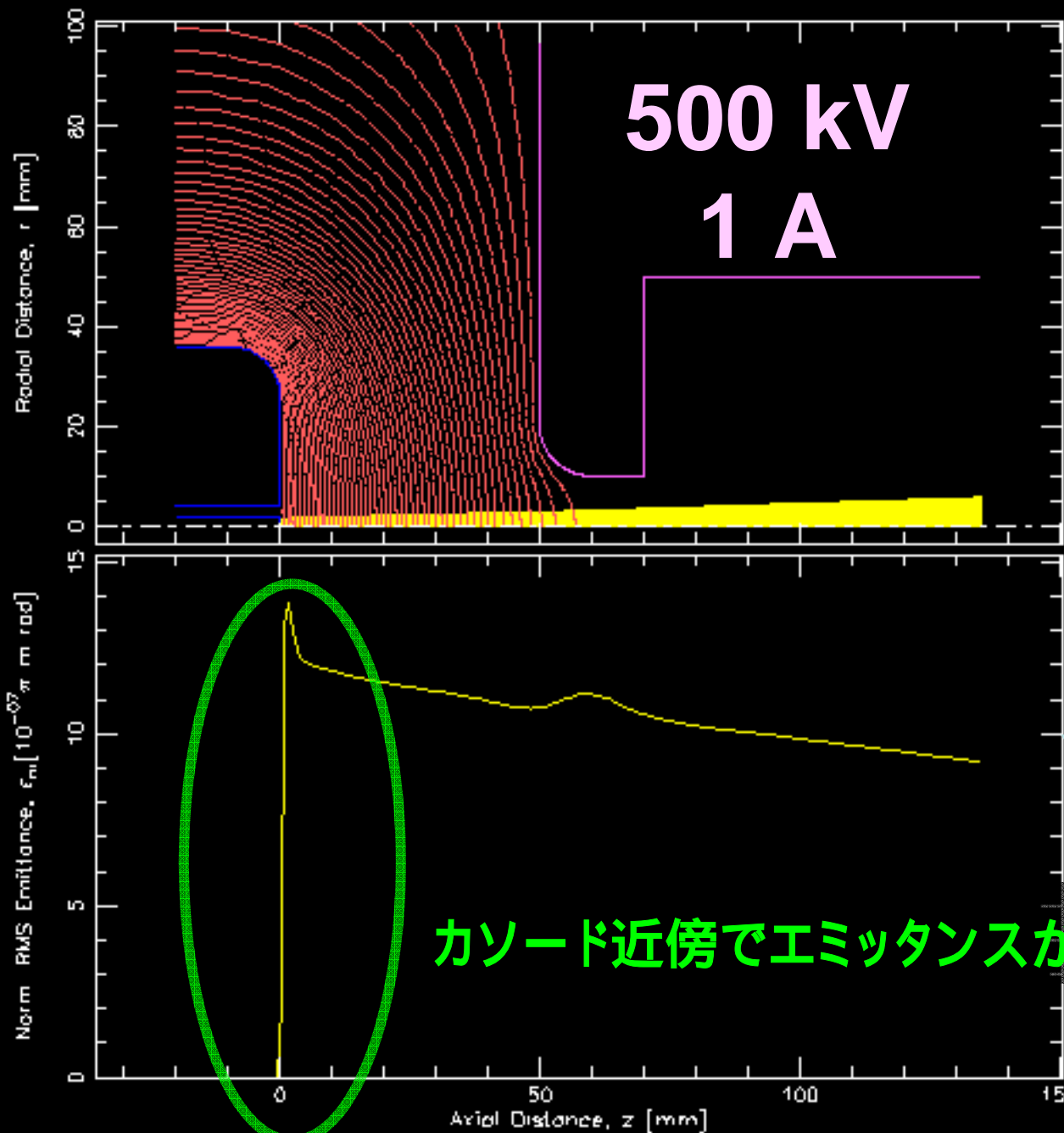
normalized rms  $r$ -emittance

$$\varepsilon_{n,r} = \sqrt{\langle r^2 \rangle \langle (\gamma\beta\alpha)^2 \rangle - \langle r\gamma\beta\alpha \rangle^2} \quad [\pi \text{ m rad}]$$

$$\alpha = \arctan\left(\frac{p_r}{p_z}\right) \quad [\text{rad}]$$

軸対称の場合 . . .

$$\varepsilon_{n,r}^2 = 2\varepsilon_{n,x}^2 = 2\varepsilon_{n,y}^2$$



KUAD2 v2.20  
dartplot

Sun Jun 18 19:36:30 2006  
driffileID, gun0101

$B^n$  iteration  
Error  
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 $H_{b,\theta}$  - 0.36[%]  
 $A_{\theta,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1  
Contour of  $\phi$   
Classification by Color  
— electron  
Injection Current  
1000.000[mA]  
Electrodes  
1 2

ここが1

カソード近傍でエミッタンスが決まっている

# カソード近傍での鏡像の効果

自由空間では...

$$\uparrow E_r \propto r$$

ビームは広がっても  
エミッタンスは増えない

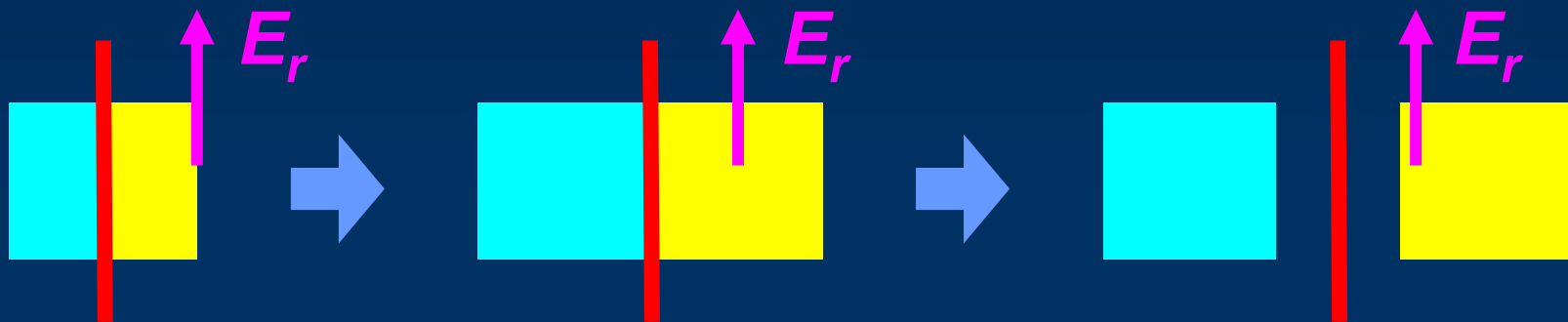
カソードがあると...

$$\uparrow E_r \text{ が } r \text{ に比例しない}$$

正電荷の鏡像

電子ビーム(負電荷)

パルスビームだと...



鏡像の影響がヘッドとテイルで異なる

# 1. コードの概要

2D, FEM, PIC

## 2. DCビームへの鏡像の影響

どの程度？

## 3. DCビームへの熱エミッタンスの影響

$$\varepsilon_{\text{total}}^2 = \varepsilon_{\text{th}}^2 + \varepsilon_{\text{SC}}^2 \quad ?$$

## 4. パルスビームへの鏡像の影響

DCビームとの違いはどの程度？

# Group of 2-D Codes

for simulating charged particle dynamics in electromagnetic fields

K. Masuda, PhD Thesis Kyoto Univ. (1997).

- Kyoto Univ. semi-Automatic meSH generator
- Kyoto Univ. EigenMode Solver
- Kyoto Univ. SOlenoidal field Solver
- Kyoto Univ. Irrotational Field Solver
- Kyoto Univ. Advanced Dart 2<sup>nd</sup> edition
- Kyoto Univ. Beam Loading Analysis

mesh generator  
**KUASH**

vacuum E-field  
**KUIFS**

electrode  
voltages

cavity eigenmode  
**KUEMS**

electrode  
voltages

**time-independent**

**time-domain**

DC beam trajectory  
beam-induced field  
Child Law  
**KUAD2**

initial  $\rho_b, J_b$   
initial  $E_b, B_b$   
injection condition

time-domain PIC  
beam-cavity interaction  
**KUBLAI**

coil  
currents

vacuum B-field  
**KUSOS**

coil  
currents

T. Shintake, NIMA 363 (1997) 83.



# Basic Equations and Formulations (1/2)

## charged particle dynamics

$$\frac{dK}{dt} = q(\mathbf{u} \cdot \mathbf{E})$$

$$K = m_0 c^2 (\gamma - 1) : \text{kinetic energy}$$

$$\gamma^2 = 1 / \{1 - (u/c)^2\}$$

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$$

$$\mathbf{p} = m_0 \gamma \mathbf{u} : \text{momentum}$$

$$\gamma^2 - (p/m_0 c)^2 = 1$$

$$\frac{\partial}{\partial \theta} = 0$$

KUBLAI

$$P_\theta = r(p_\theta + qA_\theta) = \text{const.}$$

$$u_z = u_\sigma \cos \alpha$$

$$u_r = u_\sigma \sin \alpha$$

$$\frac{dK}{dt} = q(\mathbf{u} \cdot \mathbf{E})$$

$$\frac{d\alpha}{dt} = \frac{1}{m_0 \gamma u_\sigma} \left[ -qu_\sigma B_\theta - q(E_z - u_\theta B_r) \sin \alpha + \left\{ q(E_r + u_\theta B_z) + \frac{m_0 \gamma u_\theta^2}{r} \right\} \cos \alpha \right]$$

$$\frac{\partial}{\partial t} = 0$$

KUAD2

$$P_\theta = r(p_\theta + qA_\theta) = \text{const.}$$

$$u_z = u_\sigma \cos \alpha$$

$$u_r = u_\sigma \sin \alpha$$

$$W = K + q\phi = \text{const.}$$

$$\frac{d\alpha}{dt} = \frac{1}{m_0 \gamma u_\sigma} \left[ -qu_\sigma B_\theta - q(E_z - u_\theta B_r) \sin \alpha + \left\{ q(E_r + u_\theta B_z) + \frac{m_0 \gamma u_\theta^2}{r} \right\} \cos \alpha \right]$$

# Basic Equations and Formulations (2/2)

## beam-induced electromagnetic fields

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\frac{\partial}{\partial \theta} = 0$$

### KUAD2

$$\mathbf{E} = -\nabla \phi$$

$$\mathbf{B} = \mu H_{\theta} \mathbf{i}_{\theta} + \nabla \times (A_{\theta} \mathbf{i}_{\theta})$$

$$-\nabla \cdot (\epsilon \nabla \phi) = \rho$$

$$\nabla \times \left\{ \frac{1}{\mu_s} \nabla \times (A_{\theta} \mathbf{i}_{\theta}) \right\} = \mu_0 \mathbf{J}_{\theta} \mathbf{i}_{\theta}$$

$$\nabla \times \left\{ \frac{1}{\epsilon_s} \nabla \times (H_{\theta} \mathbf{i}_{\theta}) \right\} = \nabla \times \left\{ \frac{1}{\epsilon_s} (\mathbf{J} - J_{\theta} \mathbf{i}_{\theta}) \right\}$$

### KUBLAI

$$\mathbf{E} = -\nabla \phi - \left\{ \frac{\partial (A_{\theta} \mathbf{i}_{\theta})}{\partial t} + \frac{1}{c} \nabla \times (G_{\theta} \mathbf{i}_{\theta}) \right\}$$

$$\mathbf{B} = \mu H_{\theta} \mathbf{i}_{\theta} + \nabla \times (A_{\theta} \mathbf{i}_{\theta})$$

$$-\nabla \cdot (\epsilon \nabla \phi) = \rho$$

$$\nabla \times \left\{ \frac{1}{\mu_s} \nabla \times (A_{\theta} \mathbf{i}_{\theta}) \right\} + \epsilon_s \frac{1}{c^2} \frac{\partial^2 (A_{\theta} \mathbf{i}_{\theta})}{\partial t^2} = \mu_0 \mathbf{J}_{\theta} \mathbf{i}_{\theta}$$

$$\nabla \times \left\{ \frac{1}{\epsilon_s} \nabla \times (H_{\theta} \mathbf{i}_{\theta}) \right\} + \mu_s \frac{1}{c^2} \frac{\partial^2 (H_{\theta} \mathbf{i}_{\theta})}{\partial t^2} = \nabla \times \left\{ \frac{1}{\epsilon_s} (\mathbf{J} - J_{\theta} \mathbf{i}_{\theta}) \right\}$$

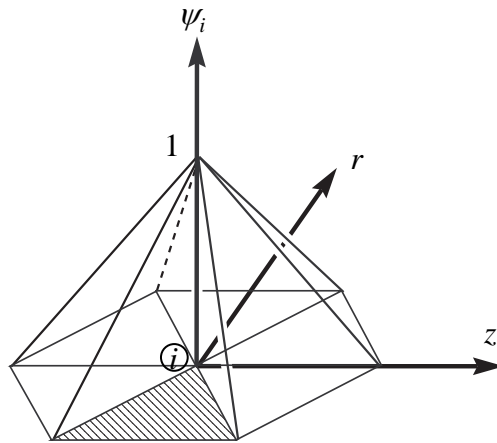
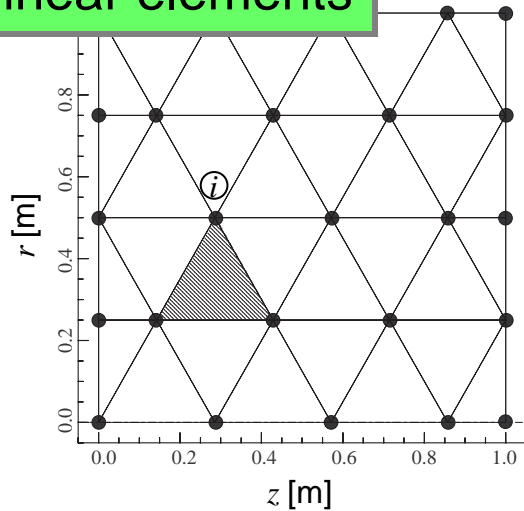
$$\nabla \times \nabla \times (G_{\theta} \mathbf{i}_{\theta}) = \mu_0 \mu_s \frac{1}{c} \frac{\partial (H_{\theta} \mathbf{i}_{\theta})}{\partial t}$$

$$\frac{\partial}{\partial t} = 0$$

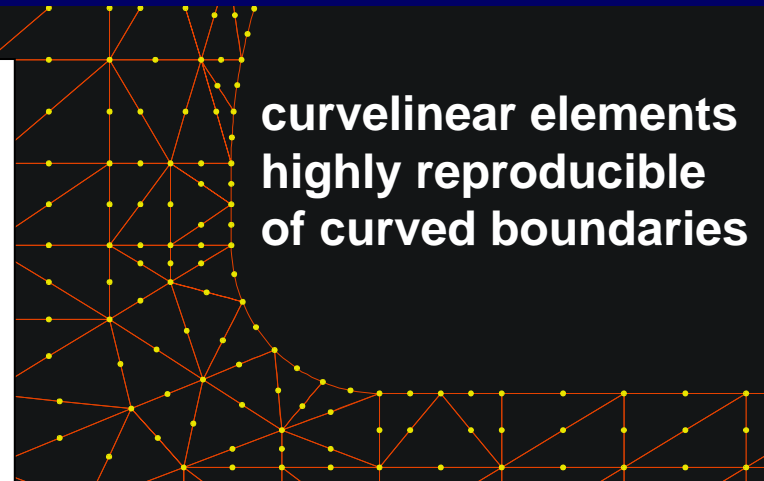
# Common Features in KU-Codes (1/3)

## FEM with with quadratic curvilinear elements

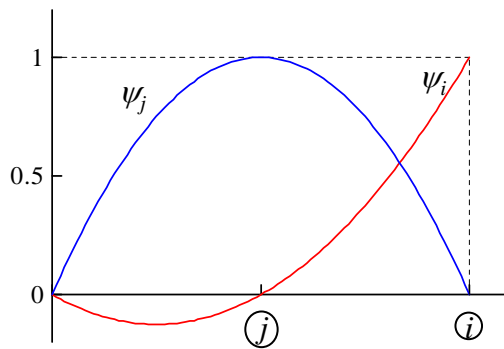
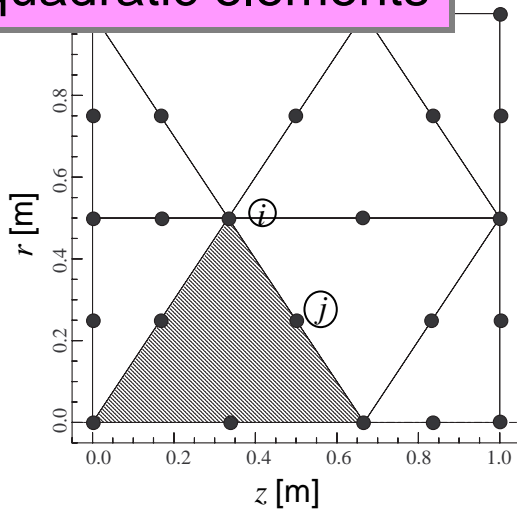
linear elements



curvilinear elements  
highly reproducible  
of curved boundaries

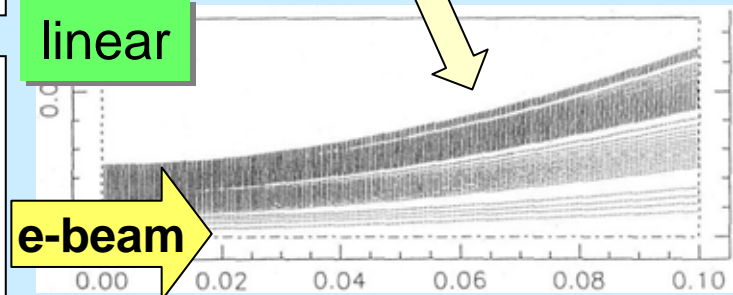


quadratic elements



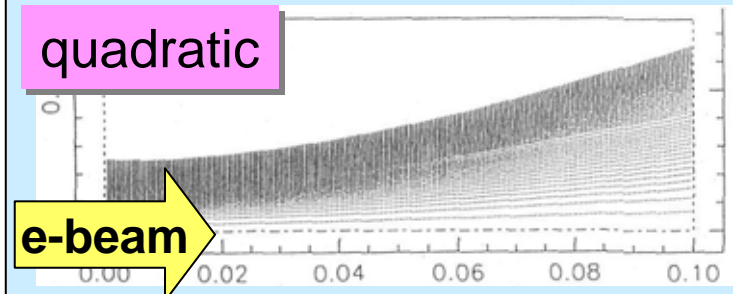
non-physical beam filamentation

linear



e-beam

quadratic



e-beam

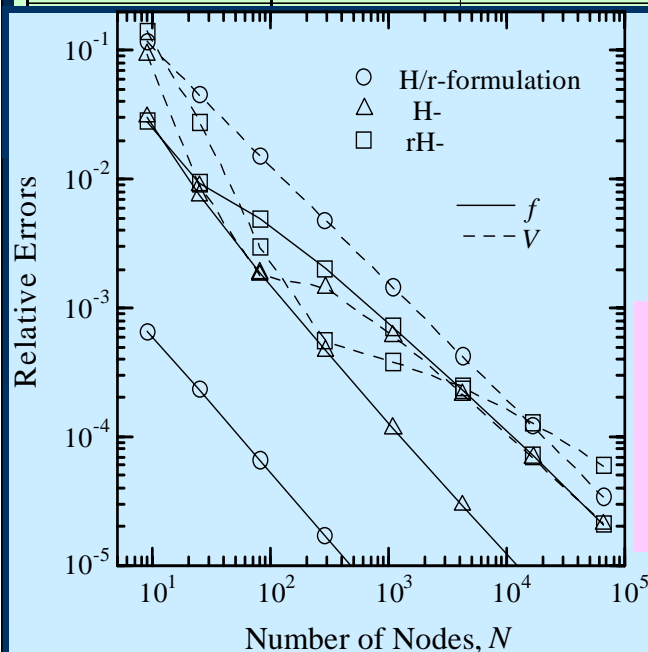
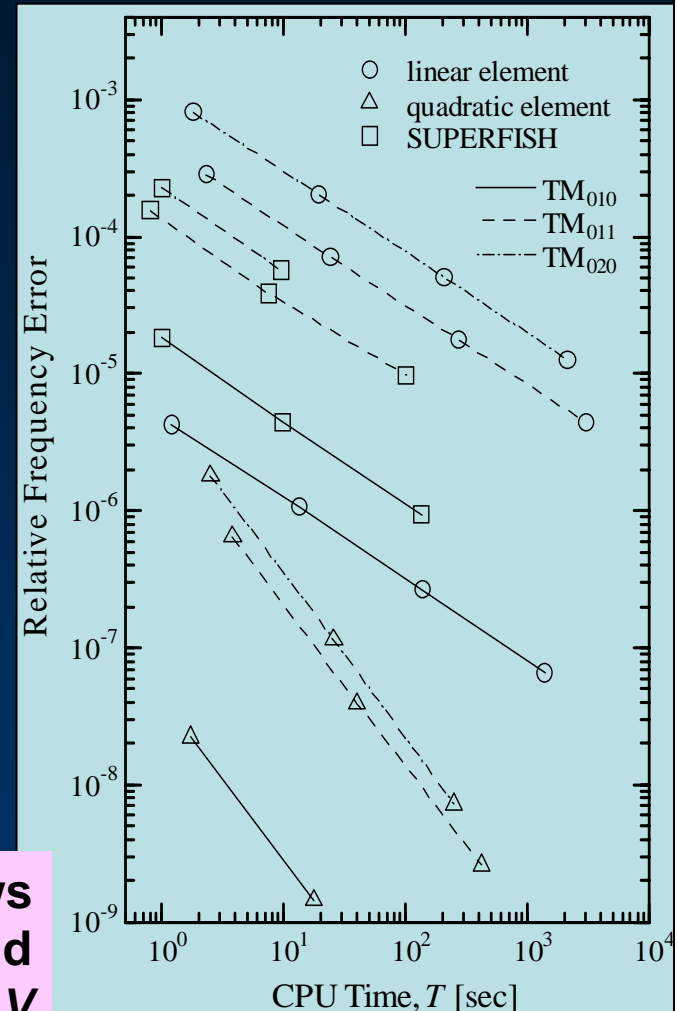
# Common Features in KU-Codes (2/3)

## high accuracy in cylindrical symmetry

Review of the existing eigenmode solvers

code	independent variable	method	basis function
LALA	$rH_\theta$	FEM	linear
SUPERFISH	$H_\theta$	FDM	linear
PRUD-W	$H_\theta$	FEM	quadratic
KUEMS	$H_\theta/r$	FEM	quadratic
MAFIA (URMEL)	$E, H$	FDM	linear

Quadratic elements show much higher precision.



The H/r-formulation shows higher accuracy in  $f$ , and smooth convergence in  $V$  with increasing  $N$ .

K. Masuda, *IEEE Trans. Microwave Theory Tech.* **46-8** (1998) 1180.

学エネルギー理工学研究所 増田 開

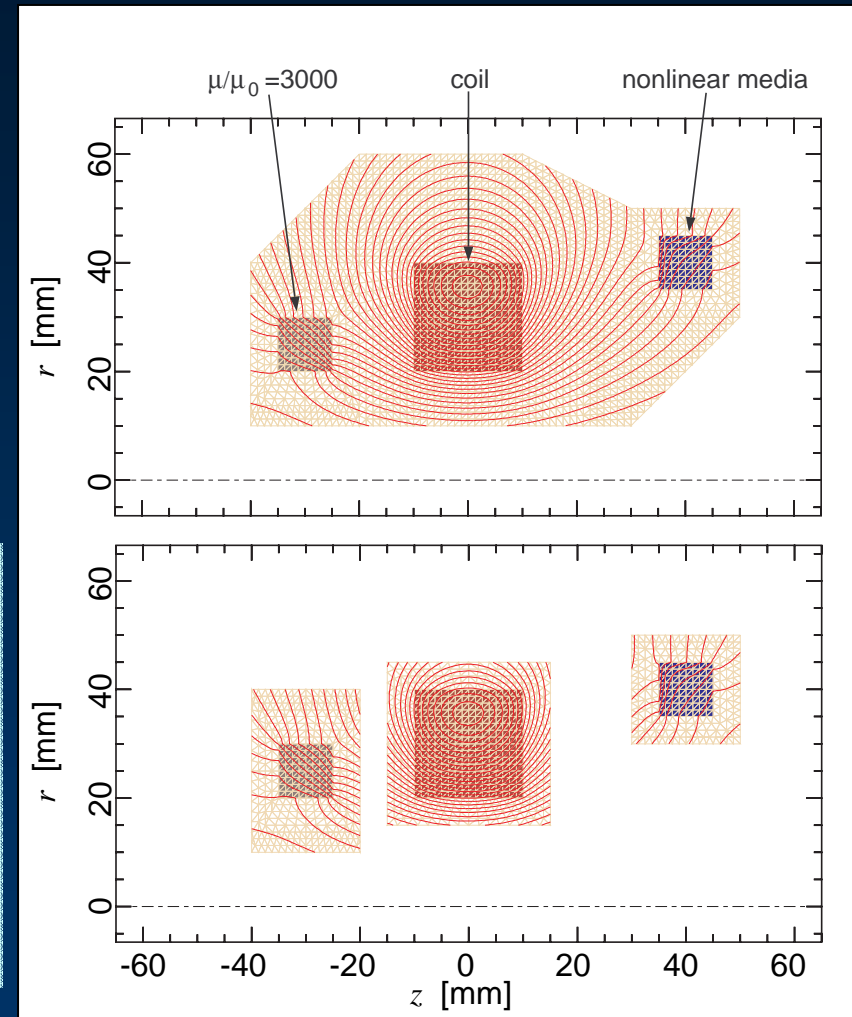
# Common Features in KU-Codes (3/3)

## unbounded fields with nonlinear media

### New FEM/MM-hybrid method

deals with unbounded problems including inhomogeneous and nonlinear media.

	unbounded problem	inhomogeneous media
finite methods (FEM, FDM)	X	O
boundary integral methods (BEM, MM)	O	X
hybrid methods	O	O



# Features of KUBLAI code

✓ full Maxwell in time-domain

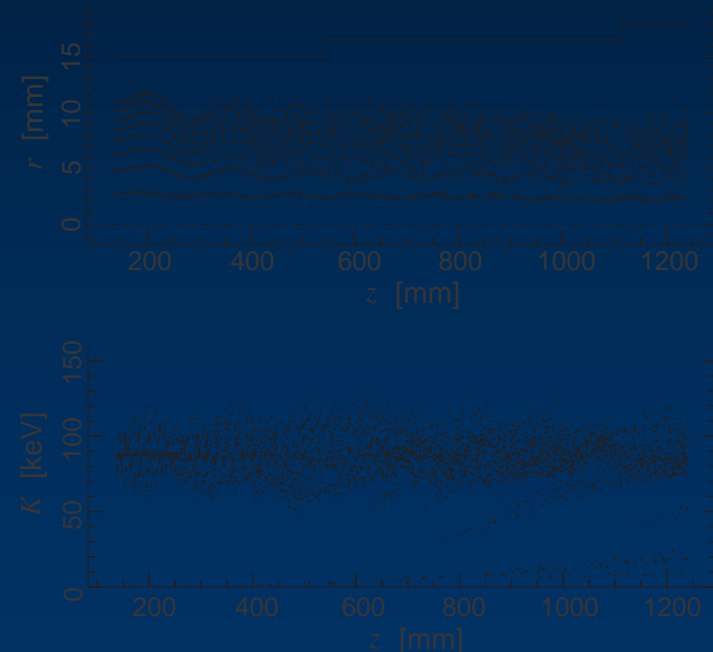
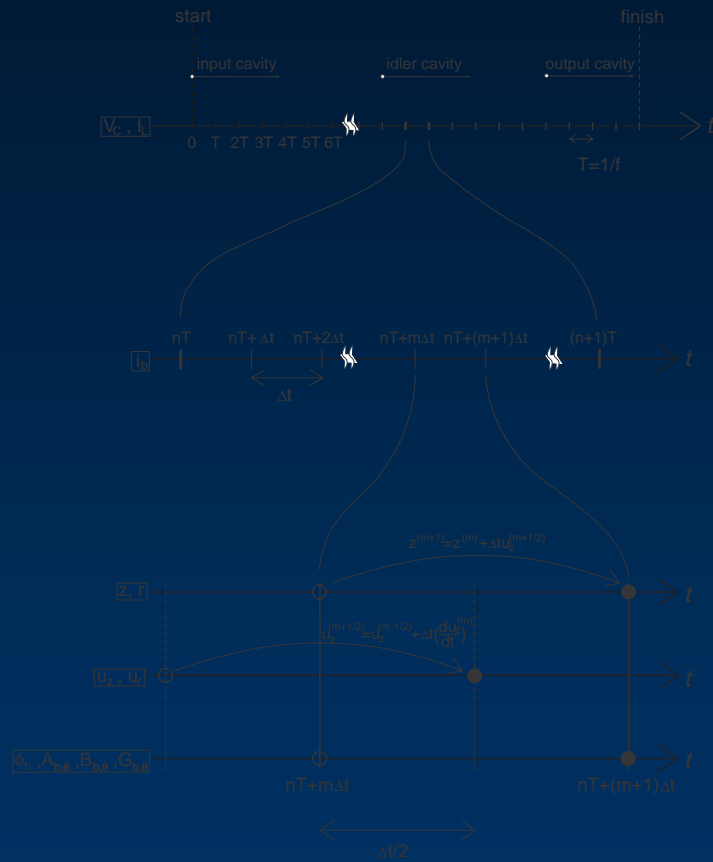
✓ modified Newmark method

new method based on Newmark method

• not requiring Courant-Freidrichs condition

$$c\Delta t \leq h\sqrt{\Delta z^2 + \Delta r^2}$$

• highly stable for intense beams



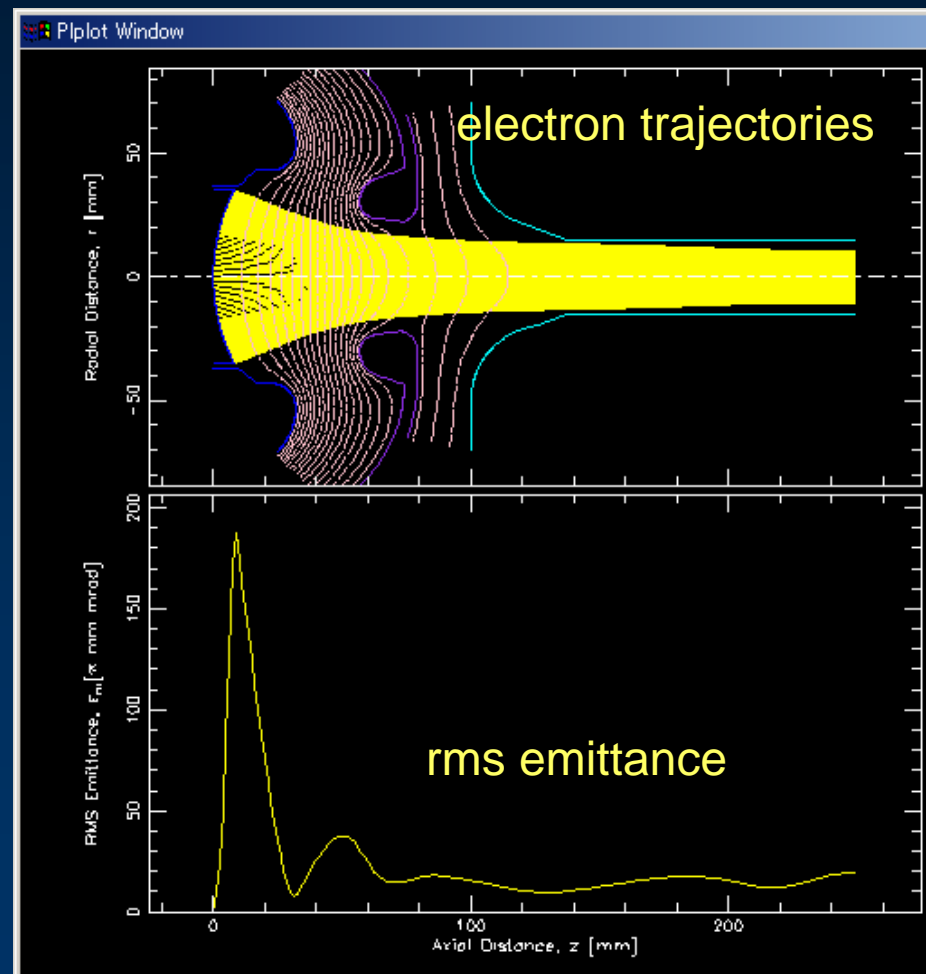
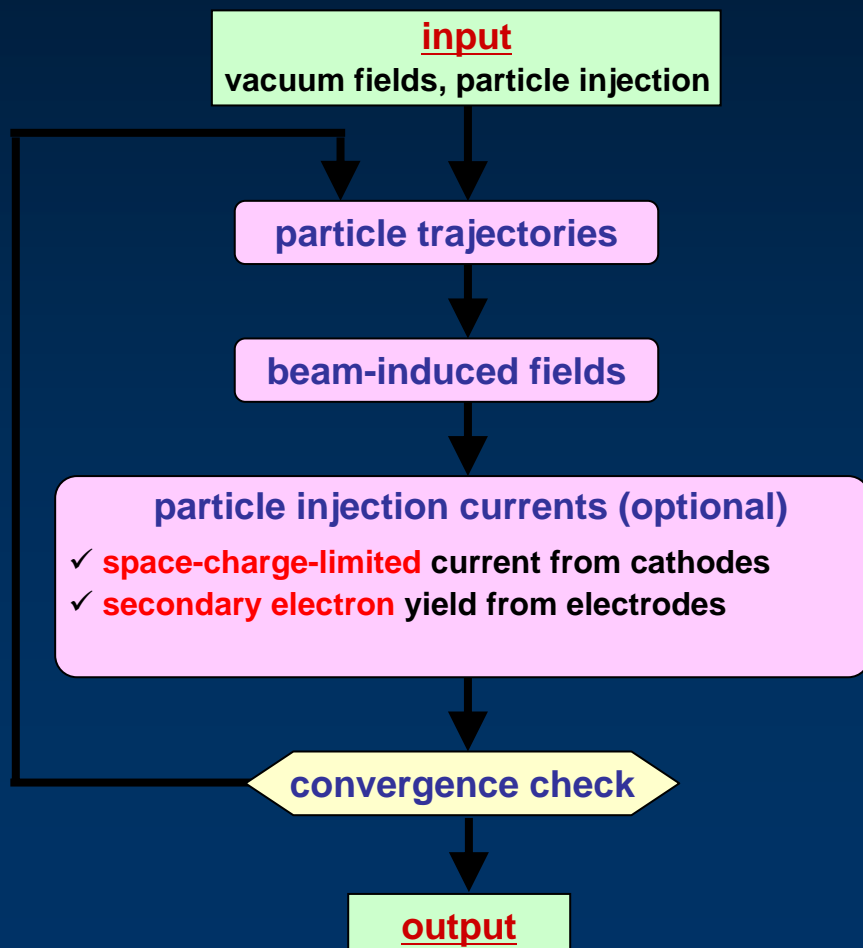
non-physical energy modulation and bunching of an intense dc beam by conventional methods

K. Masuda, *PhD Thesis Kyoto Univ. (1997) Chapter 5.*

# Features of KUAD2 code

- ✓ dc beam only (in electro-magnetostatic fields)
- ✓ much faster than KUBLAI for dc beam simulations

- ✓ deal with space-charge limited flow (Child law)

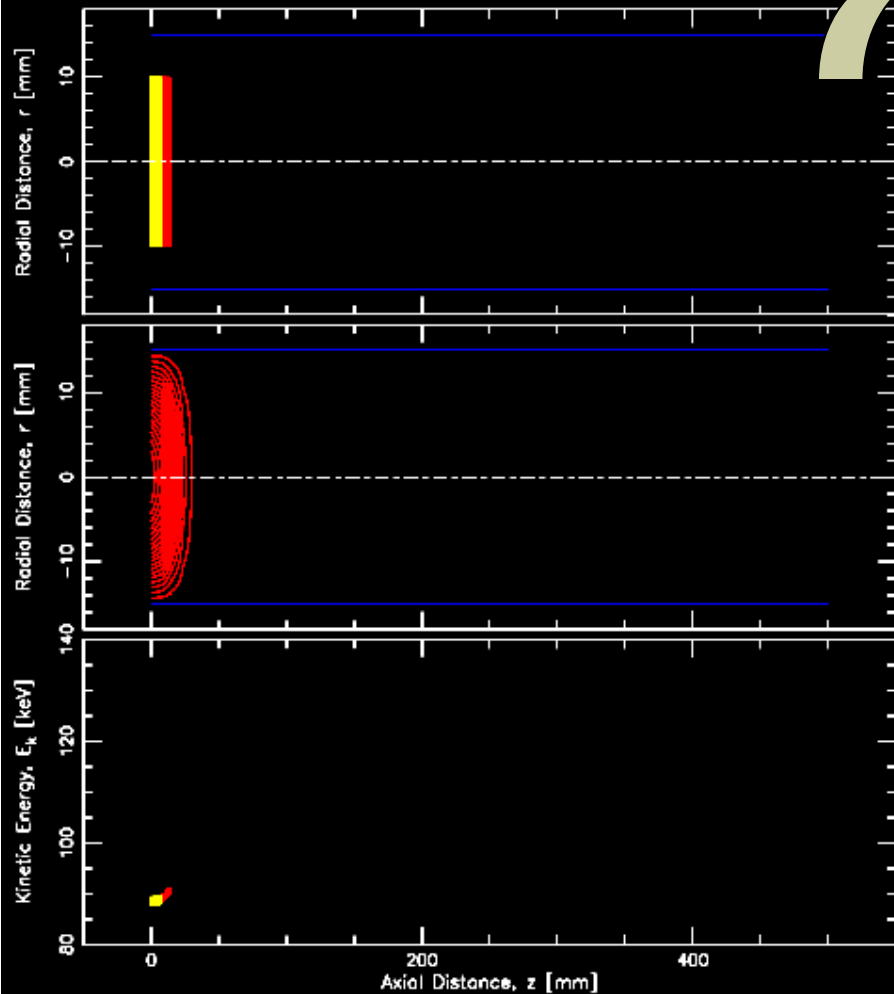


# Comparison between KUAD2 and KUBLAI

much faster convergence by KUAD2 for dc beam simulation

KUBLAI

✓ time-domain

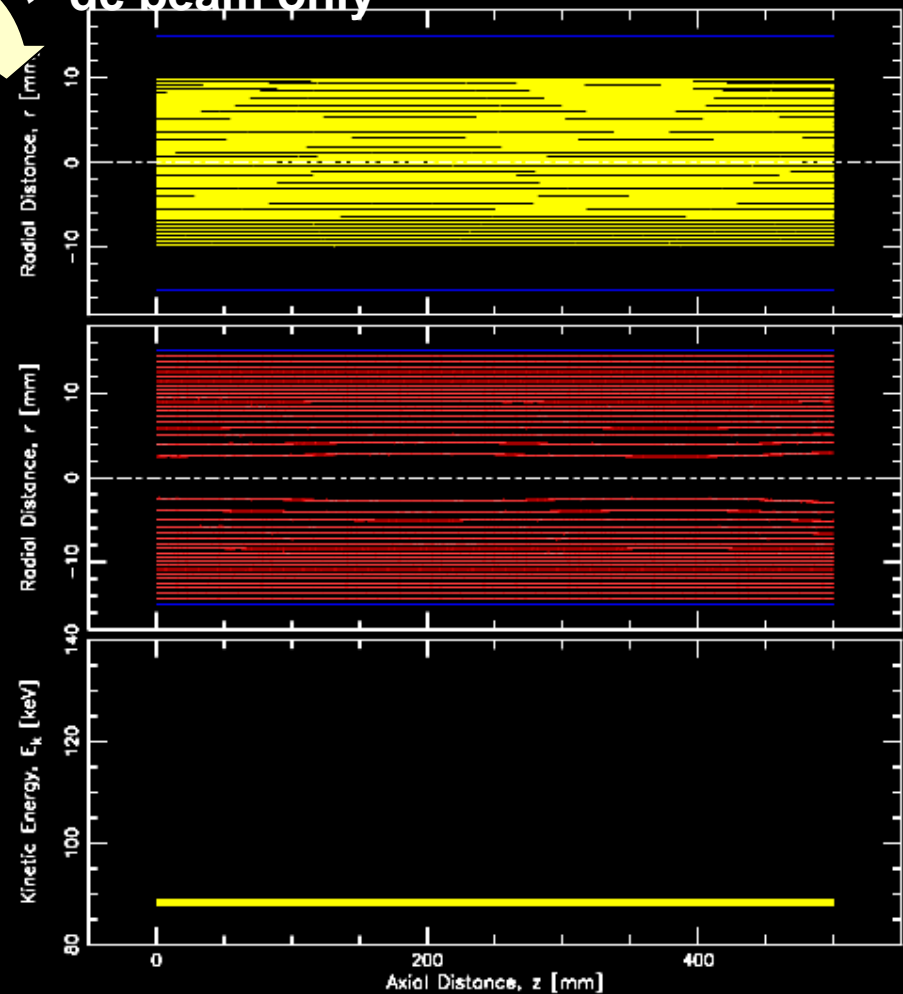


~1/30 CPU time

KUAD2

✓ electro-magnetostatic (iterative calc.)

✓ dc beam only





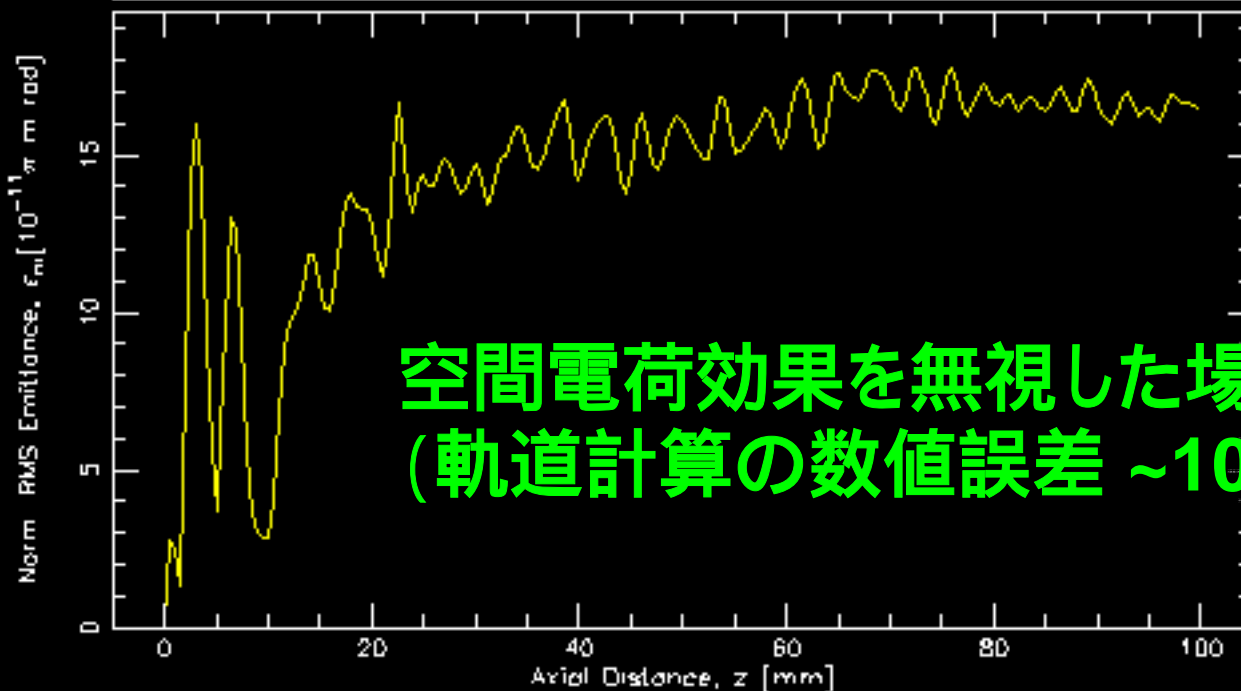
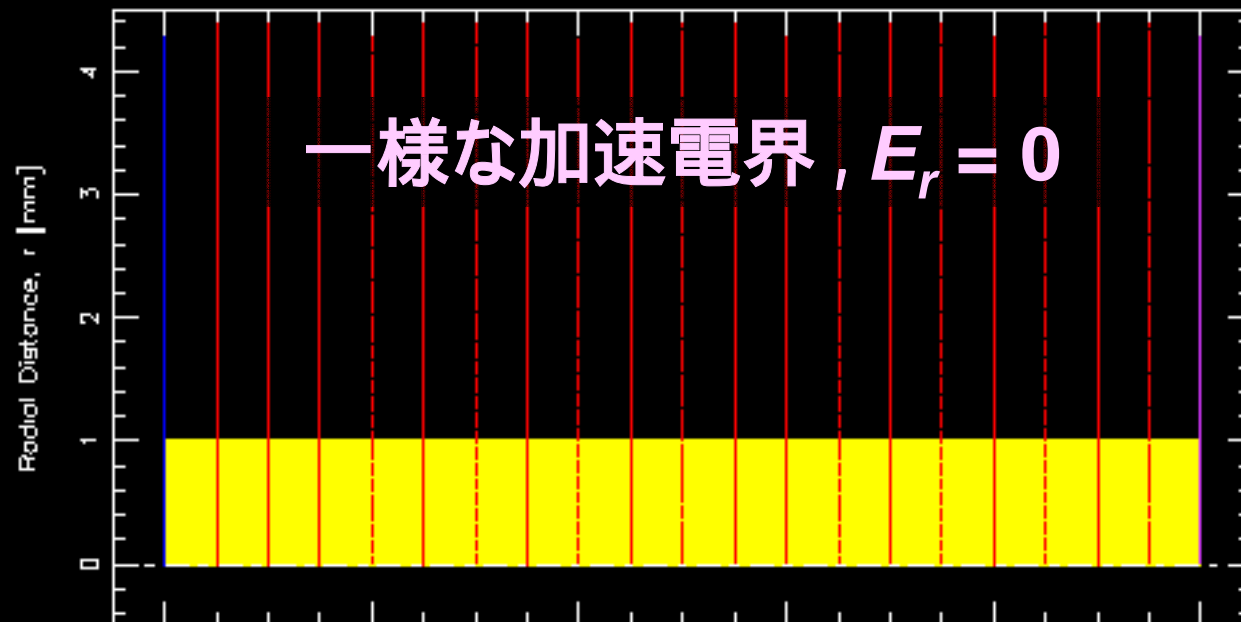
# 1. コードの概要

2. DCビームへの鏡像の影響

3. DCビームへの熱エミッタンスの影響

4. パルスビームへの鏡像の影響

一様な加速電界,  $E_r = 0$



空間電荷効果を見視した場合  
(軌道計算の数値誤差  $\sim 10^{-10} \pi$  m rad)

KUAD2 v2.30

*dartplot*

Sun Nov 26 19:00:49 2006  
driffileID, 2mmf0404D5MV2

0<sup>th</sup> iteration

Error

$\phi_b$  - 999.99[%]

$H_{b,\theta}$  - 999.99[%]

$A_{b,\theta}$  - 999.99[%]

$I_{int}$  - 999.99[%]

Plot 1

Contour of  $\phi$

Classification by Color

Injection Current

Electrodes

1 2

KUASH v2.10

meshplot

Wed Dec 6 21:16:16 2006  
meshfile10, 2mmfb

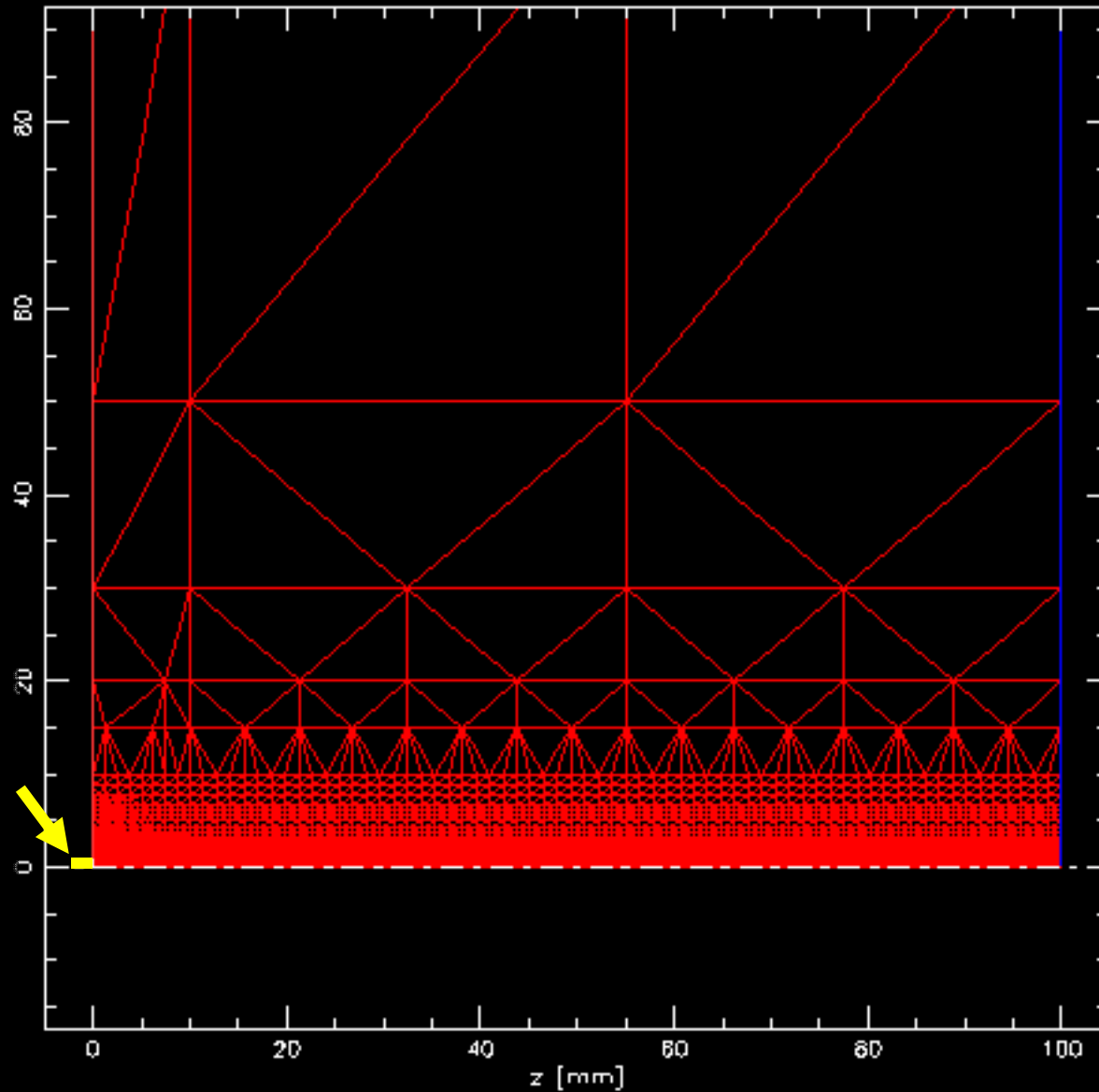
Number of Elements  
6737

Number of Nodes  
13694

Band Width of Matrices  
13532

Boundaries  
1 2 3 4

カード



メッシュ

KUASH v2.10

*meshplot*

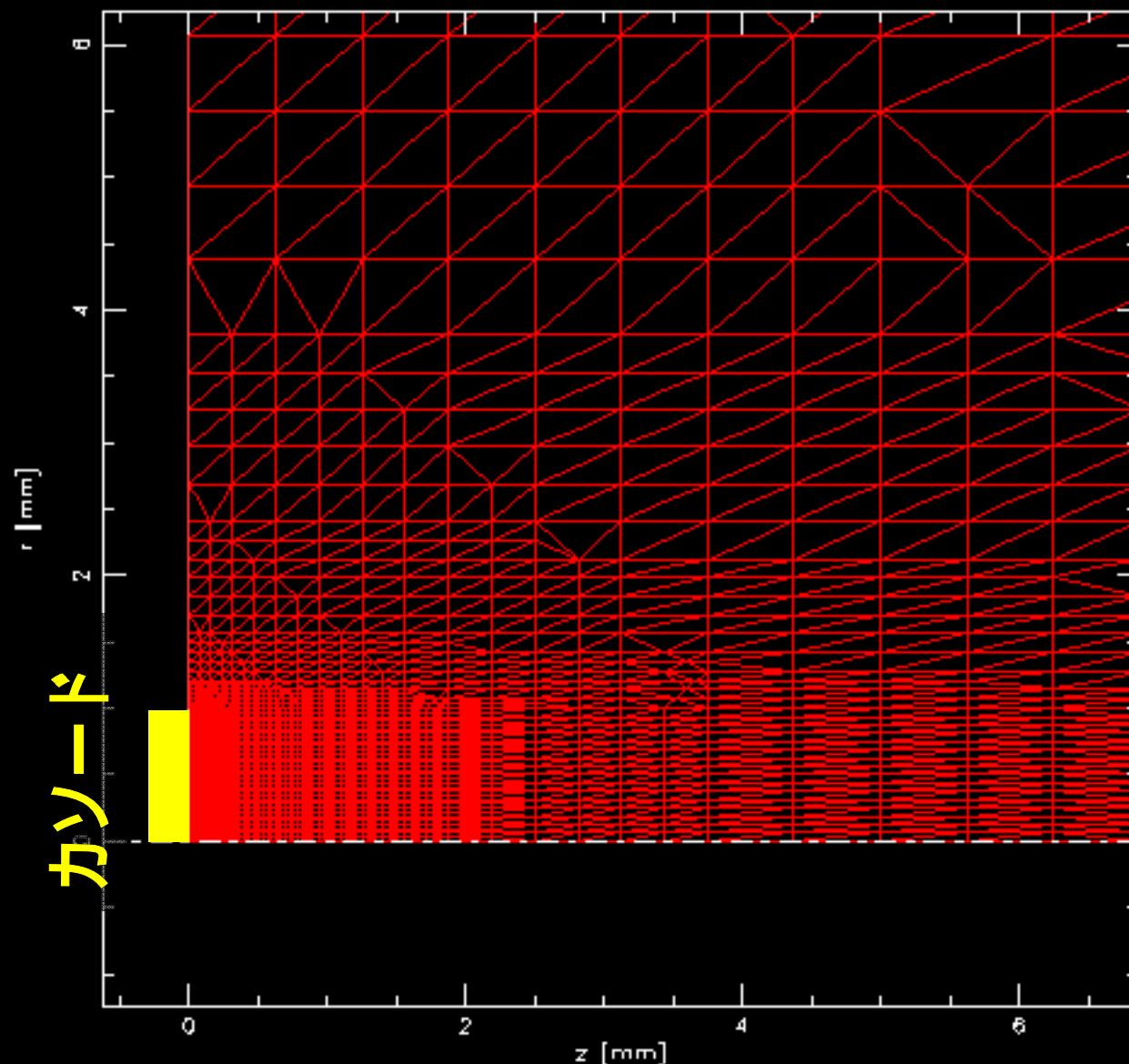
Wed Dec 6 21:17:33 2006  
meshfile10, 2mmfb

Number of Elements  
6737

Number of Nodes  
13694

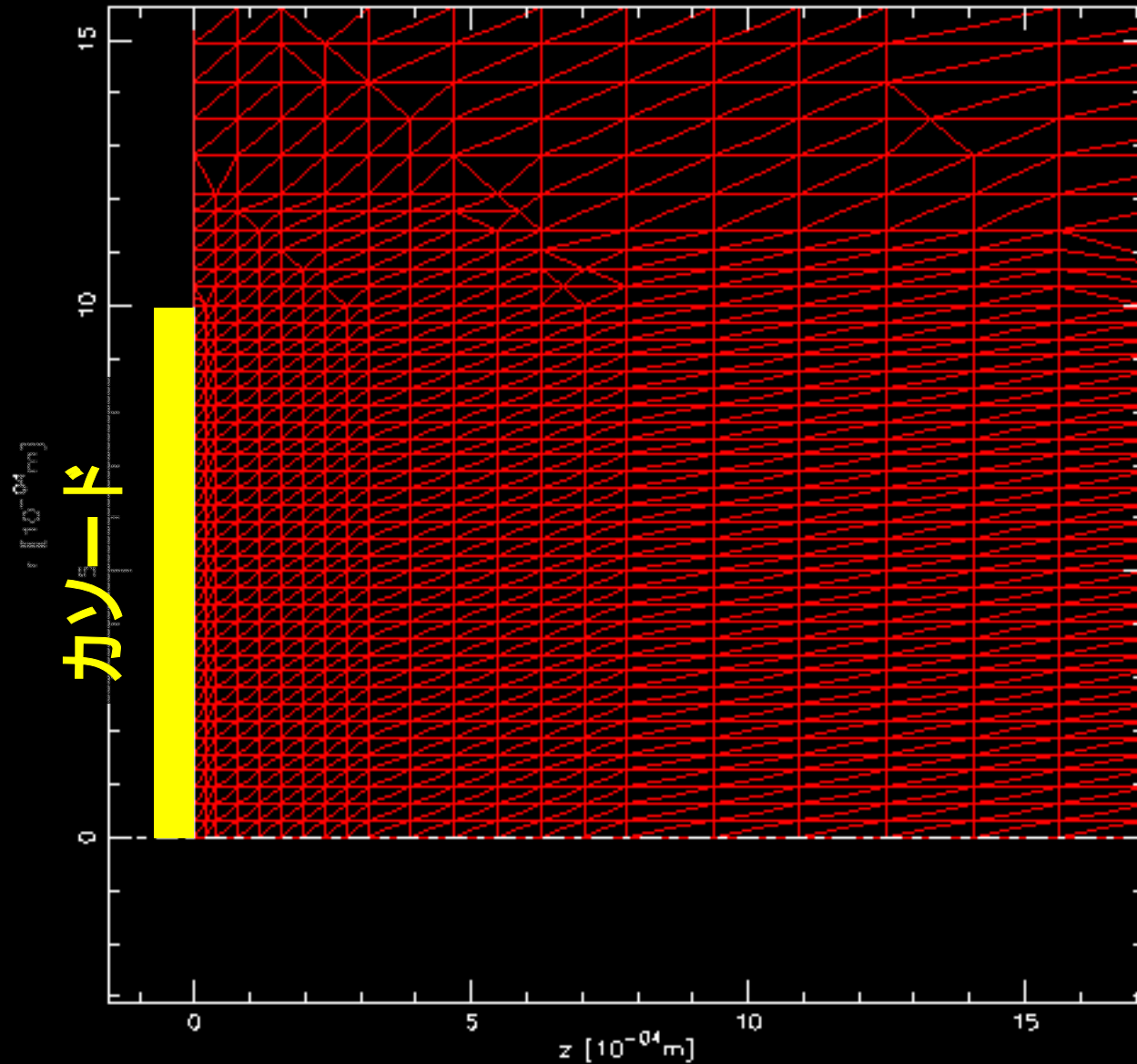
Band Width of Matrices  
13532

Boundaries  
1 2 3 4



カソード

メッシュ  
拡大



カード

KUASH v2.10

*meshplot*

Wed Dec 6 21:19:17 2006  
meshfile10, 2mmfb

Number of Elements  
6737

Number of Nodes  
13694

Band Width of Matrices  
13532

Boundaries  
1 2 3 4

メッシュ  
さらに拡大

陰極直径 : 2 mm $\phi$

電界 : 5, 10, 20, 40, 80 MV/m

電流密度 : 10, 20, 40, 80, 160 A/cm<sup>2</sup>

パルス長 : 5, 10, 20, 40 psec, DC

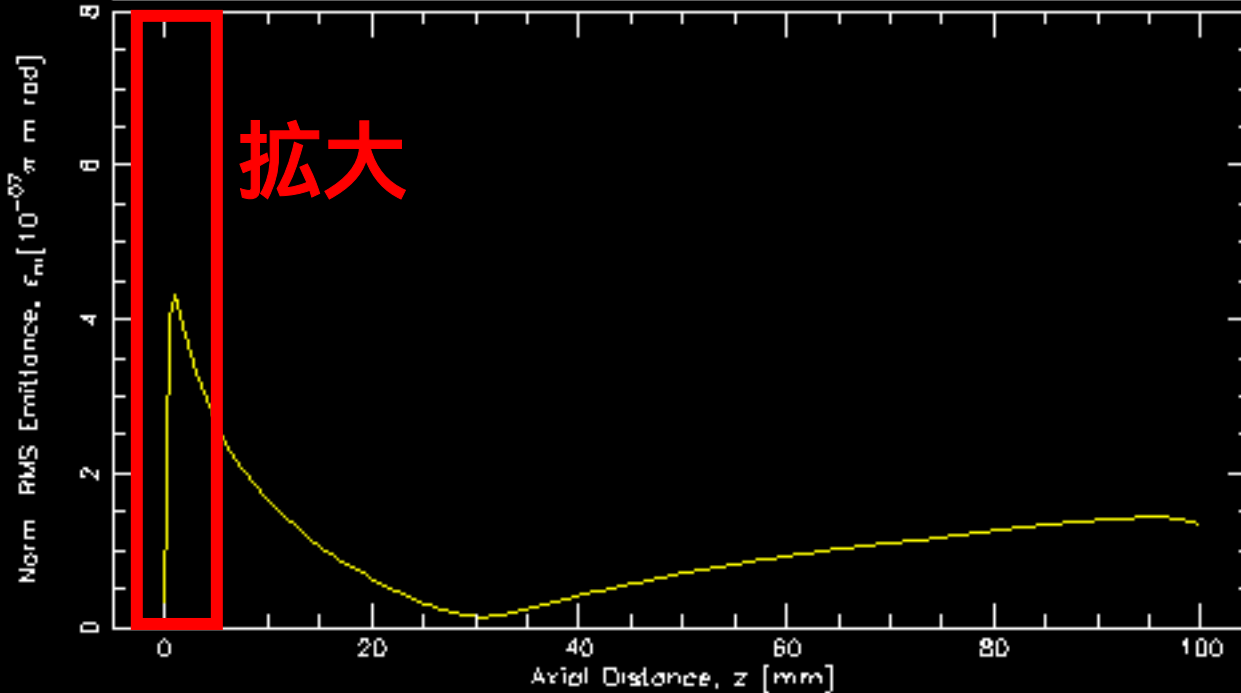
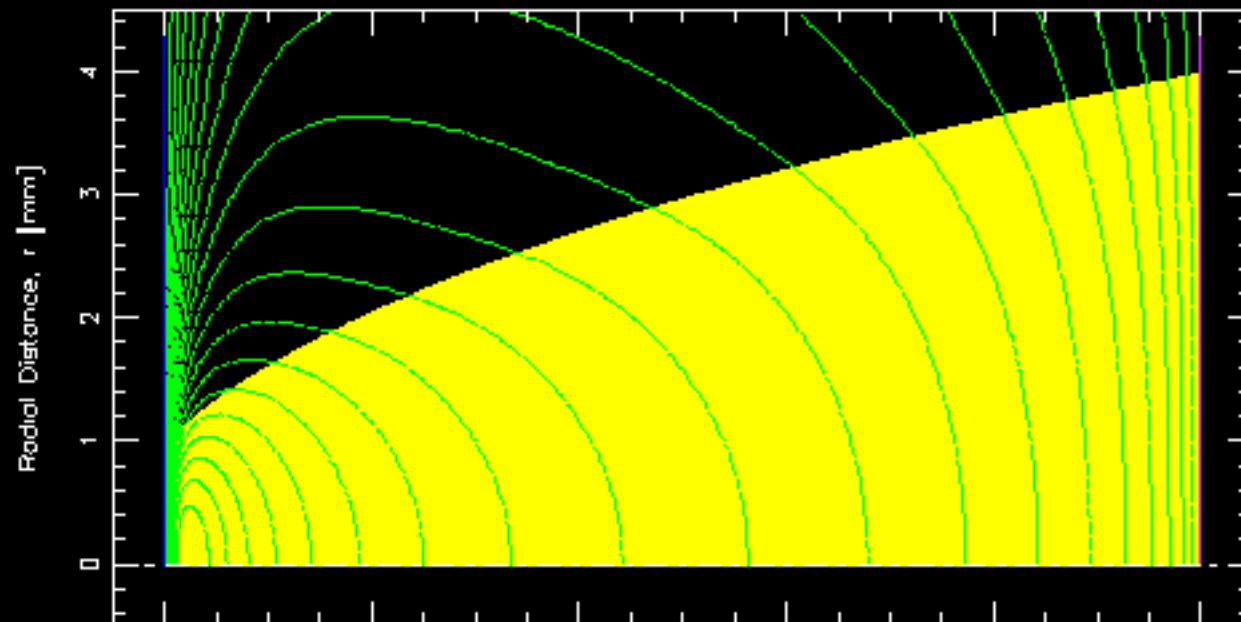
160 A/cm<sup>2</sup>, 2 mm $\phi$ , 20 psec 0.1 nC

1. コードの概要

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4. パルスビームへの鏡像の影響



KUAD2 v2.30  
dartplot

Mon Nov 27 11:13:38 2006  
driffileID, 2mmf0404D5MV

20<sup>th</sup> iteration  
Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

Injection Current

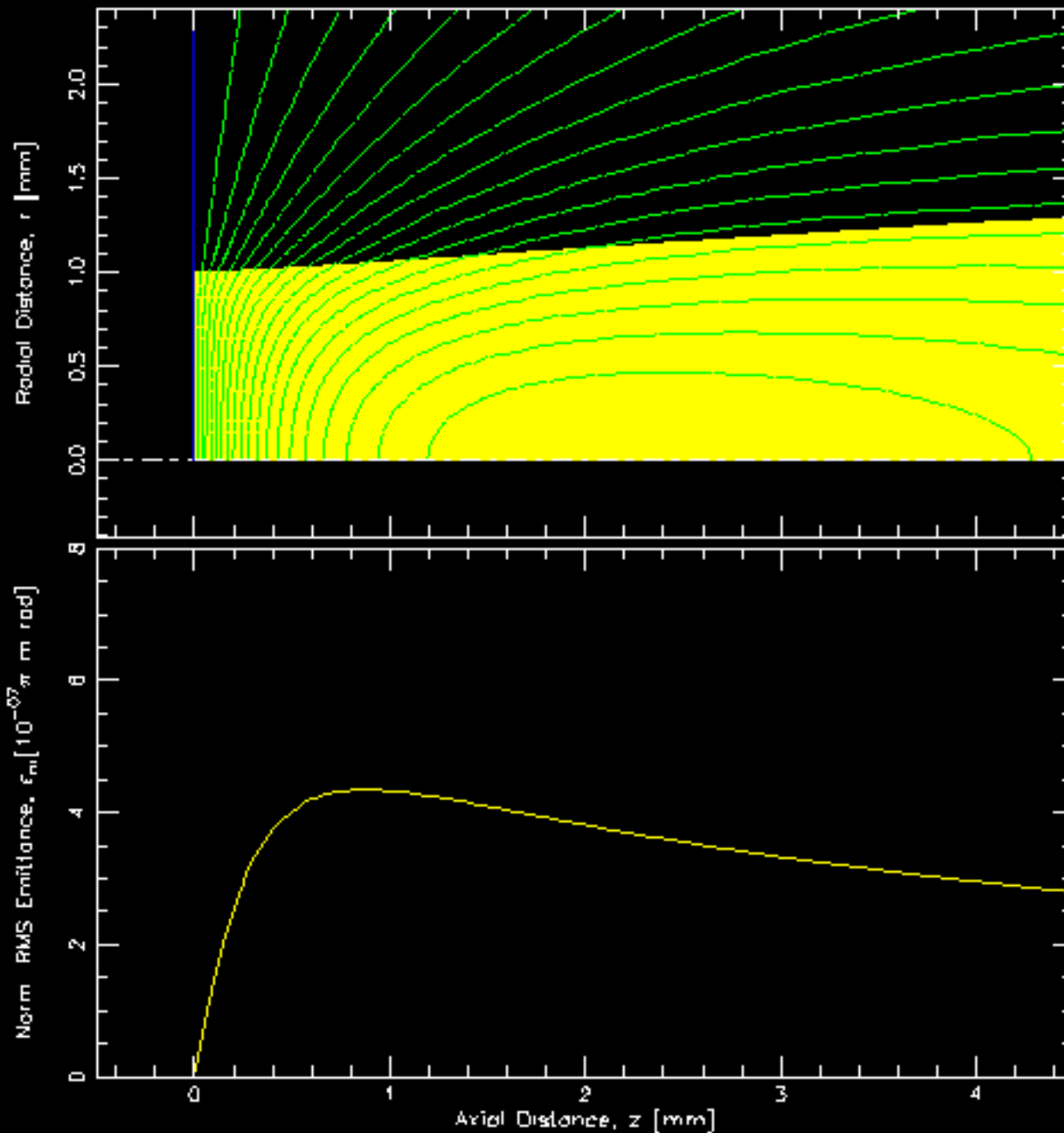
1256.637[mA]

Electrodes

1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**





KUAD2 v2.30  
*dartplot*

Sun Nov 26 20:24:25 2006  
 driffileID, 2mmf0404D5MV

20<sup>th</sup> iteration  
 Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{\theta,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

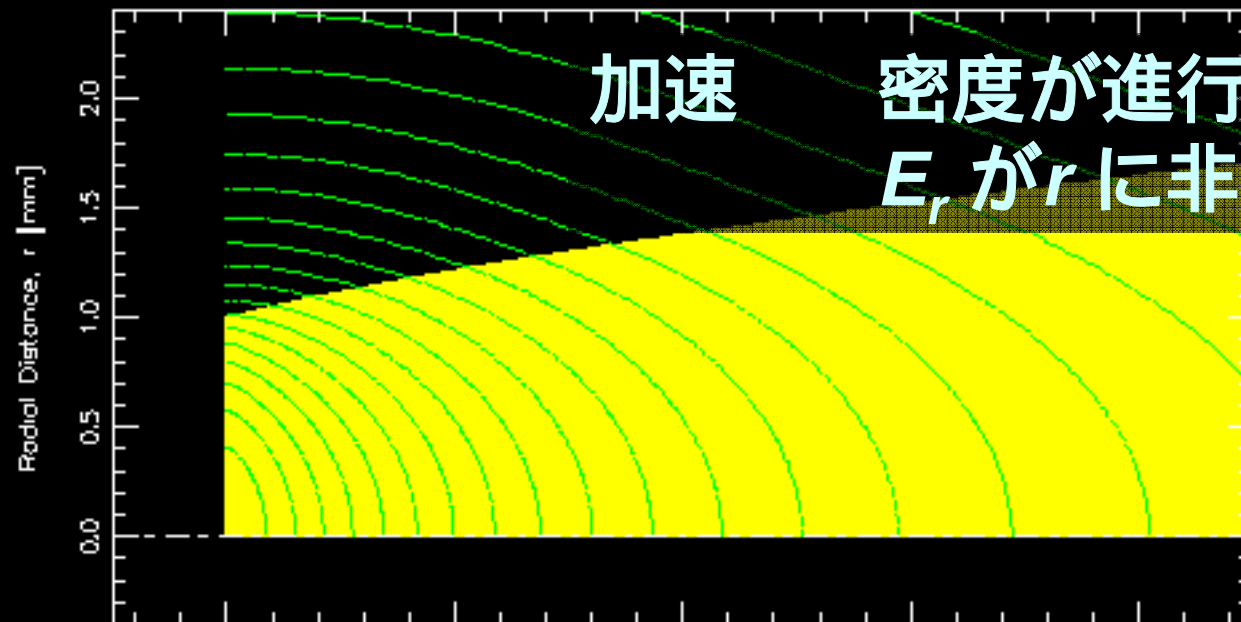
Injection Current

1256.637 [mA]

Electrodes

1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**

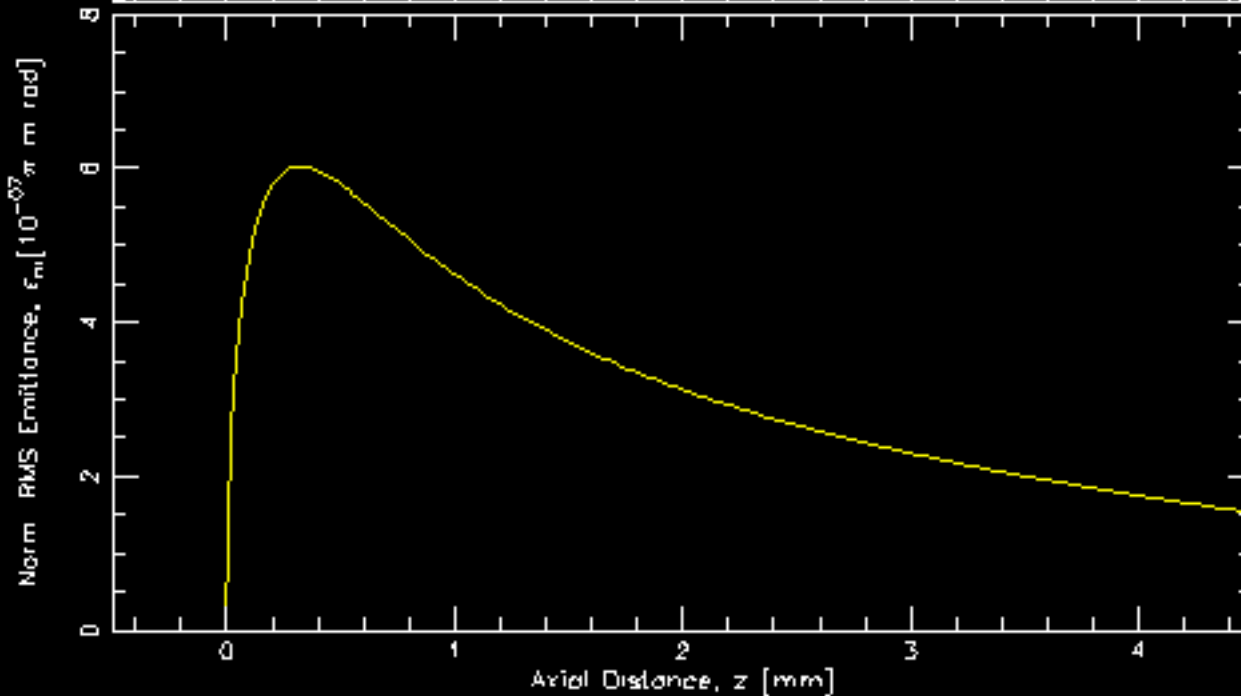


加速 密度が進行方向に非一様  
 $E_r$  が  $r$  に非線形

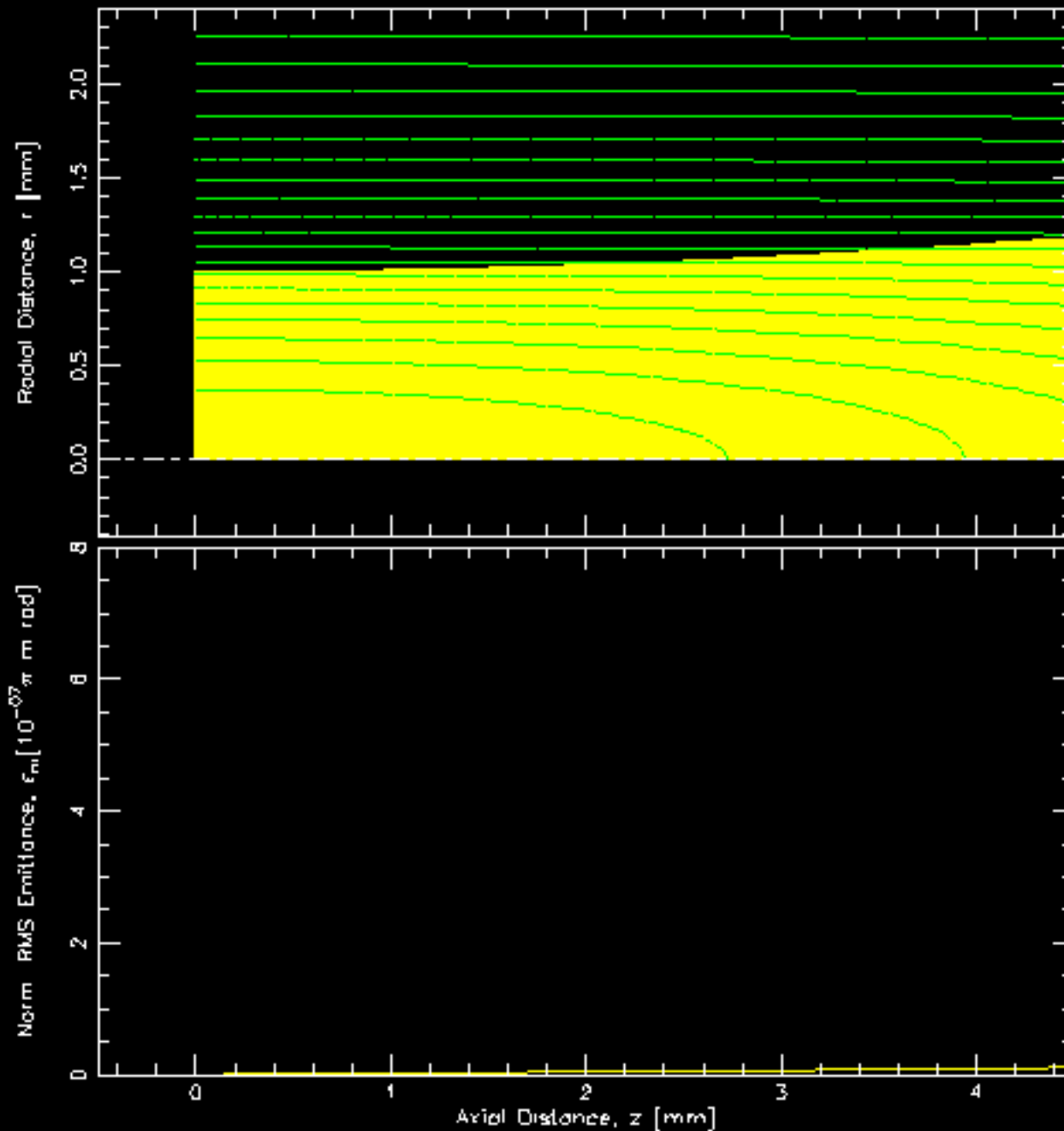
dartplot  
 Sun Nov 26 20:19:59 2006  
 driffileID, 2mmf0404D5MV2

10<sup>th</sup> iteration  
 Error  
 $\phi_b$  - 0.07[%]  
 $H_{b,\theta}$  - 0.61[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1  
 Contour of  $\phi_b$   
 Classification by Color  
 — electron  
 Injection Current  
 1256.637[mA]  
 Electrodes  
 1



5 MV/m  
 40 A/cm<sup>2</sup>  
 電極無し



KUAD2 v2.30  
*dartplot*

Mon Nov 27 11:39:41 2006  
 driffileID, 2mmf0404D5MV4

17<sup>th</sup> iteration

Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.85[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

Injection Current

1256.637[mA]

Electrodes

1

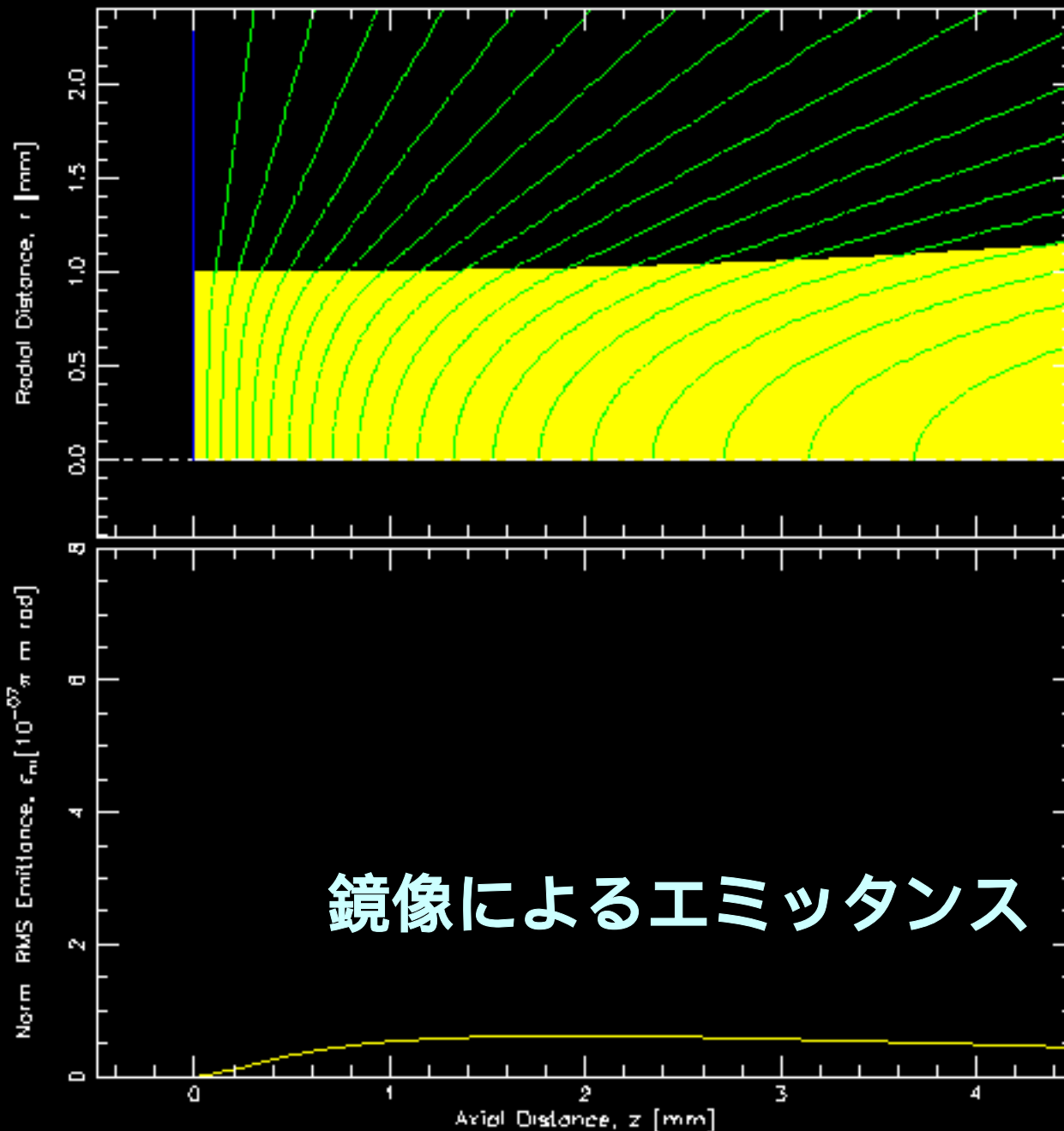
**5 MV/m**

**40 A/cm<sup>2</sup>**

**電極無し**

**加速無し**

**(10 keV入射)**



# 鏡像によるエミッタンス

KUAD2 v2.30  
dartplot

Sun Nov 26 20:33:41 2006  
driffileID, 2mmf0404D5MV5

19<sup>th</sup> iteration

Error

$\phi_b$  - 0.01[%]  
 $H_{b,\theta}$  - 0.69[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

Injection Current

1256.637[mA]

Electrodes

1 2

5 MV/m  
 40 A/cm<sup>2</sup>  
 電極有り  
 加速無し

(10 keV入射)

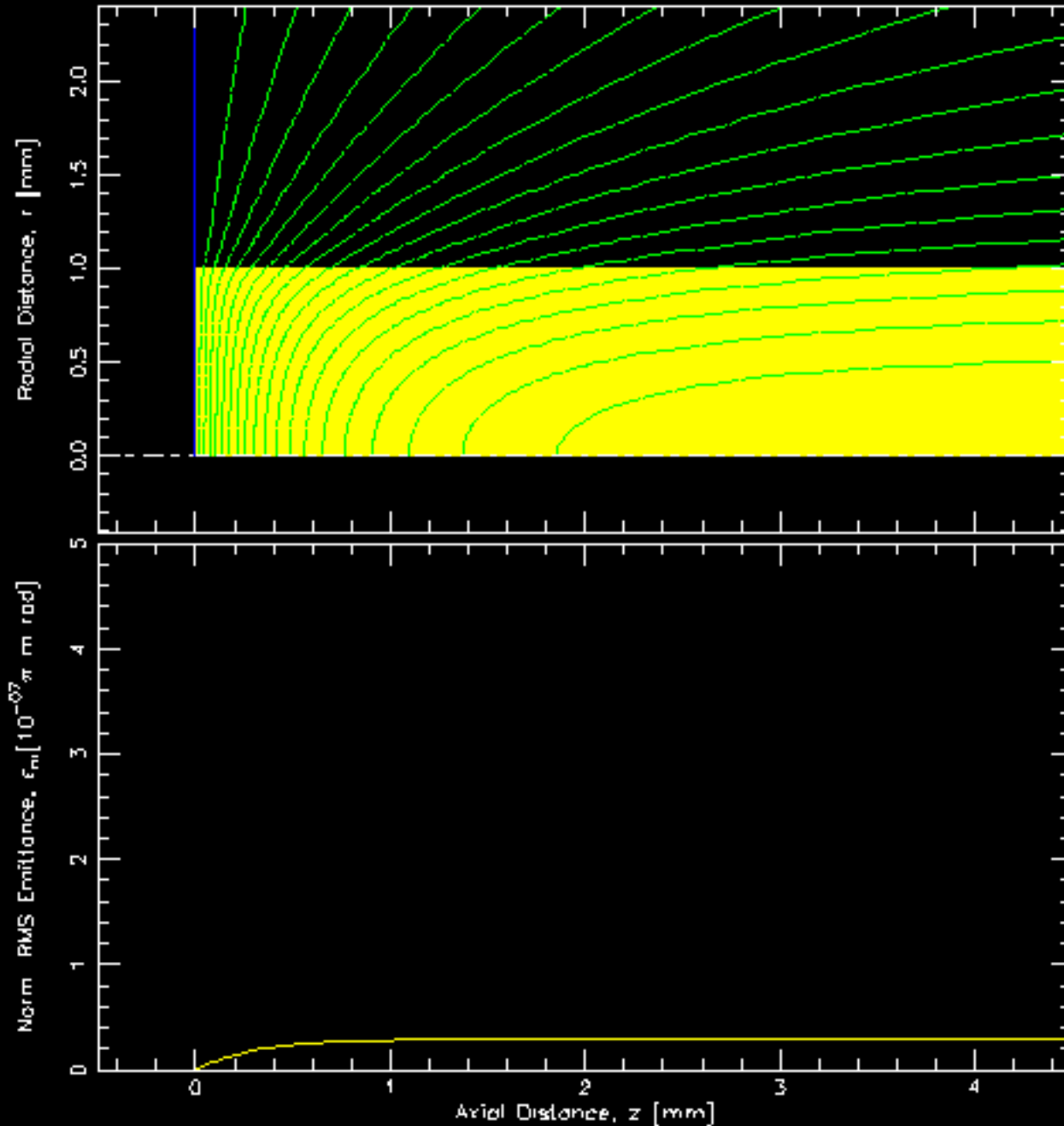
陰極近傍でエミッタンスが急激に増大

鏡像(陰極=dirichlet境界)が無くても,  
ゼロエネルギーから加速するだけで増加

加速せず, dirichlet境界も無ければ, 増えない  
(一様ビーム  $E_r$   $r$ )

加速しなくても, dirichlet境界があると, 増える  
(鏡像によるエミッタンス増)

**10, 20, 40, 80, 160 A/cm<sup>2</sup>**  
**20 MV/m**



KUAD2 v2.30

*dartplot*

Sun Nov 26 14:25:52 2006  
 driffileID, 2mmf010420MV

20<sup>th</sup> iteration

Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

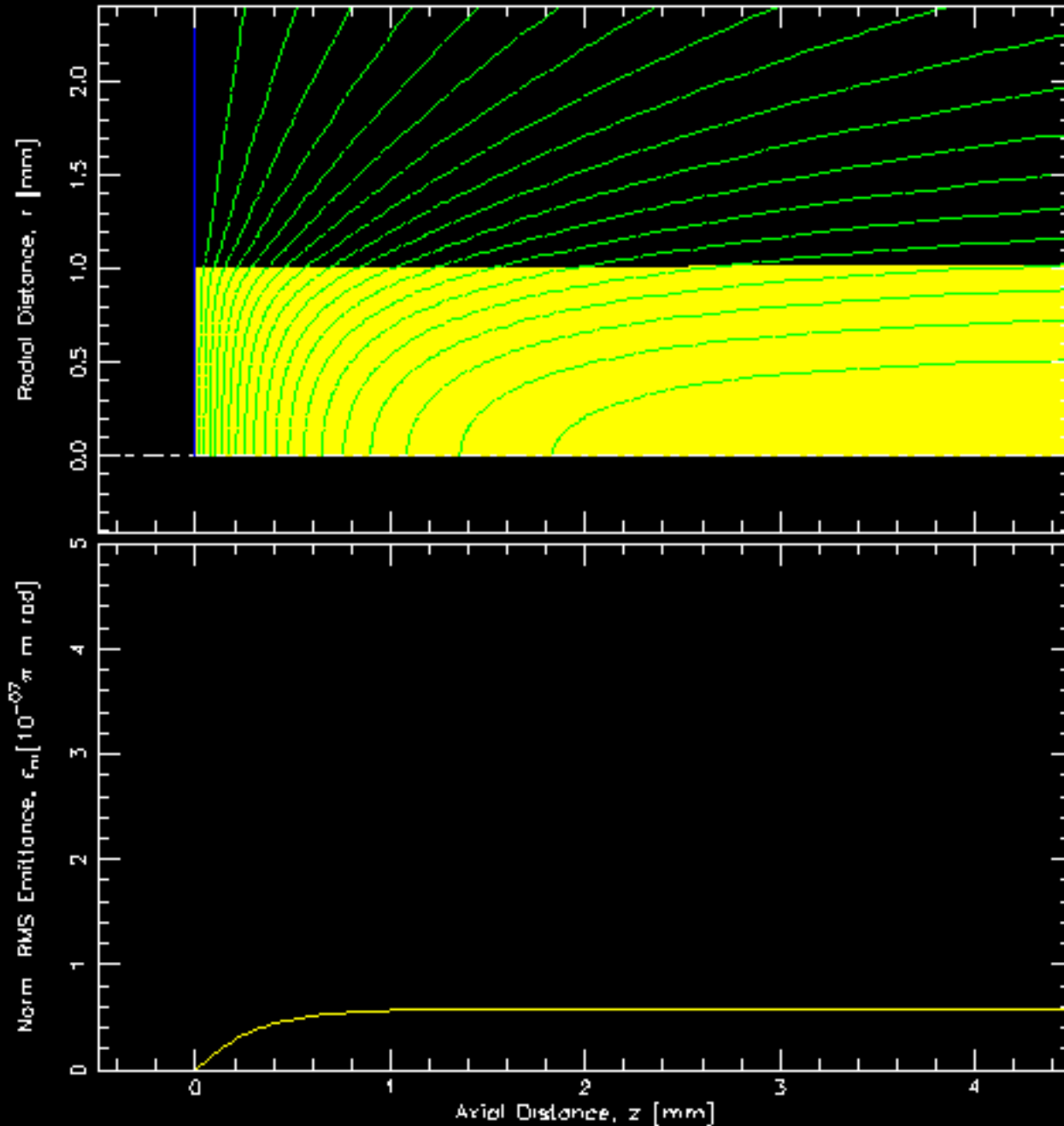
Injection Current

314.159[mA]

Electrodes

1 2

**20 MV/m**  
**10 A/cm<sup>2</sup>**



KUAD2 v2.30

*dartplot*

Sun Nov 26 14:28:21 2006  
 driffileID, 2mmf020420MV

20<sup>th</sup> iteration

Error

- $\phi_b$  - 0.00[%]
- $H_{b,\theta}$  - 0.00[%]
- $A_{b,\theta}$  - 0.00[%]
- $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

Injection Current

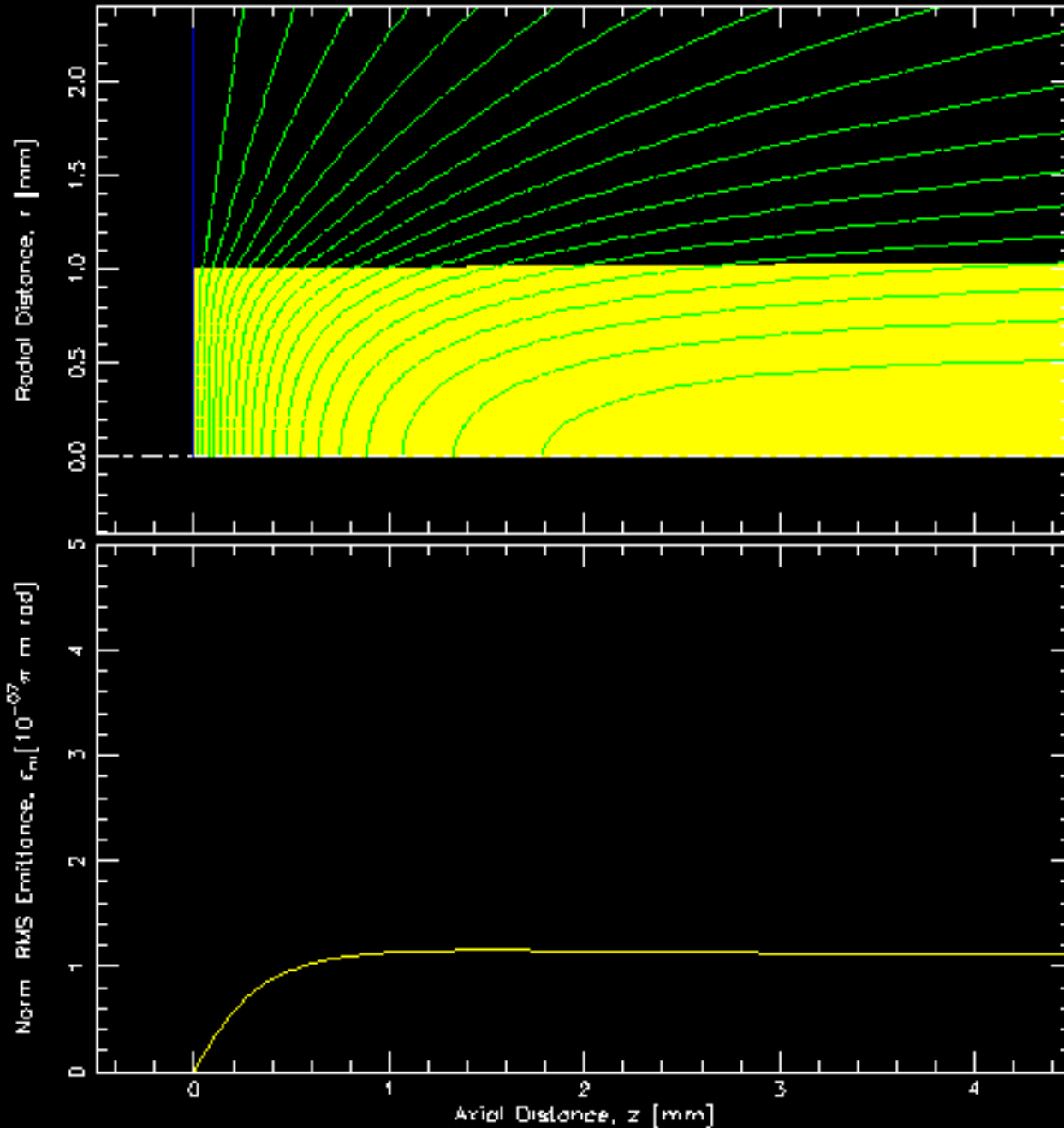
628.319[mA]

Electrodes

1 2

**20 MV/m**  
**20 A/cm<sup>2</sup>**





KUAD2 v2.30

*dartplot*

Sun Nov 26 14:30:39 2006  
 driffileID, 2mmf040420MV

20<sup>th</sup> iteration

Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

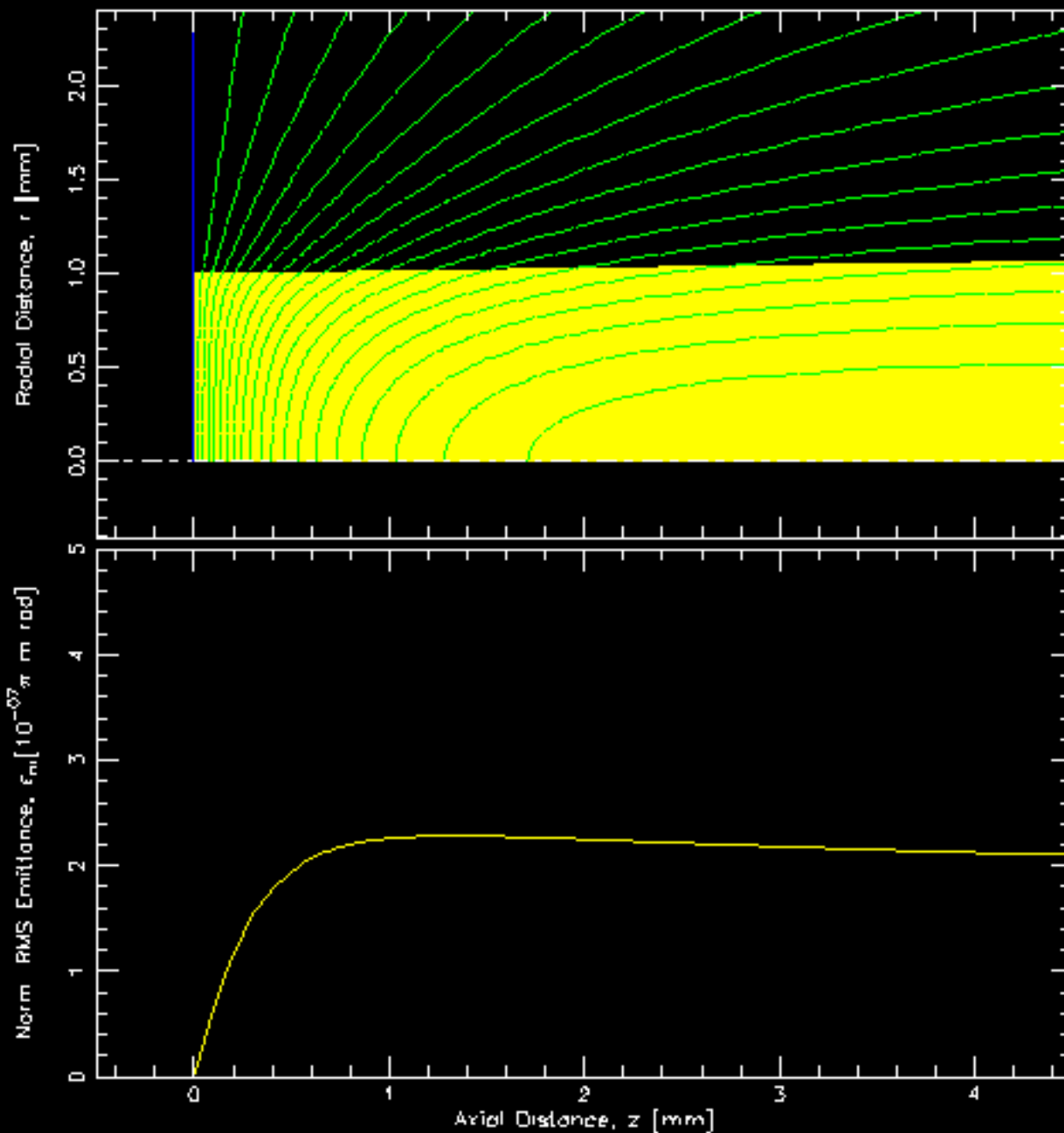
Injection Current

1256.637[mA]

Electrodes

1 2

**20 MV/m**  
**40 A/cm<sup>2</sup>**



KUAD2 v2.30  
*dartplot*

Sun Nov 26 14:32:52 2006  
 driffileID, 2mmf050420MV

20<sup>th</sup> iteration  
 Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

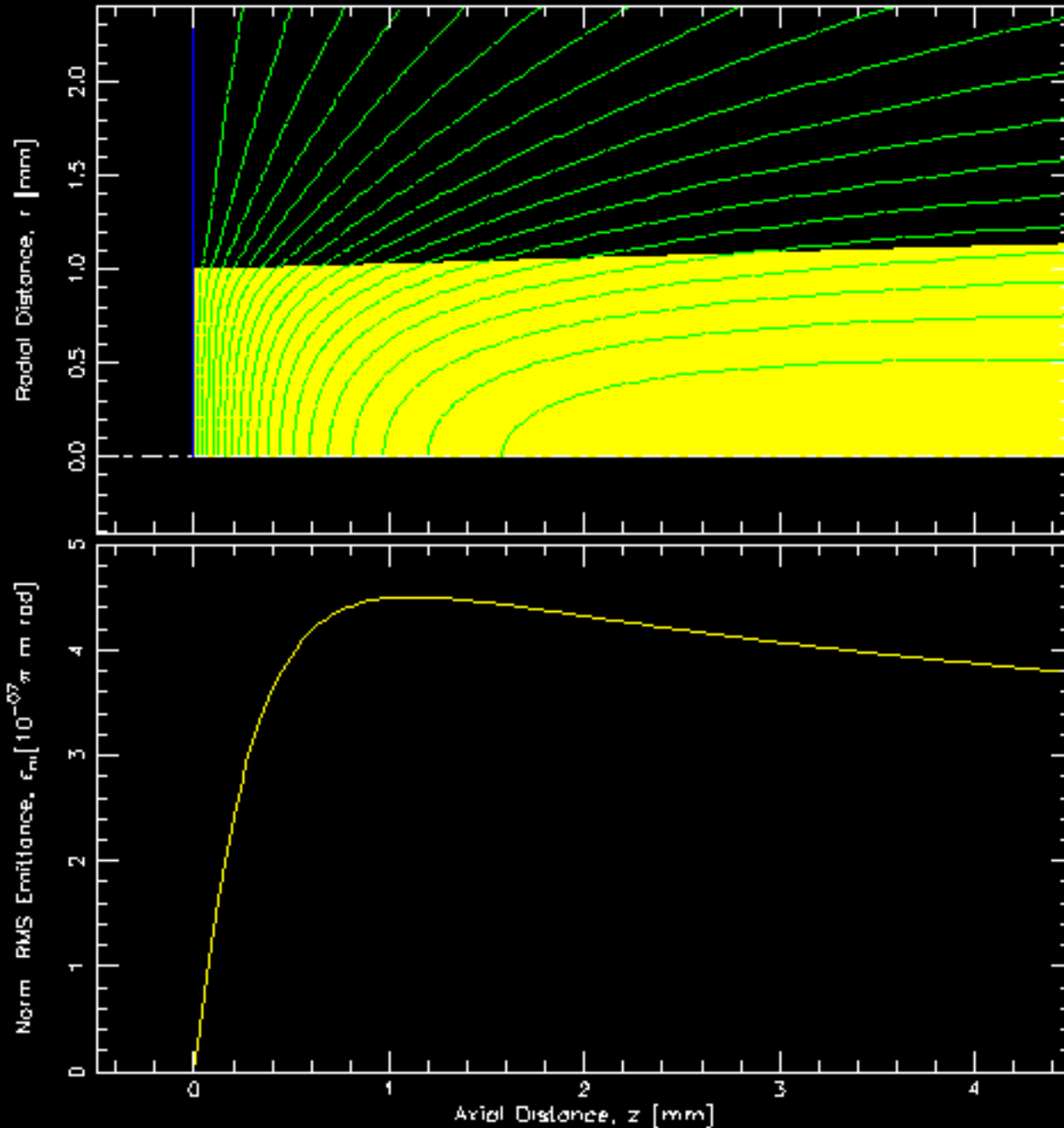
Injection Current

2.513[A]

Electrodes

1 2

**20 MV/m**  
**80 A/cm<sup>2</sup>**



KUAD2 v2.30

*dartplot*

Sun Nov 26 14:35:10 2006  
 driffileID, 2mmf160A20MV

20<sup>th</sup> iteration

Error

- $\phi_b$  - 0.00[%]
- $H_{b,\theta}$  - 0.00[%]
- $A_{b,\theta}$  - 0.00[%]
- $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

Injection Current

5.027[A]

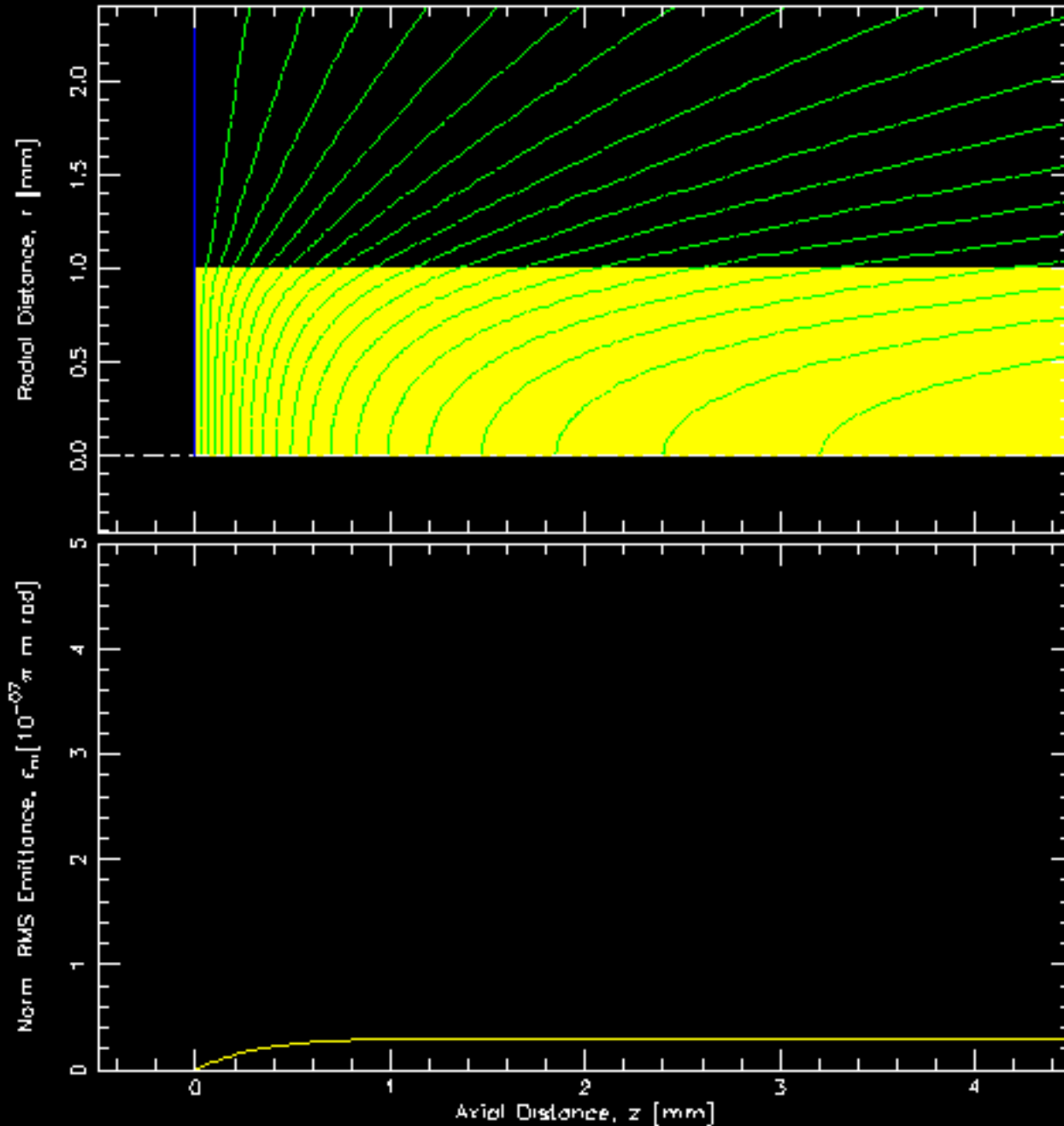
Electrodes

1 2

**20 MV/m**  
**160 A/cm<sup>2</sup>**

**40 A/cm<sup>2</sup>**

**5, 10, 20, 40, 80 MV/m**



KUAD2 v2.30

*dartplot*

Sun Nov 26 14:48:51 2006  
 driffileID, 2mmf0404B0MV

20<sup>th</sup> iteration

Error

- $\phi_b$  - 0.00[%]
- $H_{b,\theta}$  - 0.00[%]
- $A_{b,\theta}$  - 0.00[%]
- $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

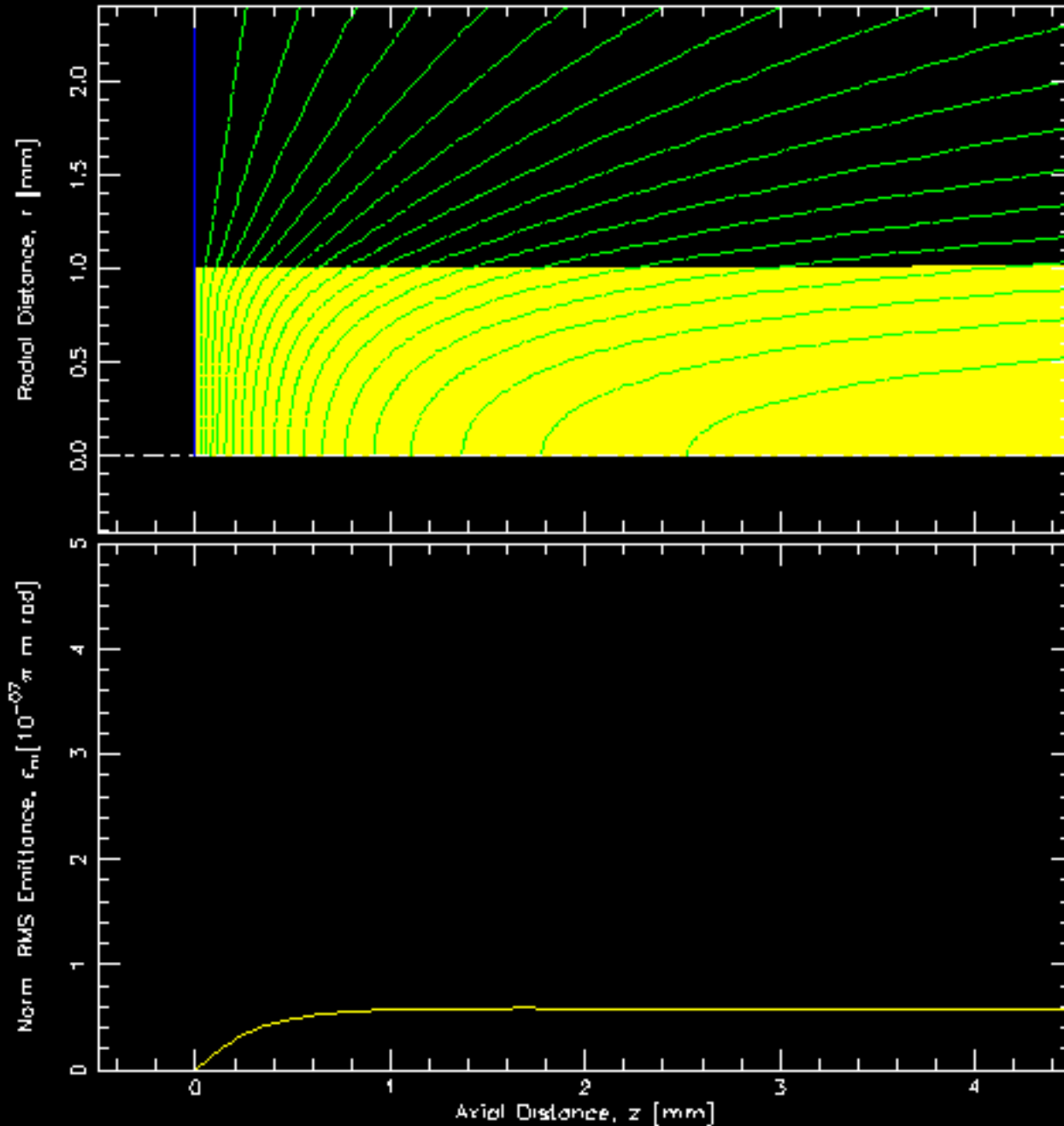
Injection Current

1256.637[mA]

Electrodes

1 2

**80 MV/m**  
**40 A/cm<sup>2</sup>**



KUAD2 v2.30

*dartplot*

Sun Nov 26 14:46:30 2006  
 driffileID, 2mmf040A40MV

20<sup>th</sup> iteration

Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

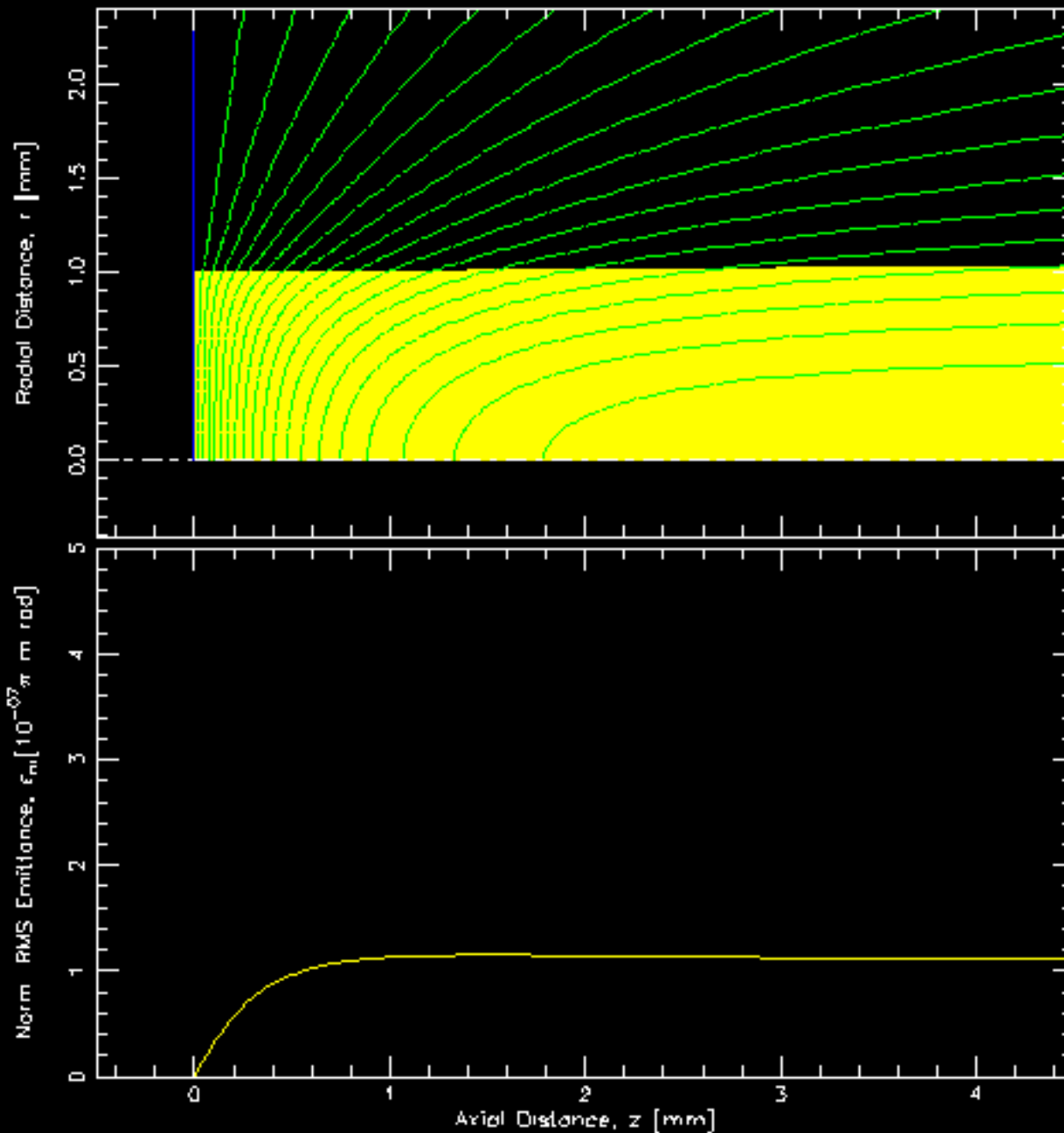
Injection Current

1256.637[mA]

Electrodes

1 2

**40 MV/m**  
**40 A/cm<sup>2</sup>**



KUAD2 v2.30  
*dartplot*

Sun Nov 26 14:43:50 2006  
 driffileID, 2mmf040420MV

20<sup>th</sup> iteration  
 Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

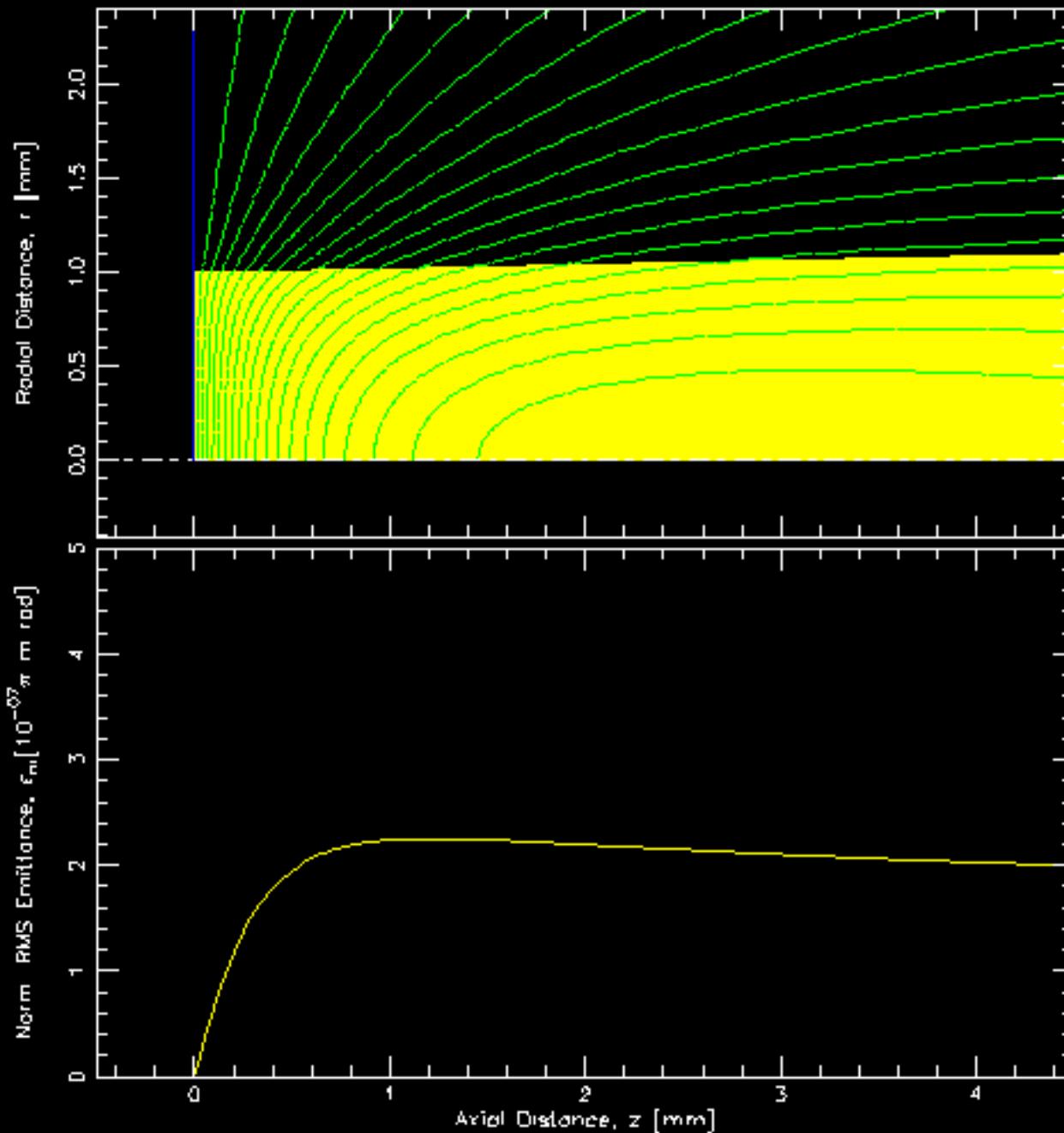
Injection Current

1256.637[mA]

Electrodes

1 2

**20 MV/m**  
**40 A/cm<sup>2</sup>**



KUAD2 v2.30

*dartplot*

Sun Nov 26 14:41:17 2006  
driffileID, 2mmf040A10MV

20<sup>th</sup> iteration

Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

Injection Current

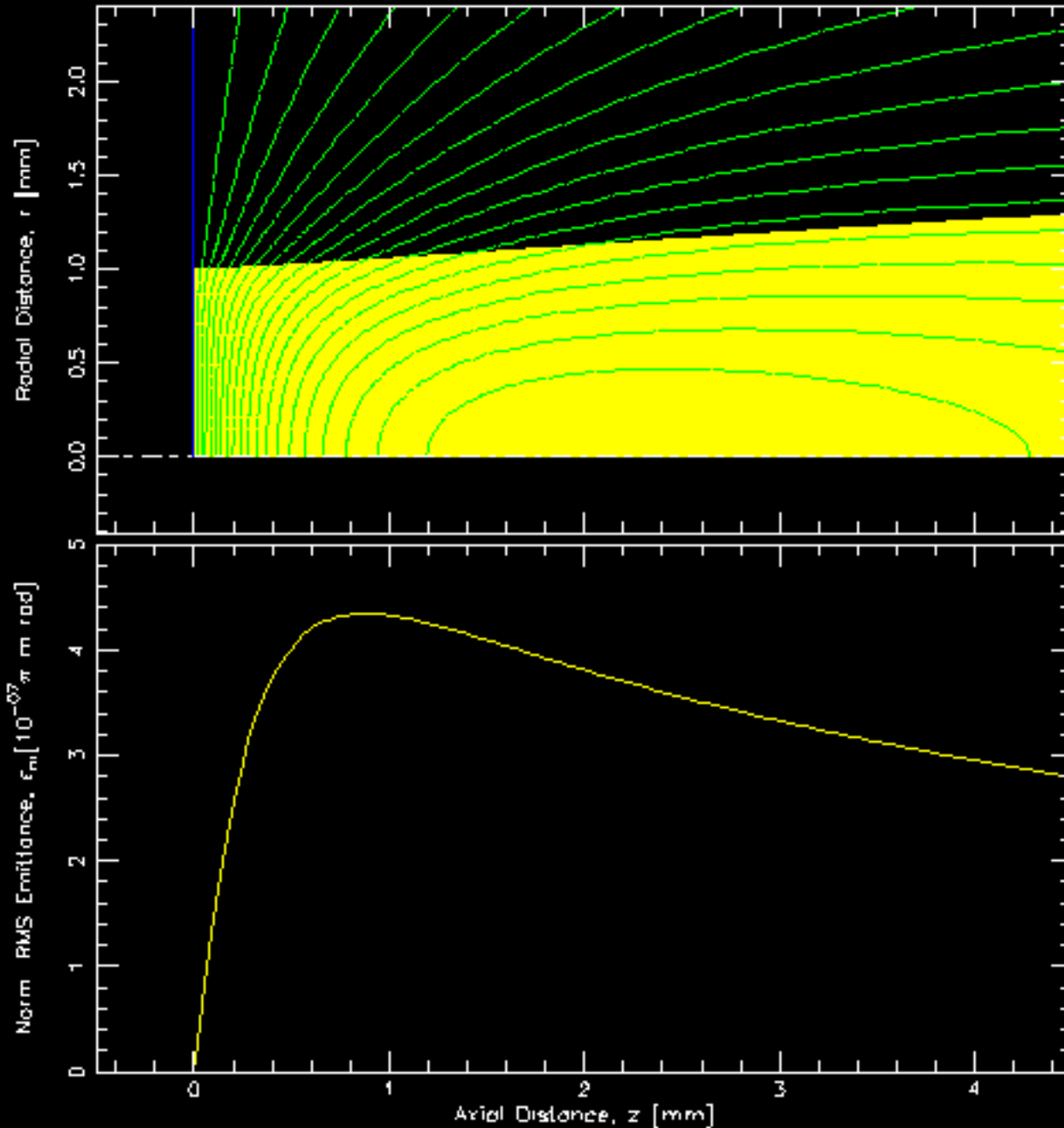
1256.637[mA]

Electrodes

1 2

**10 MV/m**  
**40 A/cm<sup>2</sup>**





KUAD2 v2.30

*dartplot*

Sun Nov 26 14:39:04 2006  
 driffileID, 2mmf0404D5MV

20<sup>th</sup> iteration

Error

- $\phi_b$  - 0.00[%]
- $H_{b,\theta}$  - 0.00[%]
- $A_{b,\theta}$  - 0.00[%]
- $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

Injection Current

1256.637[mA]

Electrodes

1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**

# norm. rms r-emittance @ $z = 1$ mm

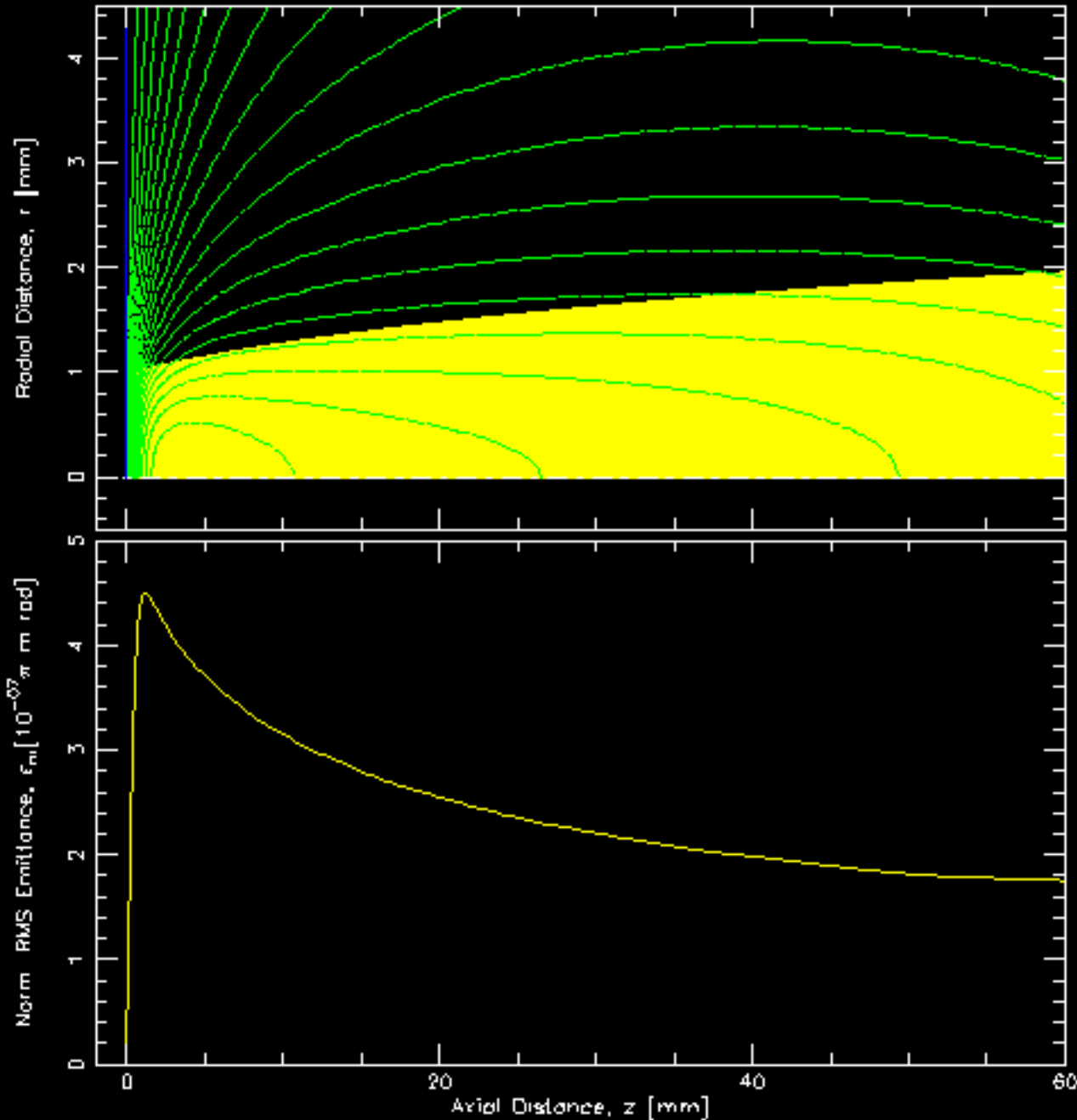
[ $\pi$  mm mrad]

	10 A/cm <sup>2</sup>	20	40	80	160
05 MV/m	0.11	0.22	0.43	S.C.L.	S.C.L.
10	0.056	0.11	0.22	0.44	0.82
20	0.028	0.057	0.11	0.23	0.45
40	0.014	0.029	0.057	0.11	0.23
80	0.0074	0.015	0.029	0.059	0.12

160 A/cm<sup>2</sup>, 2 mm $\phi$ , 20 psec 0.1 nC

陰極近傍 ( $z < 1$  mm) でエミッタンスが急激に増大

(エミッタンス@ $z=1$  mm)      (電流密度) / (電界)



KUAD2 v2.30

*dartplot*

Sun Nov 26 15:12:24 2006  
 driffileID, 2mmf160A20MV

20<sup>th</sup> iteration

Error

- $\phi_b$  - 0.00[%]
- $H_{b,\theta}$  - 0.00[%]
- $A_{b,\theta}$  - 0.00[%]
- $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

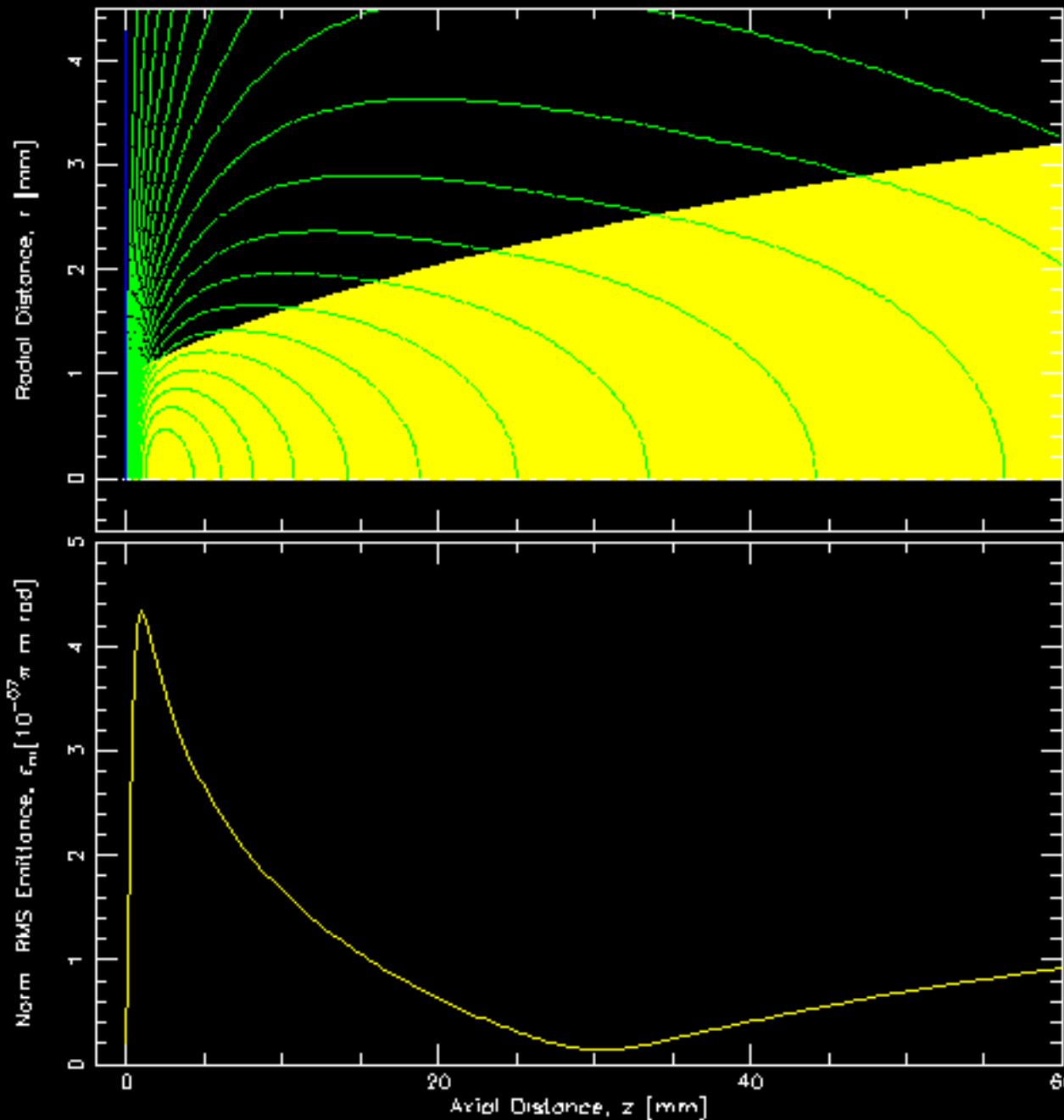
Injection Current

5.027[A]

Electrodes

1 2

**20 MV/m**  
**160 A/cm<sup>2</sup>**



KUAD2 v2.30

*dartplot*

Sun Nov 26 15:25:02 2006  
 driffileID, 2mmf0404D5MV

20<sup>th</sup> iteration

Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

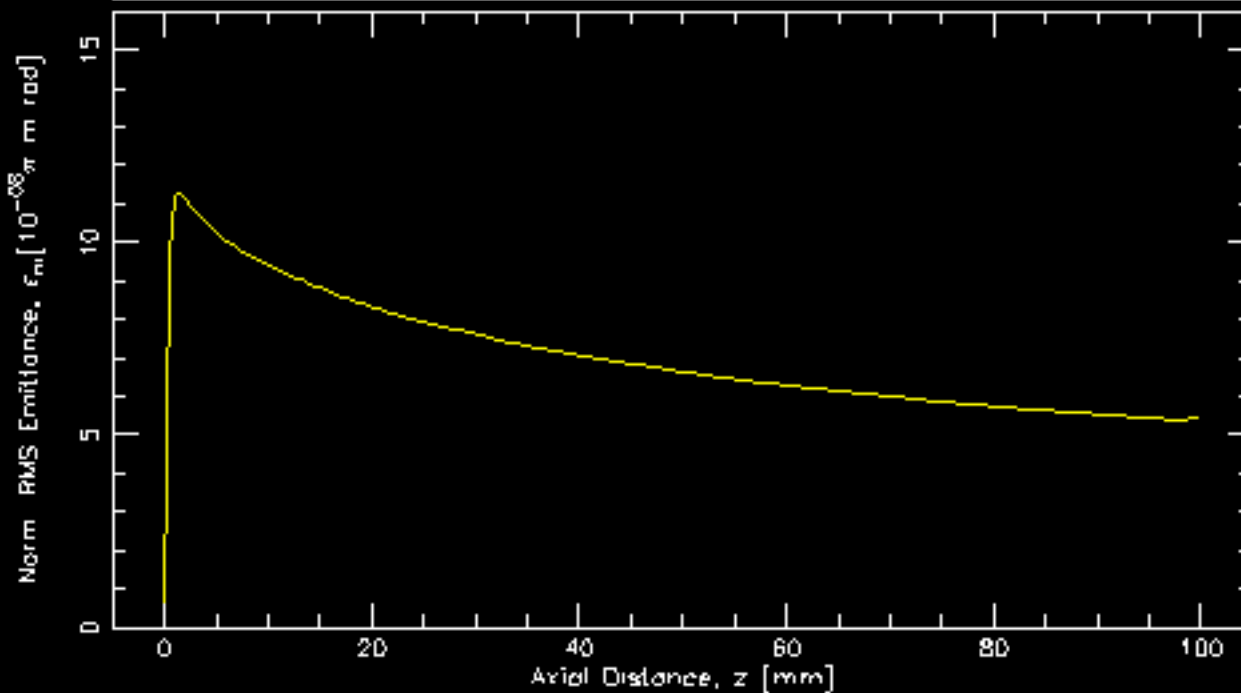
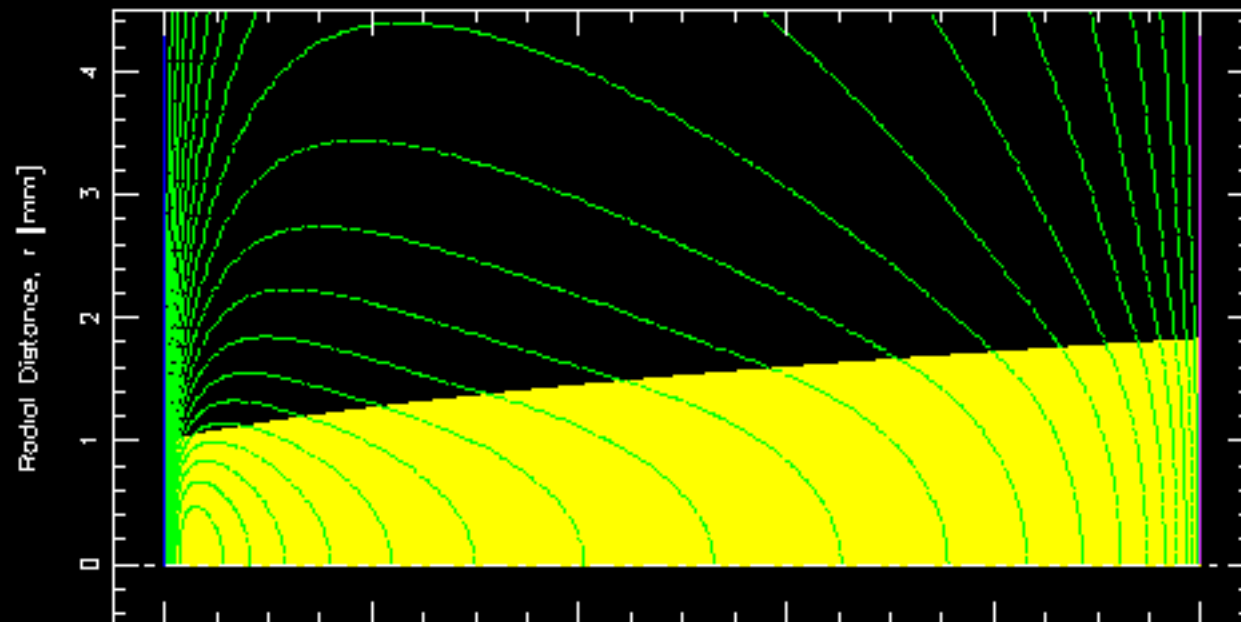
Injection Current

1256.637[mA]

Electrodes

1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**



KUAD2 v2.30

*dartplot*

Thu Nov 30 12:06:01 2006  
 driffileID, 2mmf0104D5MV

20<sup>th</sup> iteration

Error

- $\phi_b$  - 0.00[%]
- $H_{b,\theta}$  - 0.00[%]
- $A_{b,\theta}$  - 0.00[%]
- $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

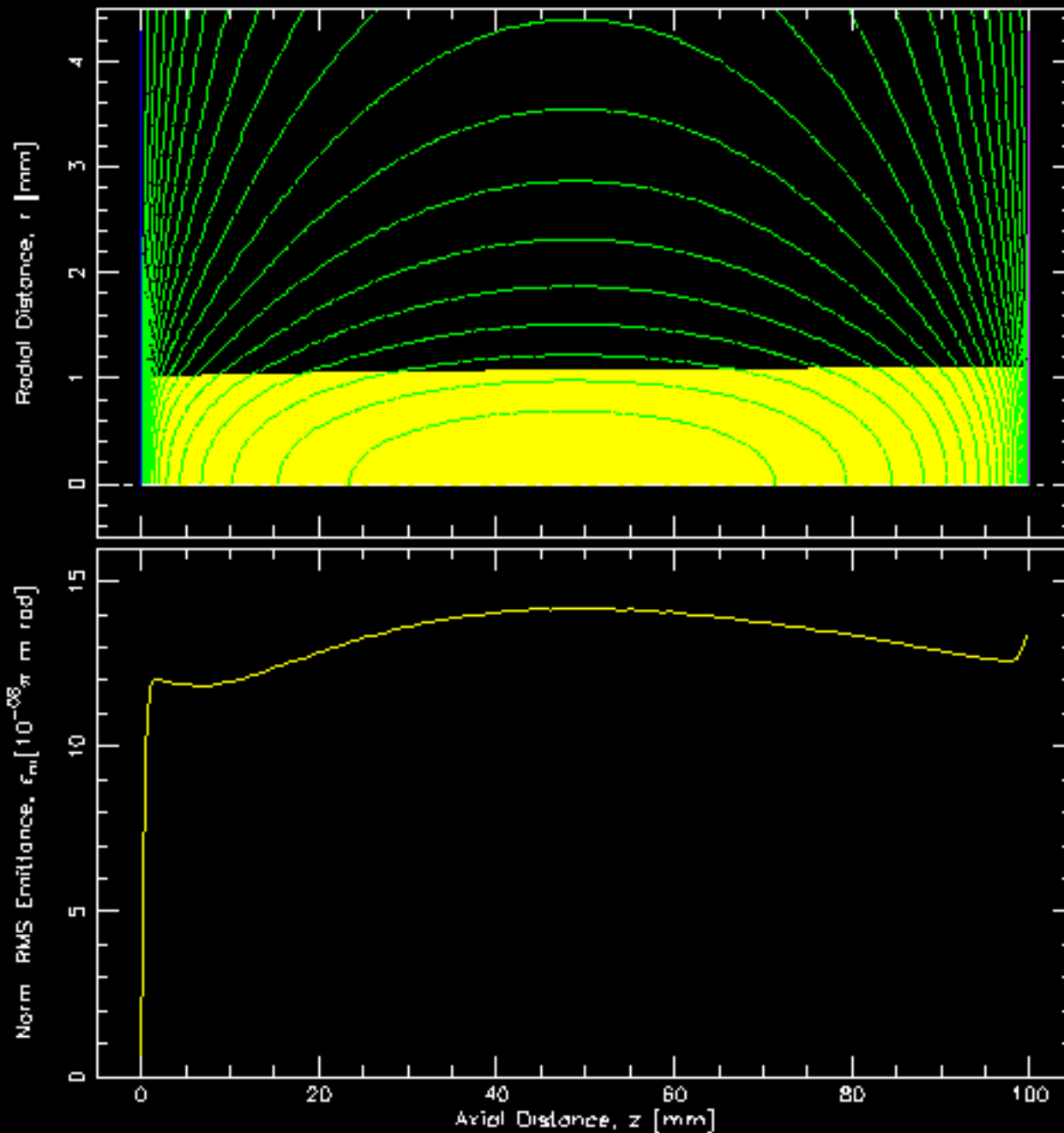
Injection Current

314.159[mA]

Electrodes

1 2

**5 MV/m**  
**10 A/cm<sup>2</sup>**



KUAD2 v2.30

*dartplot*

Thu Nov 30 12:04:09 2006  
driffileID, 2mmf160A60MV

20<sup>th</sup> iteration

Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

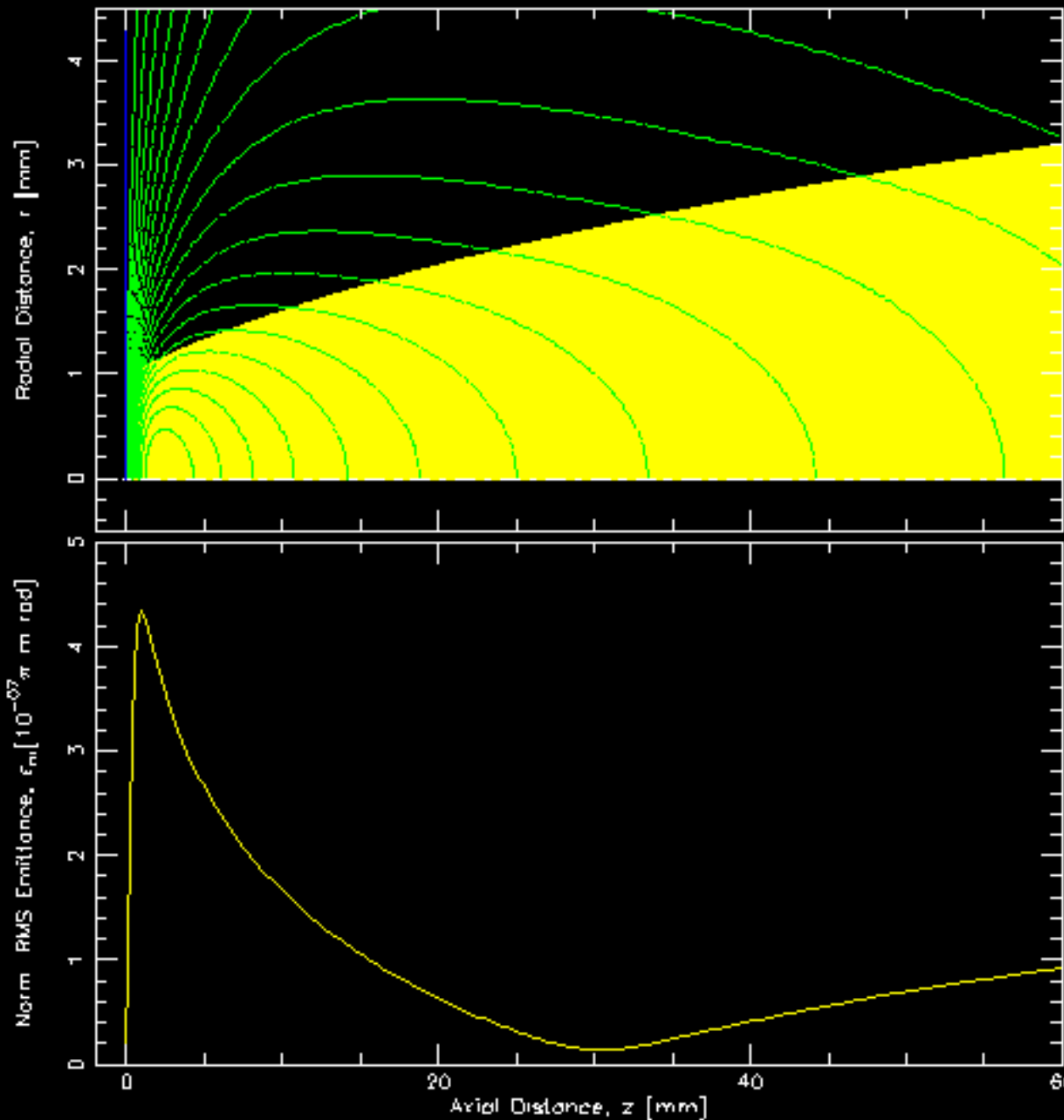
Injection Current

5.027[A]

Electrodes

1 2

**80 MV/m**  
**160 A/cm<sup>2</sup>**



KUAD2 v2.30

*dartplot*

Sun Nov 26 15:25:02 2006  
driffileID, 2mmf0404D5MV

20<sup>th</sup> iteration

Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

Injection Current

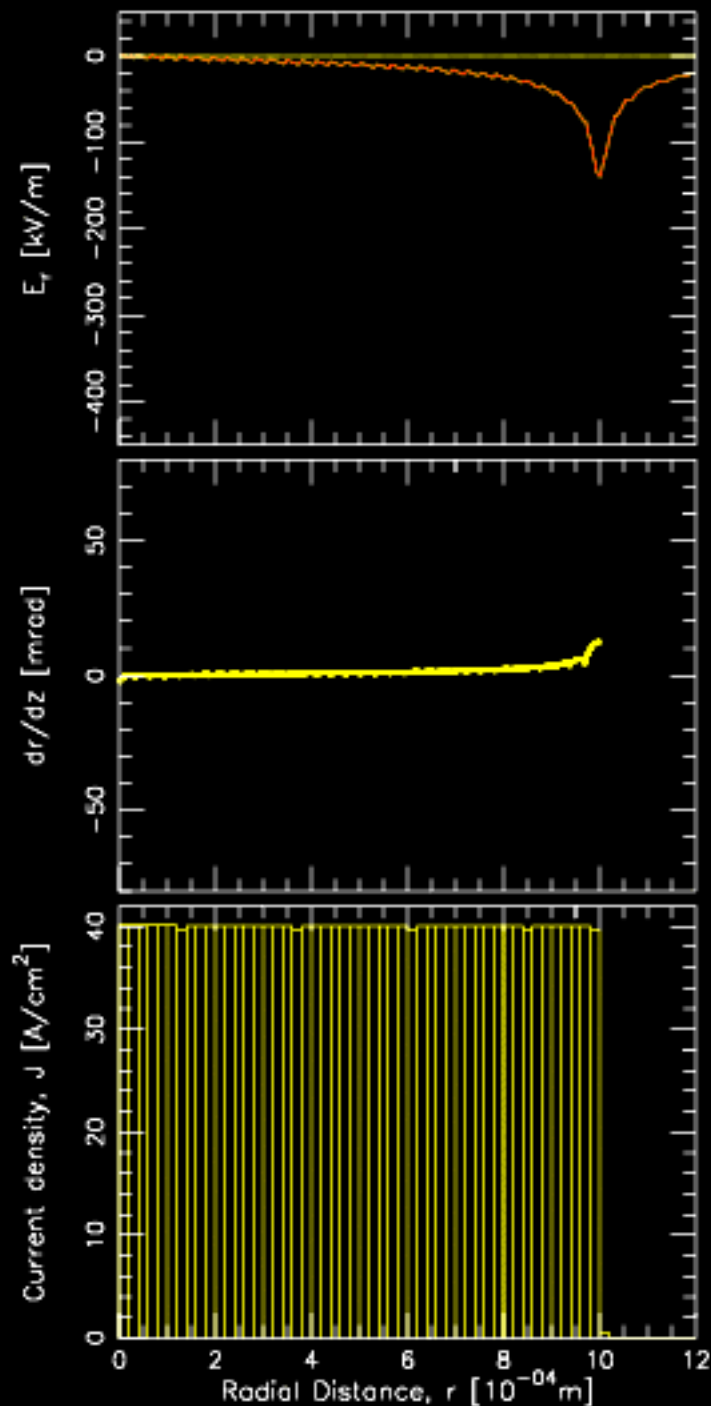
1256.637[mA]

Electrodes

1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**





maximum  $E_{0,r}$   
0.022 [kV/m]

minimum  $E_{0,r}$   
-0.001 [kV/m]

maximum  $E_{b,r}$   
1.489 [kV/m]

minimum  $E_{b,r}$   
-139.526 [kV/m]

maximum  $dr/dz$   
12.411 [mrad]

minimum  $dr/dz$   
-2.439 [mrad]

$\sigma_r$   
3.491 [mrad]

$E_{n,r}$   
2.421 [ $10^{-08} \pi$  m rad]

maximum J  
40.078 [ $A/cm^2$ ]

minimum J  
0.000 [ $A/cm^2$ ]

KUAD2 v2.30

dartprof

Sun Nov 26 17:08:50 2006  
drfileID: 2mmf040AD5MV

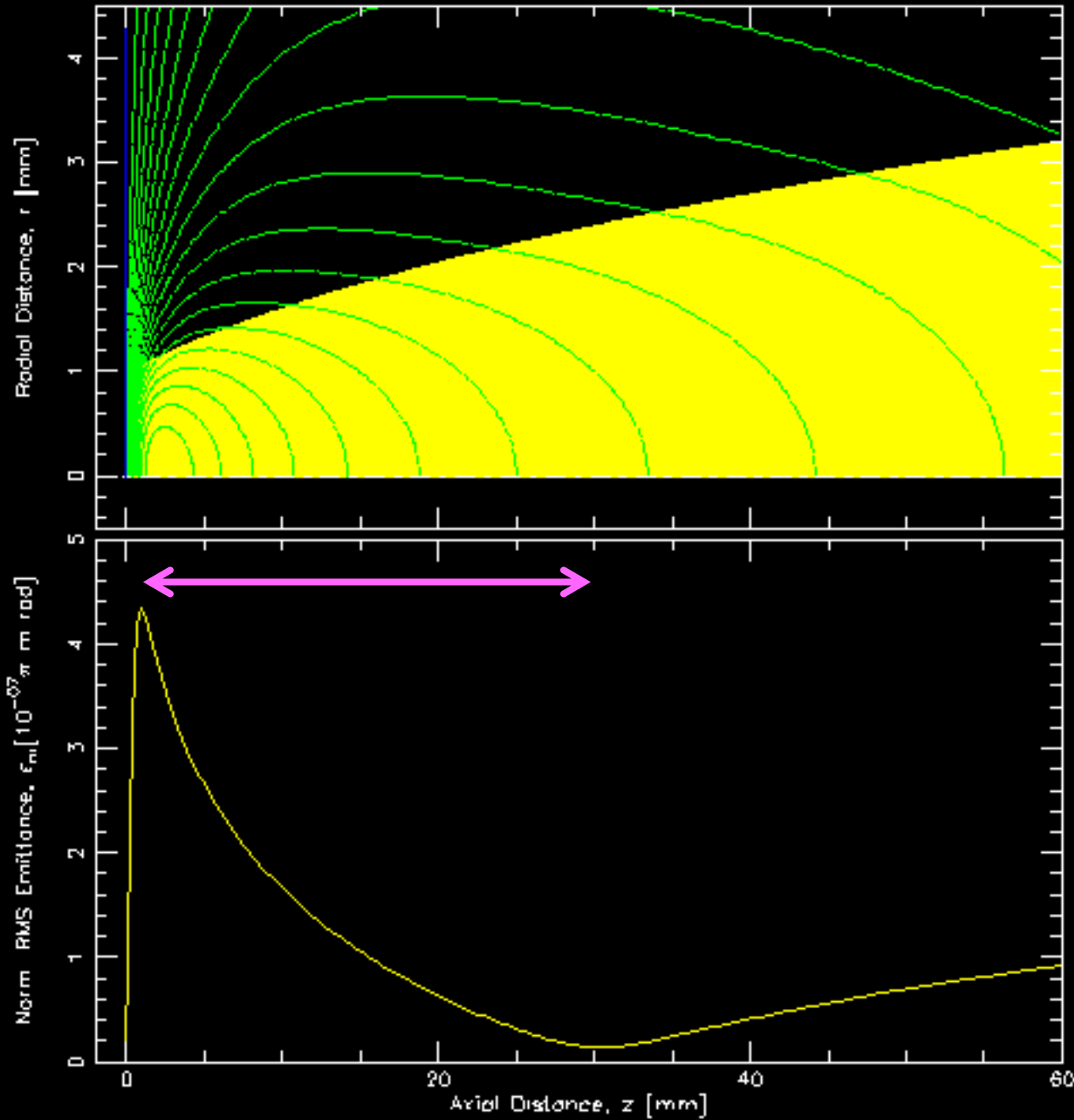
Monitor No. 1  
electron  
forward  
z = 0.020[mm]

Number of Particles  
4096

Current, Beam Radius  
I = 1256.637[mA]  
 $r_{max} = 10.000[10^{-04} m]$   
 $\sigma_r = 7.071[10^{-04} m]$

Classification by Color  
—electron  
—Total Field  
—Vacuum Field  
—Beam-Induced Field

[Link to animation](#)



KUAD2 v2.30

*dartplot*

Sun Nov 26 15:25:02 2006  
 driffileID, 2mmf0404D5MV

20<sup>th</sup> iteration

Error

- $\phi_b$  - 0.00[%]
- $H_{b,\theta}$  - 0.00[%]
- $A_{b,\theta}$  - 0.00[%]
- $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

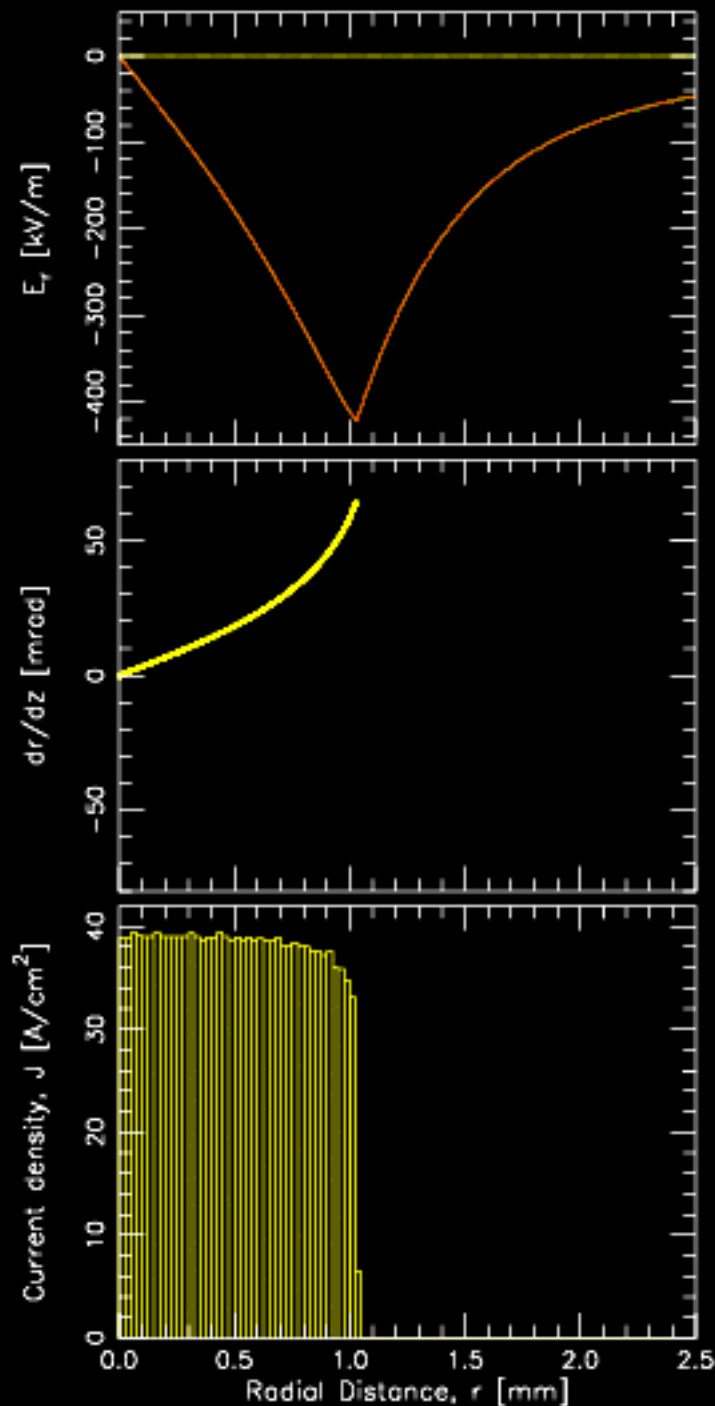
Injection Current

1256.637[mA]

Electrodes

1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**



maximum  $E_{0,r}$   
0.016 [kV/m]

minimum  $E_{0,r}$   
-0.030 [kV/m]

maximum  $E_{b,r}$   
-0.141 [kV/m]

minimum  $E_{b,r}$   
-421.386 [kV/m]

maximum  $dr/dz$   
63.910 [mrad]

minimum  $dr/dz$   
-0.387 [mrad]

$\sigma_r$   
34.759 [mrad]

$\epsilon_{n,r}$   
4.195 [ $10^{-07} \pi$  m rad]

maximum  $J$   
39.389 [A/cm<sup>2</sup>]

minimum  $J$   
0.000 [A/cm<sup>2</sup>]

KUAD2 v2.30

dartprof

Sun Nov 26 17:16:01 2006  
drfileID: 2mmf040AD5MV

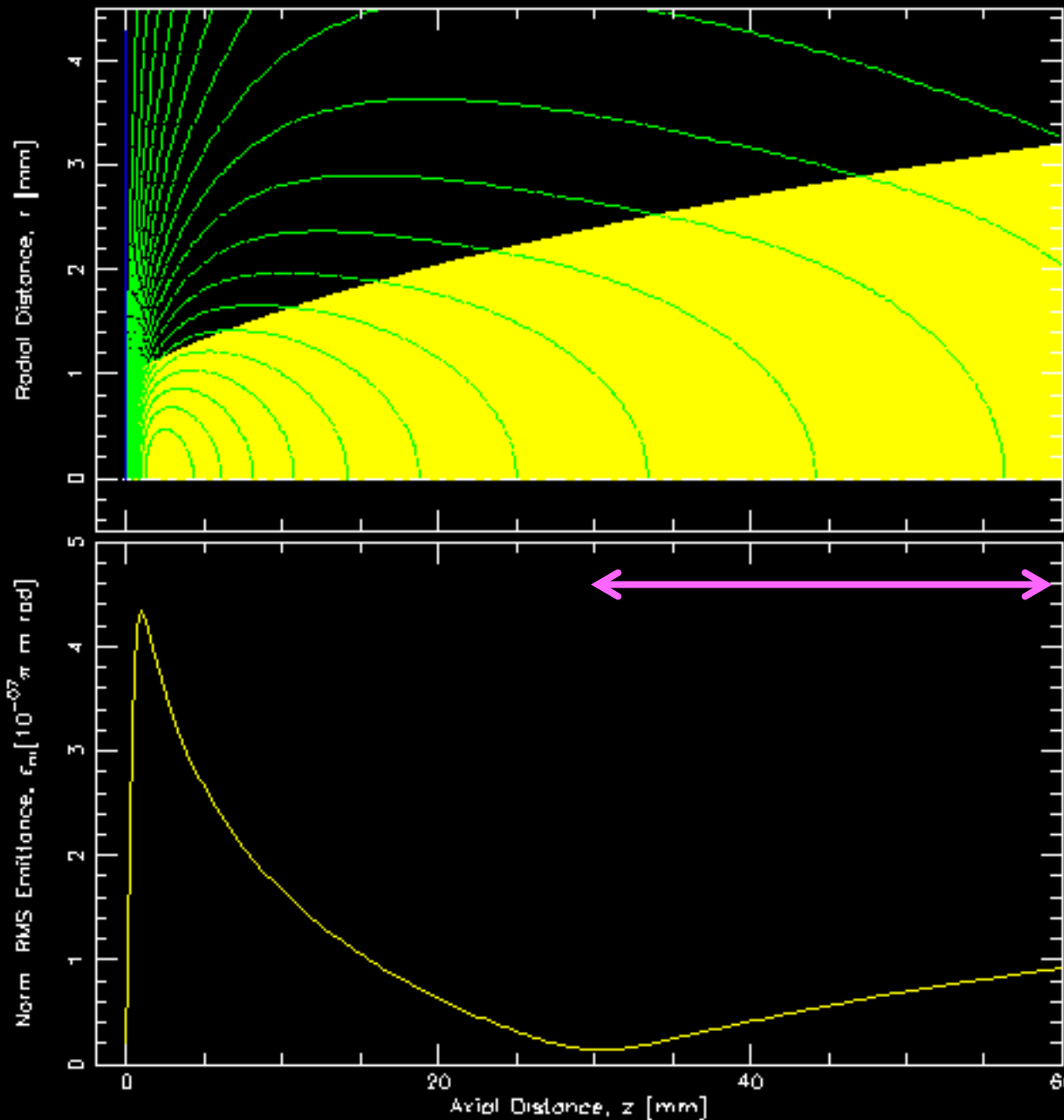
Monitor No. 1  
electron  
forward  
z = 0.600[mm]

Number of Particles  
4096

Current, Beam Radius  
I = 1256.637[mA]  
 $r_{max} = 1.029$ [mm]  
 $\sigma_r = 0.720$ [mm]

Classification by Color  
—electron  
—Total Field  
—Vacuum Field  
—Beam-Induced Field

[Link to animation](#)



KUAD2 v2.30  
dartplot

Sun Nov 26 15:25:02 2006  
driffileID, 2mmf0404D5MV

20<sup>th</sup> iteration  
Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

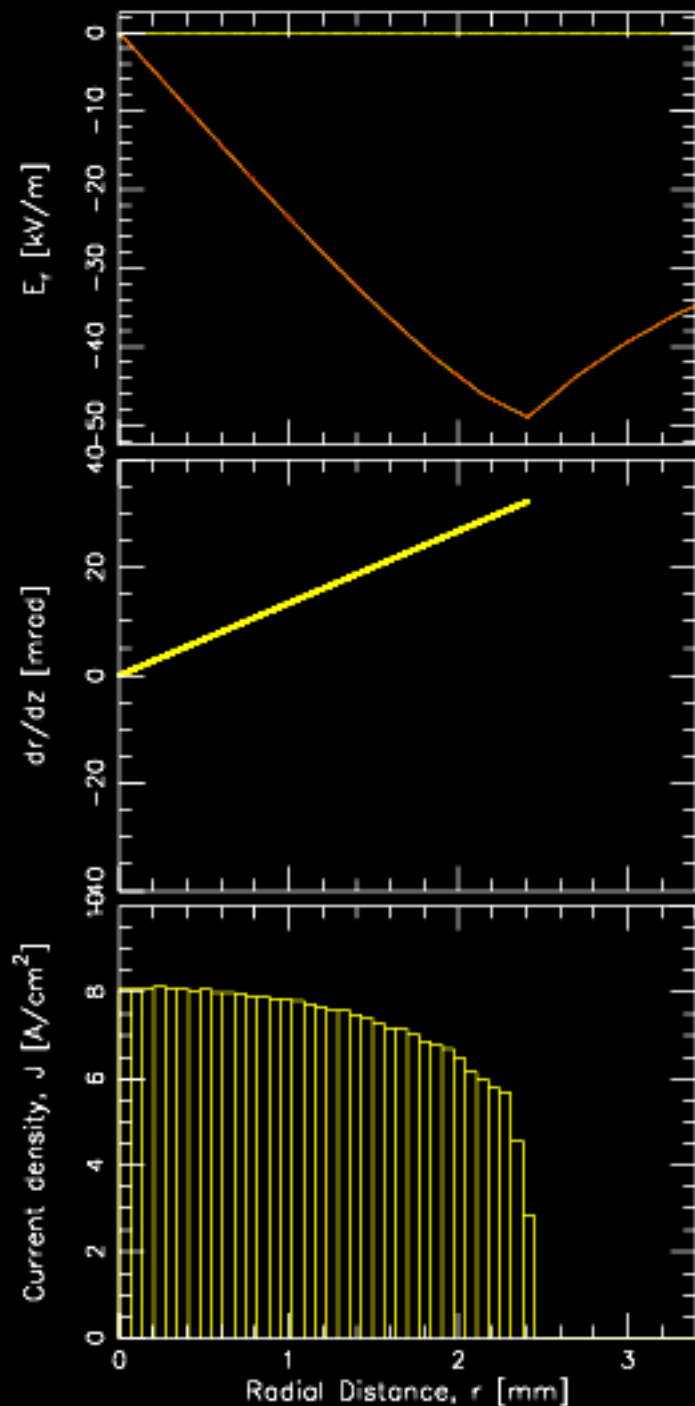
Injection Current

1256.637[mA]

Electrodes

1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**



maximum  $E_{0,r}$   
0.022 [kV/m]

minimum  $E_{0,r}$   
-0.014 [kV/m]

maximum  $E_{b,r}$   
-0.040 [kV/m]

minimum  $E_{b,r}$   
-48.940 [kV/m]

maximum  $dr/dz$   
32.141 [mrad]

minimum  $dr/dz$   
0.005 [mrad]

$\sigma_r$   
21.973 [mrad]

$E_{n,r}$   
13.668 [ $10^{-09} \pi$  m rad]

maximum J  
8.130 [A/cm<sup>2</sup>]

minimum J  
0.000 [A/cm<sup>2</sup>]

**KUAD2 v2.30**

*dartprof*

Sun Nov 26 16:45:36 2006  
drfileID: 2mmf040AD5MV

Monitor No. 1  
electron  
forward  
z = 30.600[mm]

Number of Particles  
4096

Current, Beam Radius  
I = 1256.637[mA]  
 $r_{\max} = 2.409$ [mm]  
 $\sigma_r = 1.642$ [mm]

Classification by Color  
—electron  
—Total Field  
—Vacuum Field  
—Beam-Induced Field

[Link to animation](#)

陰極近傍 ( $z < 1$  mm) の空間電荷効果で、

電流密度分布 が 一様ではなくなり...

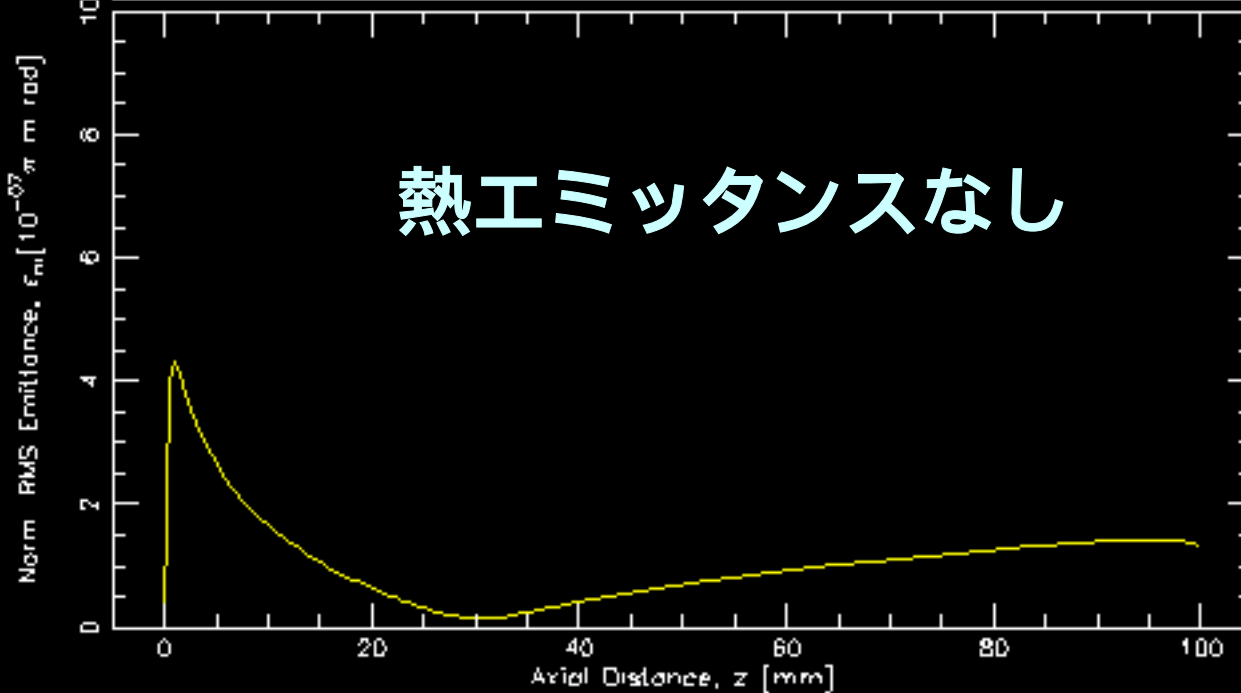
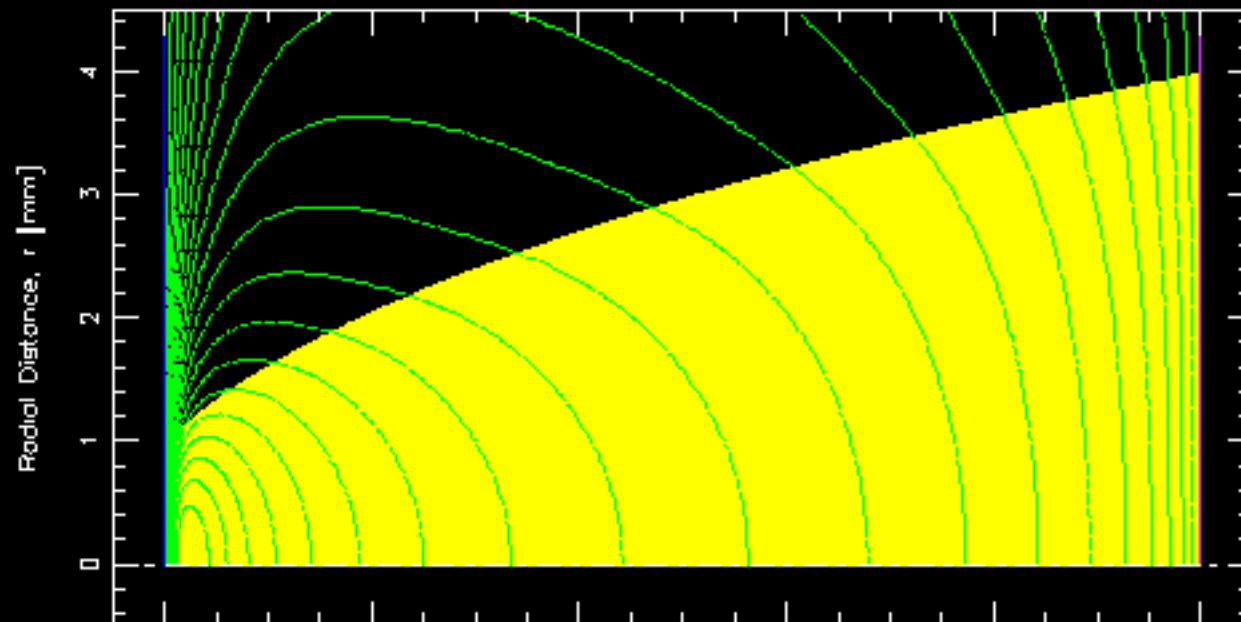
陰極を離れた後の エミッタンスの増減 に影響

1. コードの概要

2. DCビームへの鏡像の影響

3. DCビームへの熱エミッタンスの影響

4. パルスビームへの鏡像の影響



熱エミッタンスなし

KUAD2 v2.31  
dartplot

Wed Dec 6 18:27:14 2006  
driffileID, 2mmf0404D5MV

20<sup>th</sup> iteration  
Error

$\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.00[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

Injection Current

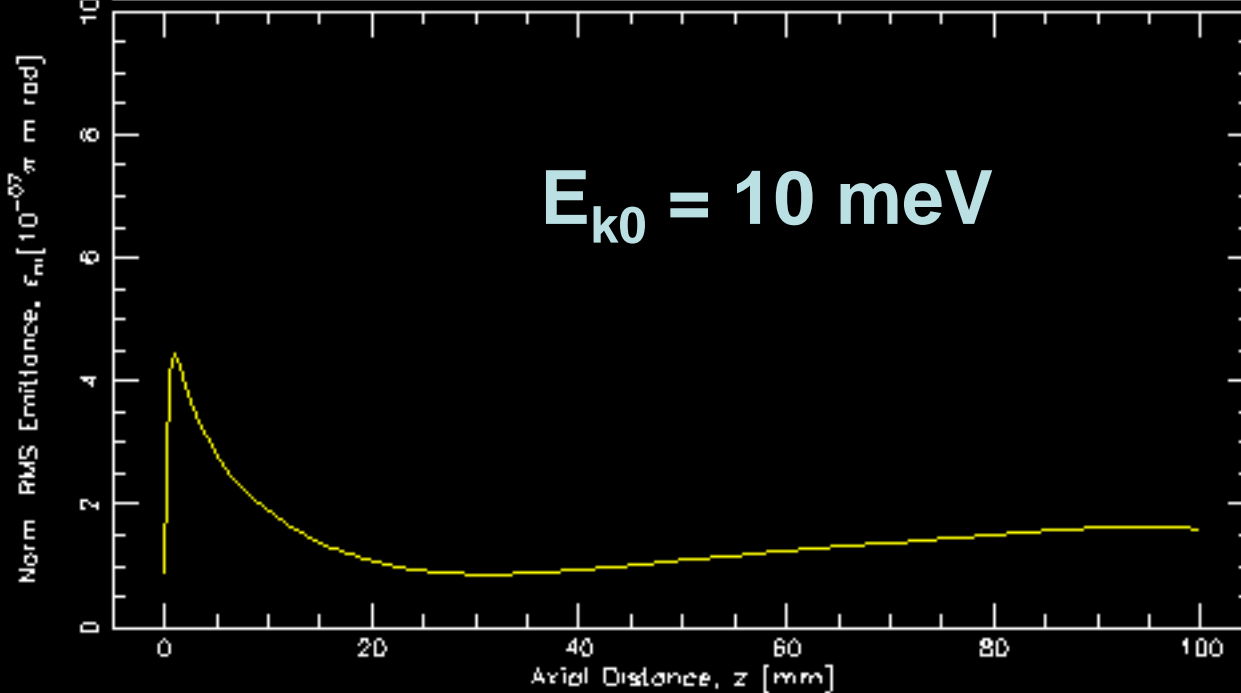
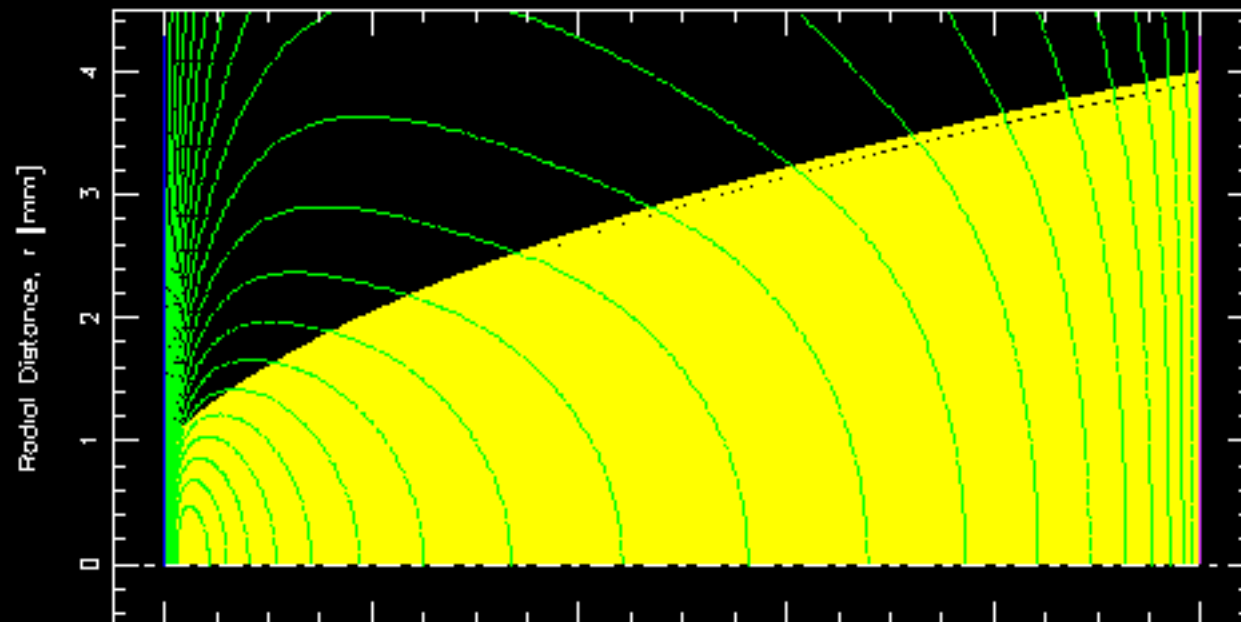
1256.637[mA]

Electrodes

1 2

5 MV/m  
40 A/cm<sup>2</sup>





KUAD2 v2.31  
dartplot

Thu Nov 30 16:10:17 2006  
driffileID, 2mmf0404D5mVo

17<sup>th</sup> iteration

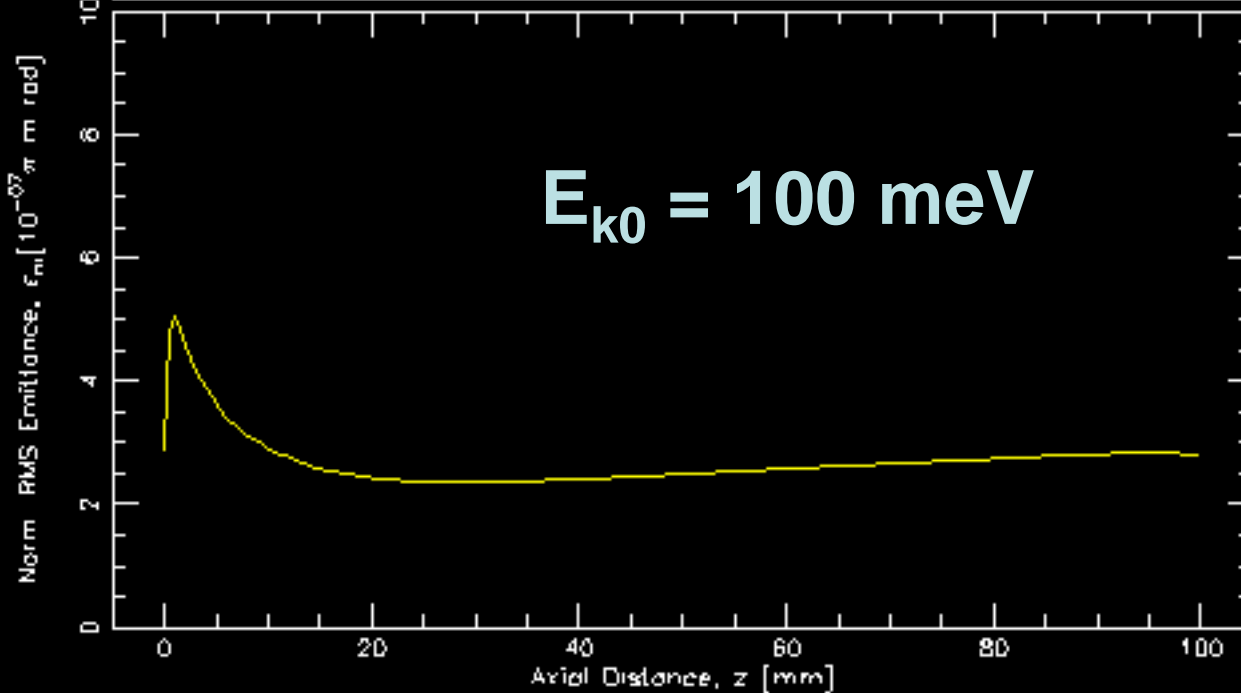
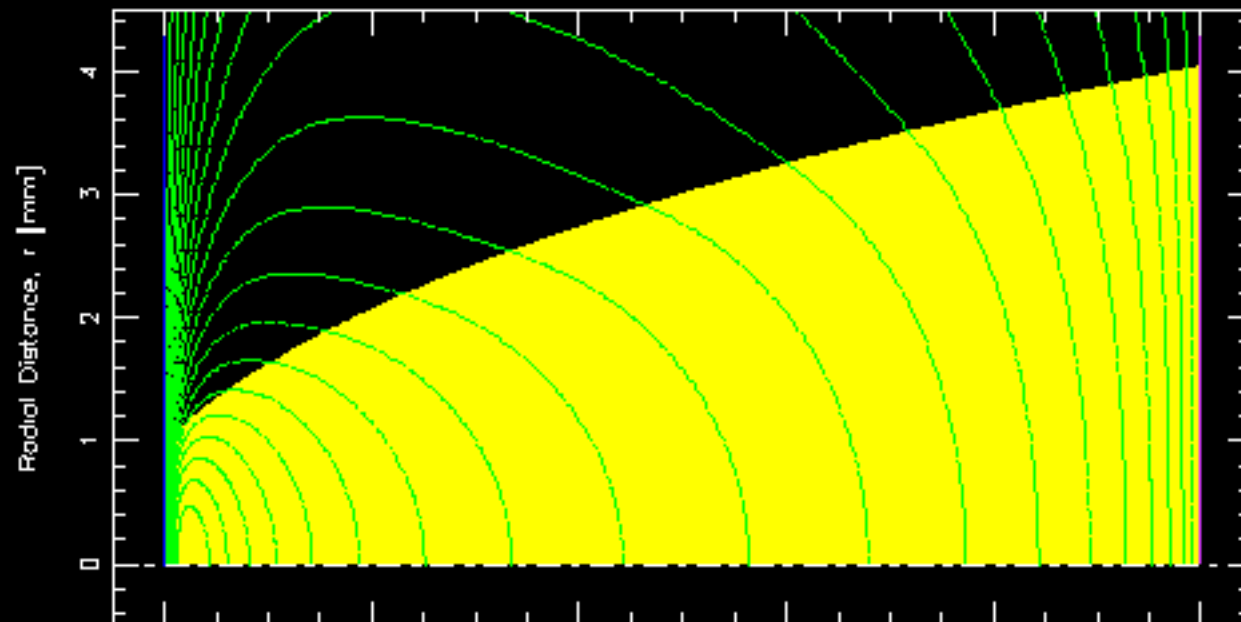
Error  
 $\phi_b$  - 0.00[%]  
 $H_{b,\theta}$  - 0.42[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1  
 Contour of  $\phi_b$   
 Classification by Color

— electron  
 Injection Current  
 1256.637[mA]

Electrodes  
 1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**



KUAD2 v2.31  
*dartplot*

Thu Nov 30 16:13:17 2006  
 driffileID, 2mmf0404D5mVb

g<sup>th</sup> iteration

Error  
 $\phi_b$  - 0.21[%]  
 $H_{b,\theta}$  - 0.81[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

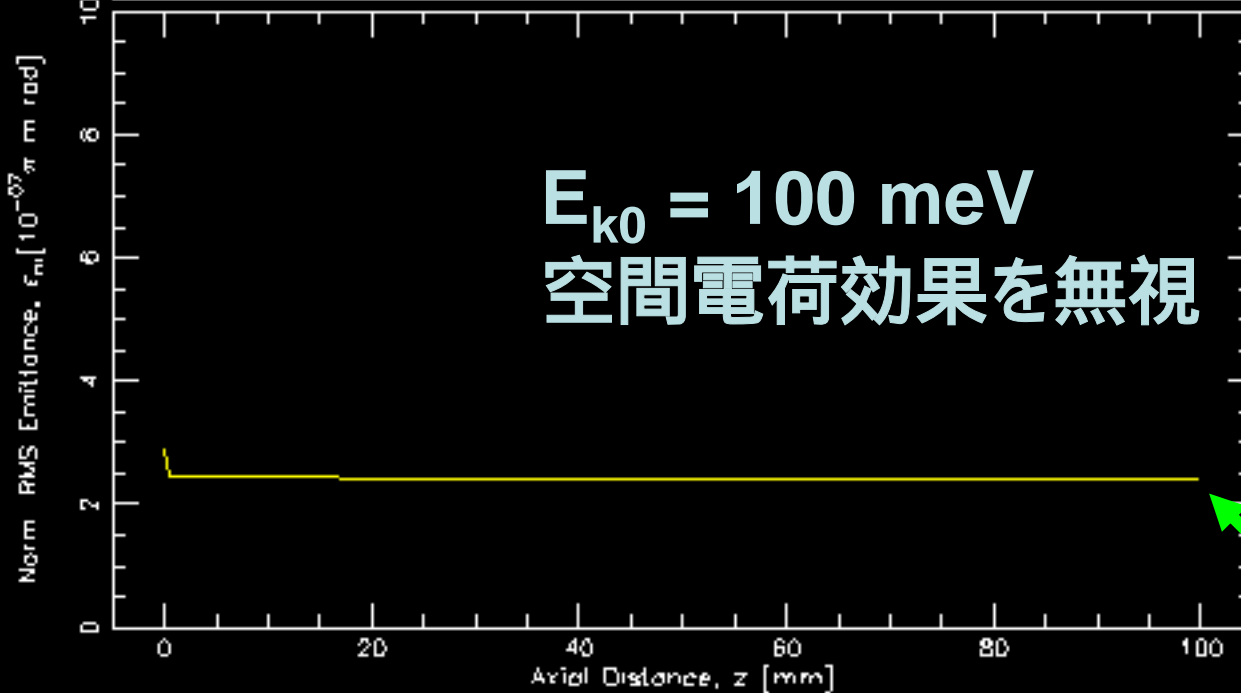
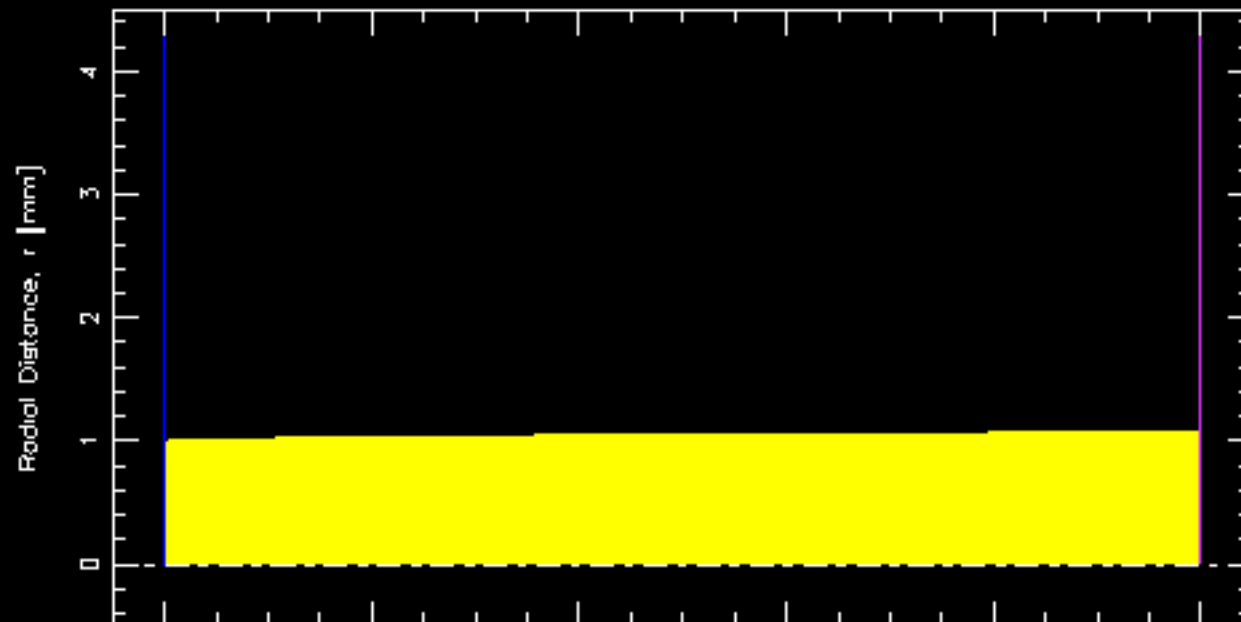
Injection Current

1256.637[mA]

Electrodes

1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**



$E_{k0} = 100$  meV  
空間電荷効果を無視

KUAD2 v2.31  
*dartplot*

Thu Nov 30 16:47:20 2006  
driffileID, 2mmf0404D5mVb2

$\phi^{th}$  iteration

Error

- $\phi_b$  - 999.99[%]
- $H_{b,\theta}$  - 999.99[%]
- $A_{b,\theta}$  - 999.99[%]
- $I_{int}$  - 999.99[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

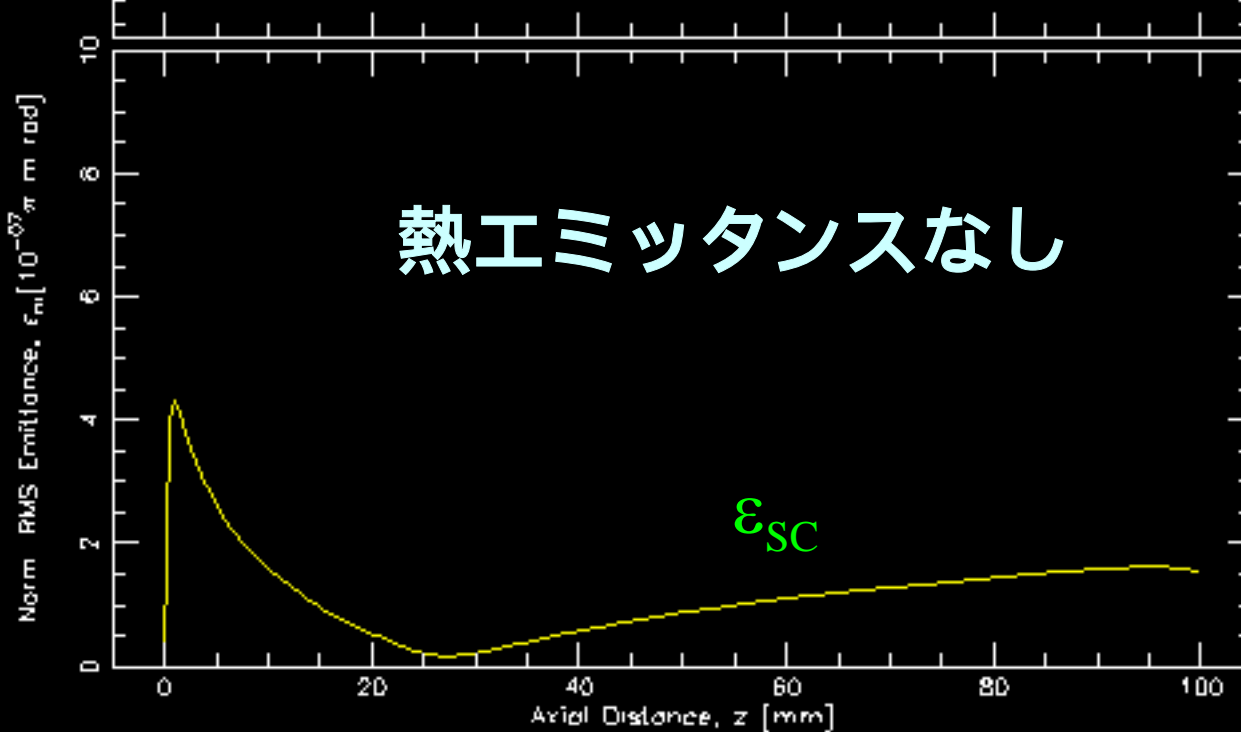
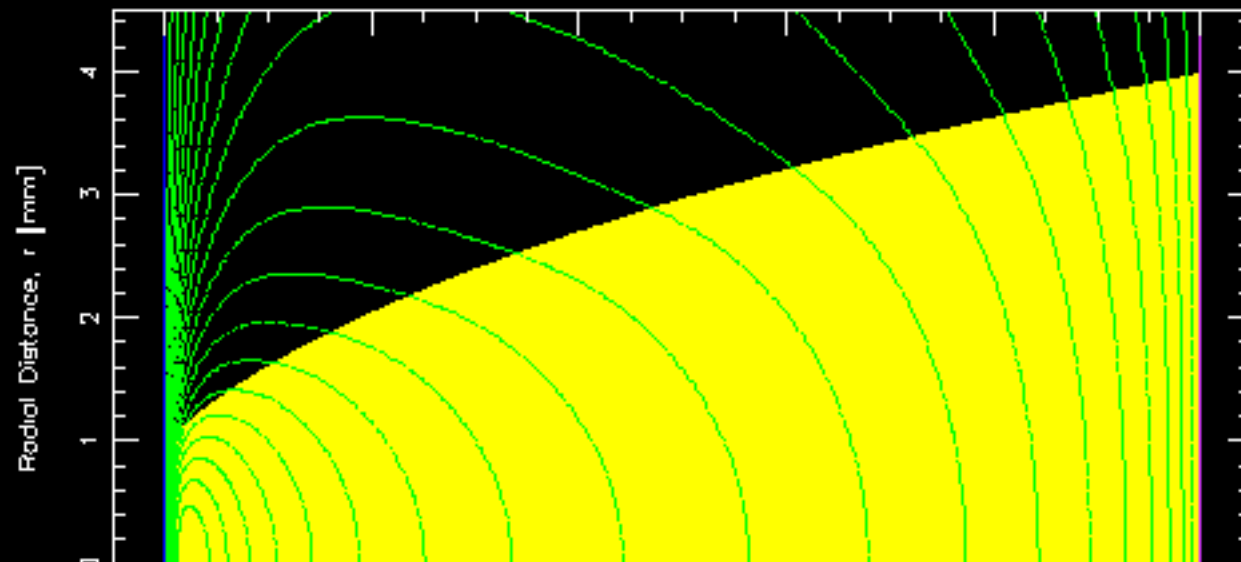
Injection Current

Electrodes

- 1
- 2

5 MV/m  
40 A/cm<sup>2</sup>

$\epsilon_{th} \equiv 0.24$



熱エミッタンスなし

$\epsilon_{SC}$

KUAD2 v2.31  
dartplot

Thu Nov 30 17:02:18 2006  
driffileID, 2mmf0404D5mVh

9<sup>th</sup> iteration

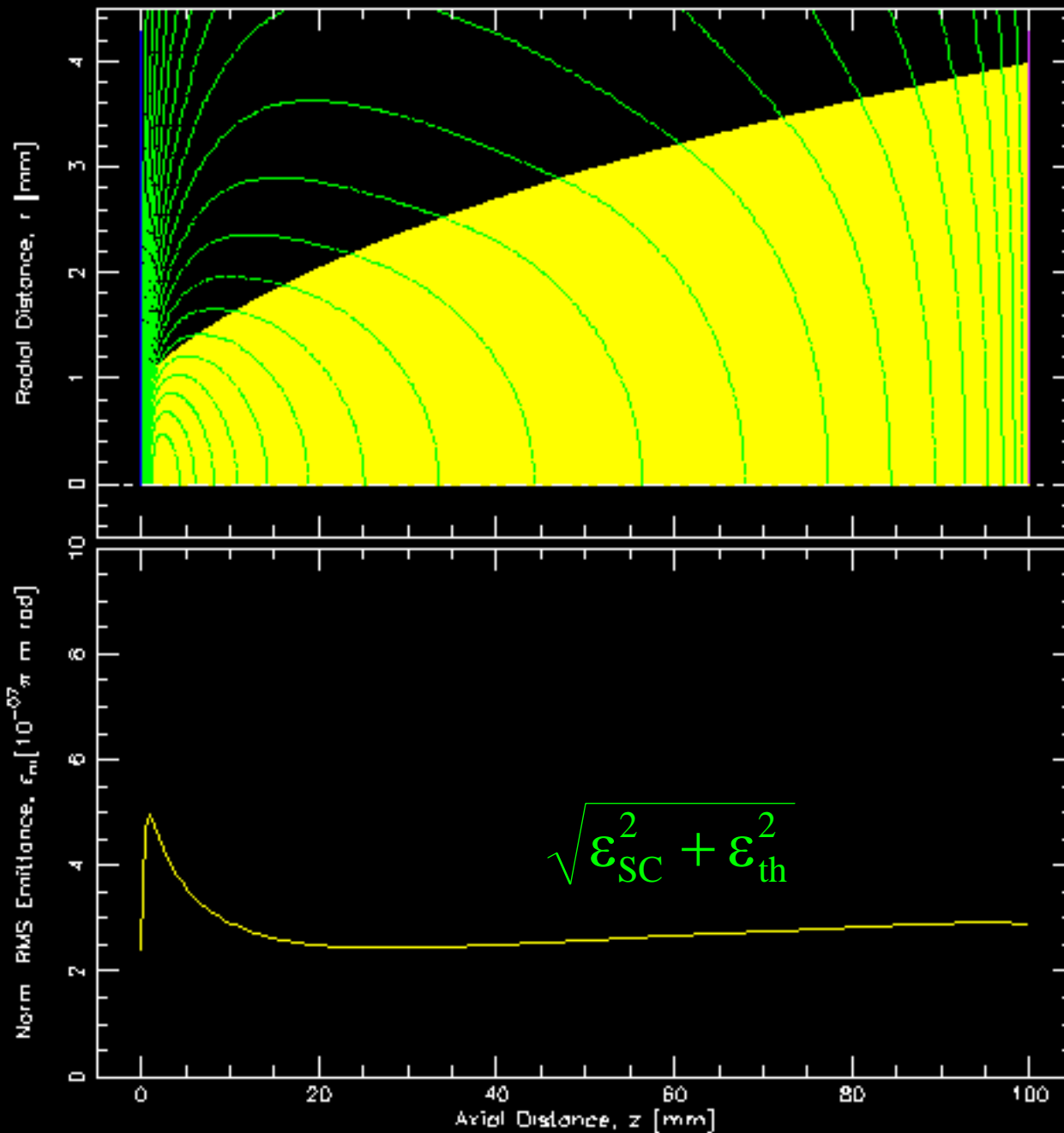
Error  
 $\phi_b$  - 0.21[%]  
 $H_{b,\theta}$  - 0.77[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1  
 Contour of  $\phi_b$   
 Classification by Color

— electron  
 Injection Current  
 1256.637[mA]

Electrodes  
 1 2

5 MV/m  
 40 A/cm<sup>2</sup>



KUAD2 v2.31  
*dartplot*

Thu Nov 30 17:07:34 2006  
 driffileID, 2mmf0404D5mVh

$g^{th}$  iteration

Error

$\phi_b$  - 0.21[%]  
 $H_{b,\theta}$  - 0.77[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

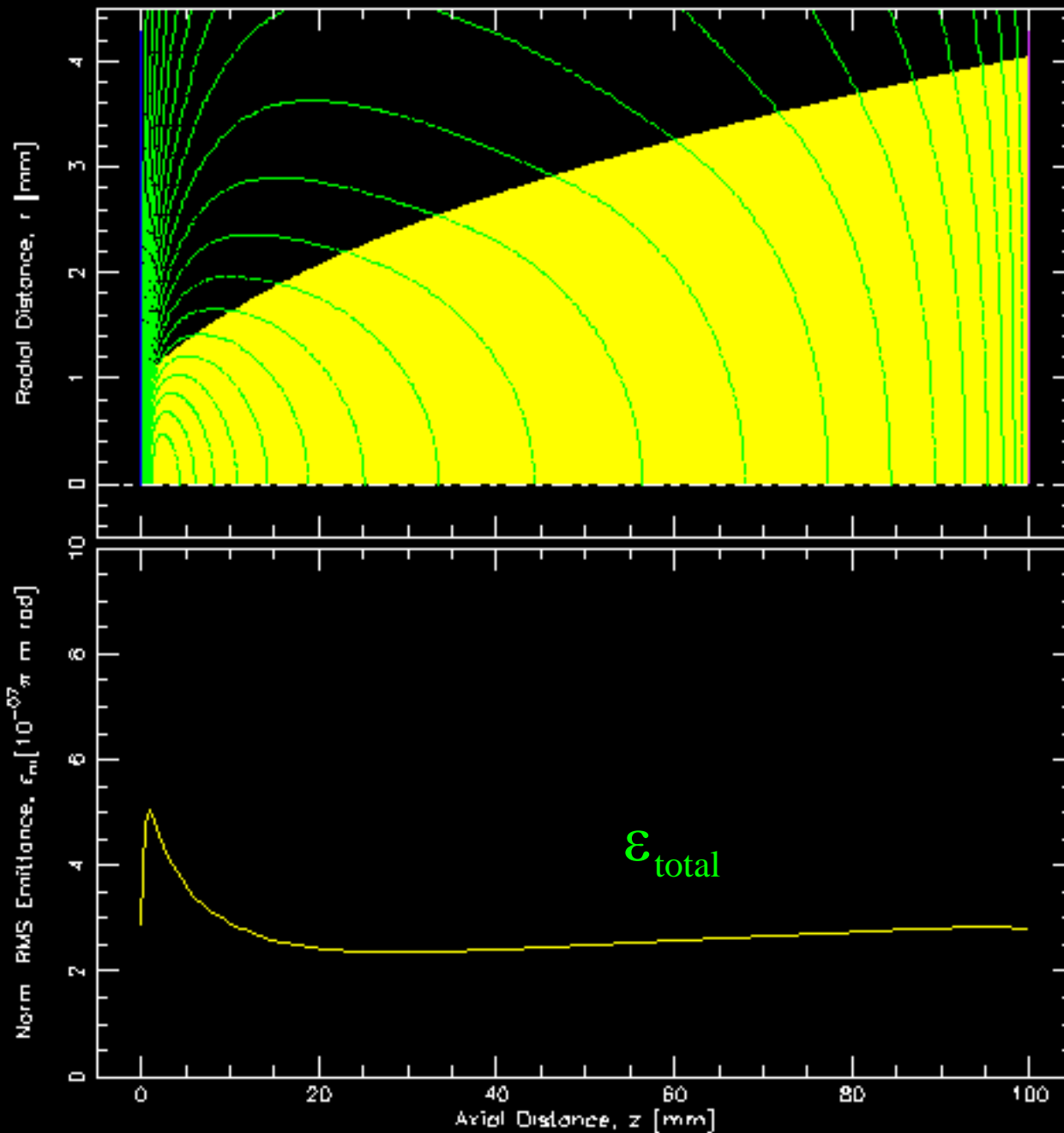
Injection Current

1256.637 [mA]

Electrodes

1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**



KUAD2 v2.31  
*dartplot*

Thu Nov 30 16:13:17 2006  
 driffileID, 2mmf0404D5mVb

$g^{th}$  iteration

Error

$\phi_b$  - 0.21[%]  
 $H_{b,\theta}$  - 0.81[%]  
 $A_{b,\theta}$  - 0.00[%]  
 $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

Injection Current

1256.637[mA]

Electrodes

1 2

**5 MV/m**  
**40 A/cm<sup>2</sup>**

$$\varepsilon_{\text{total}}^2 = \varepsilon_{\text{th}}^2 + \varepsilon_{\text{SC}}^2 \quad \text{は成り立っている}$$

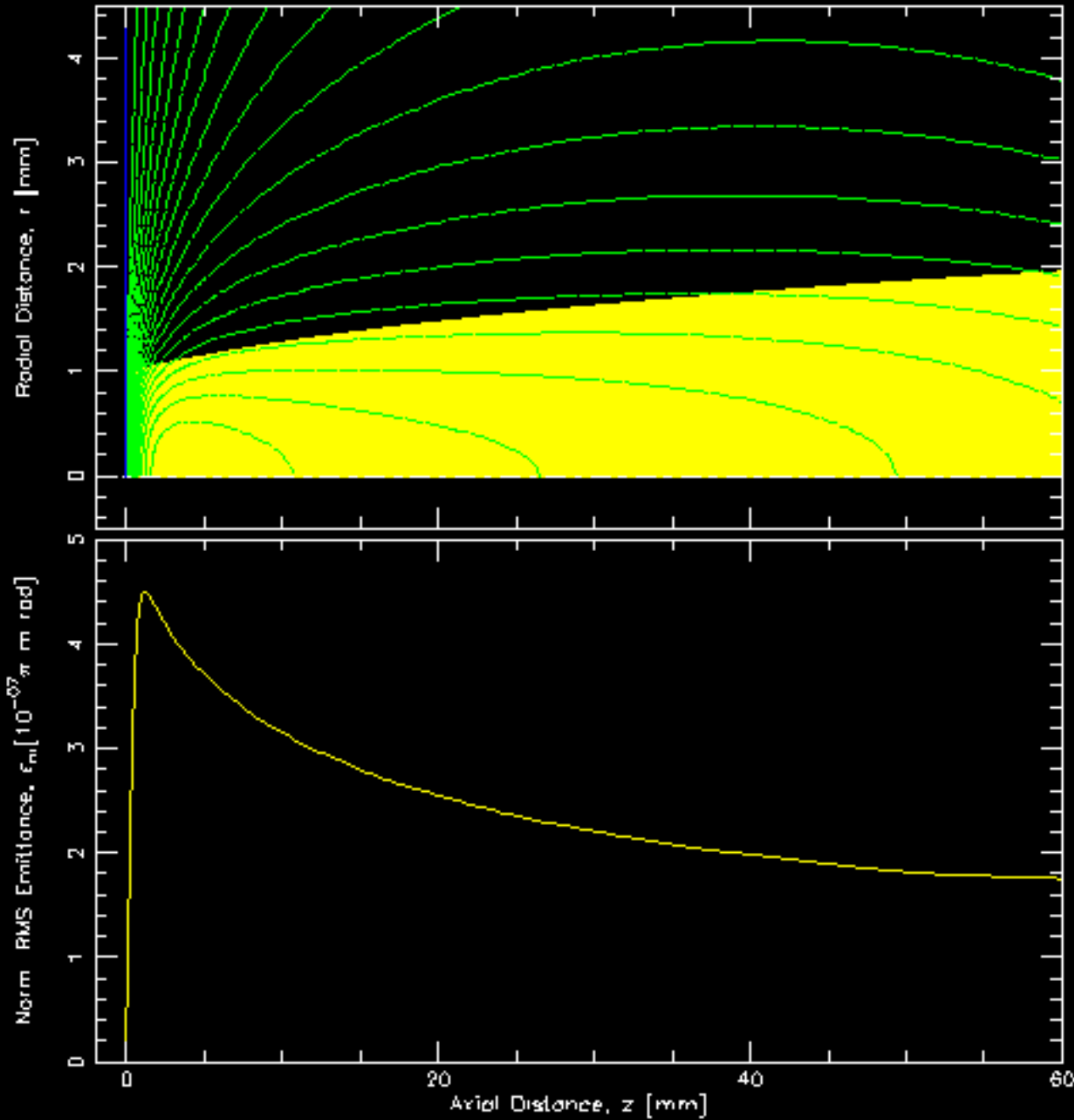
1. コードの概要

2. DCビームへの鏡像の影響

3. DCビームへの熱エミッタンスの影響

4. パルスビームへの鏡像の影響





KUAD2 v2.30

*dartplot*

Sun Nov 26 15:12:24 2006  
 driffileID, 2mmf160A20MV

20<sup>th</sup> iteration

Error

- $\phi_b$  - 0.00[%]
- $H_{b,\theta}$  - 0.00[%]
- $A_{b,\theta}$  - 0.00[%]
- $I_{inj}$  - 0.00[%]

Plot 1

Contour of  $\phi_b$

Classification by Color

— electron

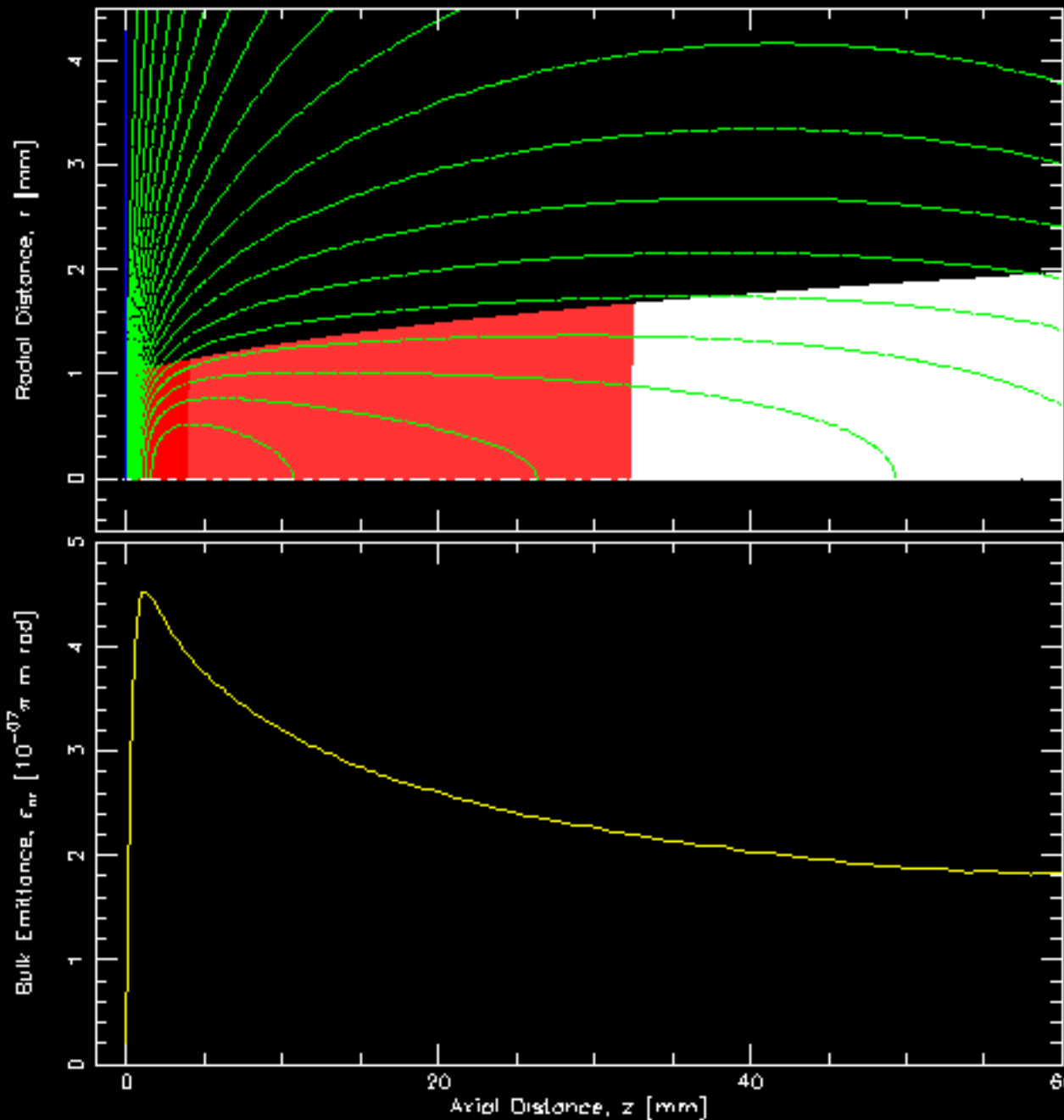
Injection Current

5.027[A]

Electrodes

1 2

**20 MV/m**  
**160 A/cm<sup>2</sup>**  
**DCビーム**



KUBLAI v1.60

*beamplot*

Thu Dec 7 09:59:28 2006  
 blofileID: 2mmf160A20MVb

2<sup>nd</sup> rf cycle

rf phase  
 1 / 20 [ $2\pi$ rad]

# of particles  
 total : 105483  
 electron : 105483

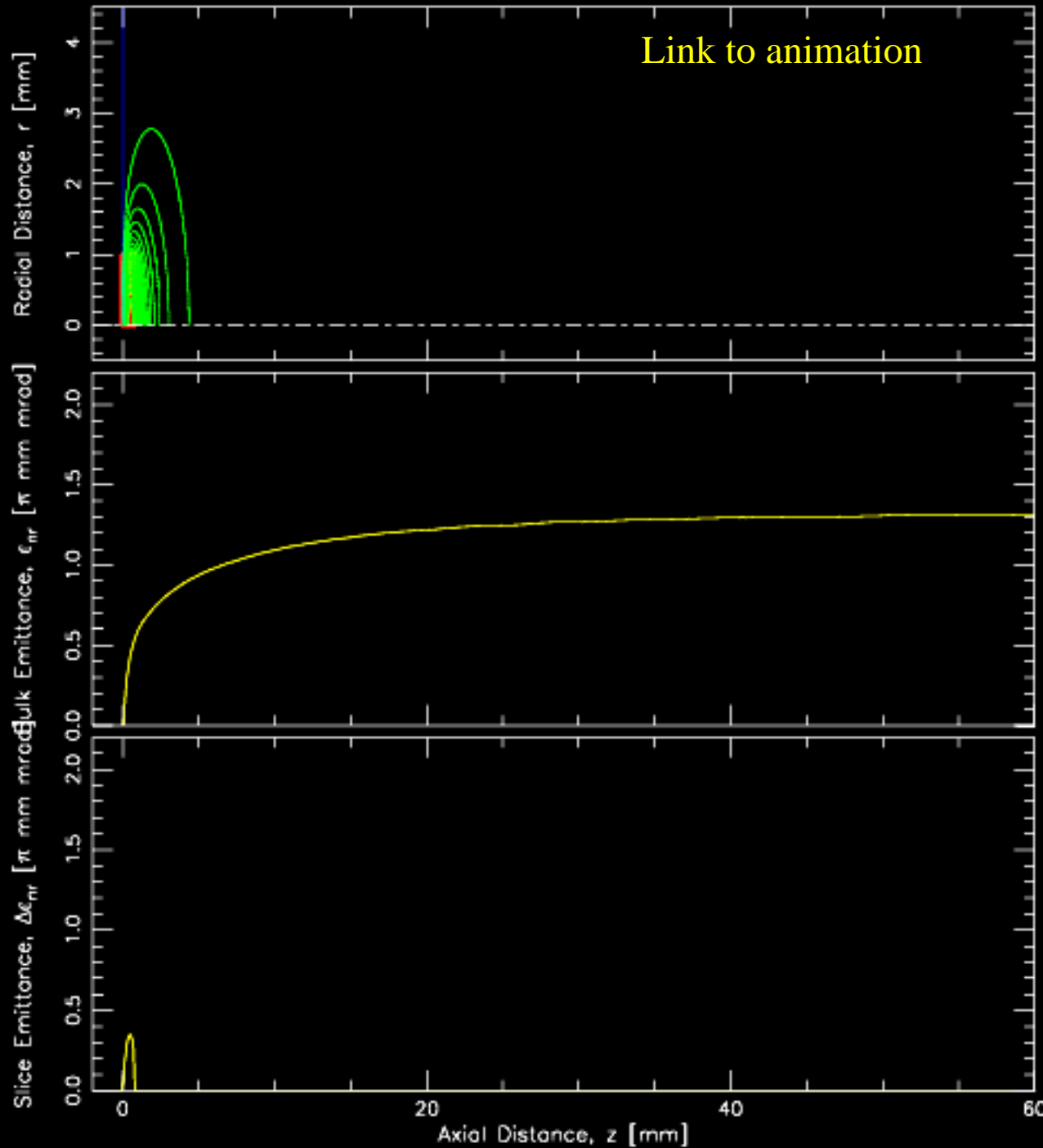
Plot 1  
 Contour of  $\phi_b$

Classification by Color  
 — Total Field  
 — Vacuum Field  
 — Cavity Field  
 — Beam-Induced Field  
 — Mesh

Electrodes

**20 MV/m**  
**160 A/cm<sup>2</sup>**  
**DCビーム**

[Link to animation](#)



**KUBLAI v1.60**

*beamplot*

Tue Dec 5 11:33:09 2006  
blfileID: 2mmf160A20MV20p

2<sup>nd</sup> rf cycle

rf phase  
2/ 50 [ $2\pi$ rad]

# of particles  
total : 40960  
electron : 40960

Plot 1

Contour of  $\phi_b$

Classification by Color

- Total Field
- Vacuum Field
- Cavity Field
- Beam-Induced Field
- Mesh

Electrodes

**20<sup>2</sup> MV/m**

**160 A/cm<sup>2</sup>**

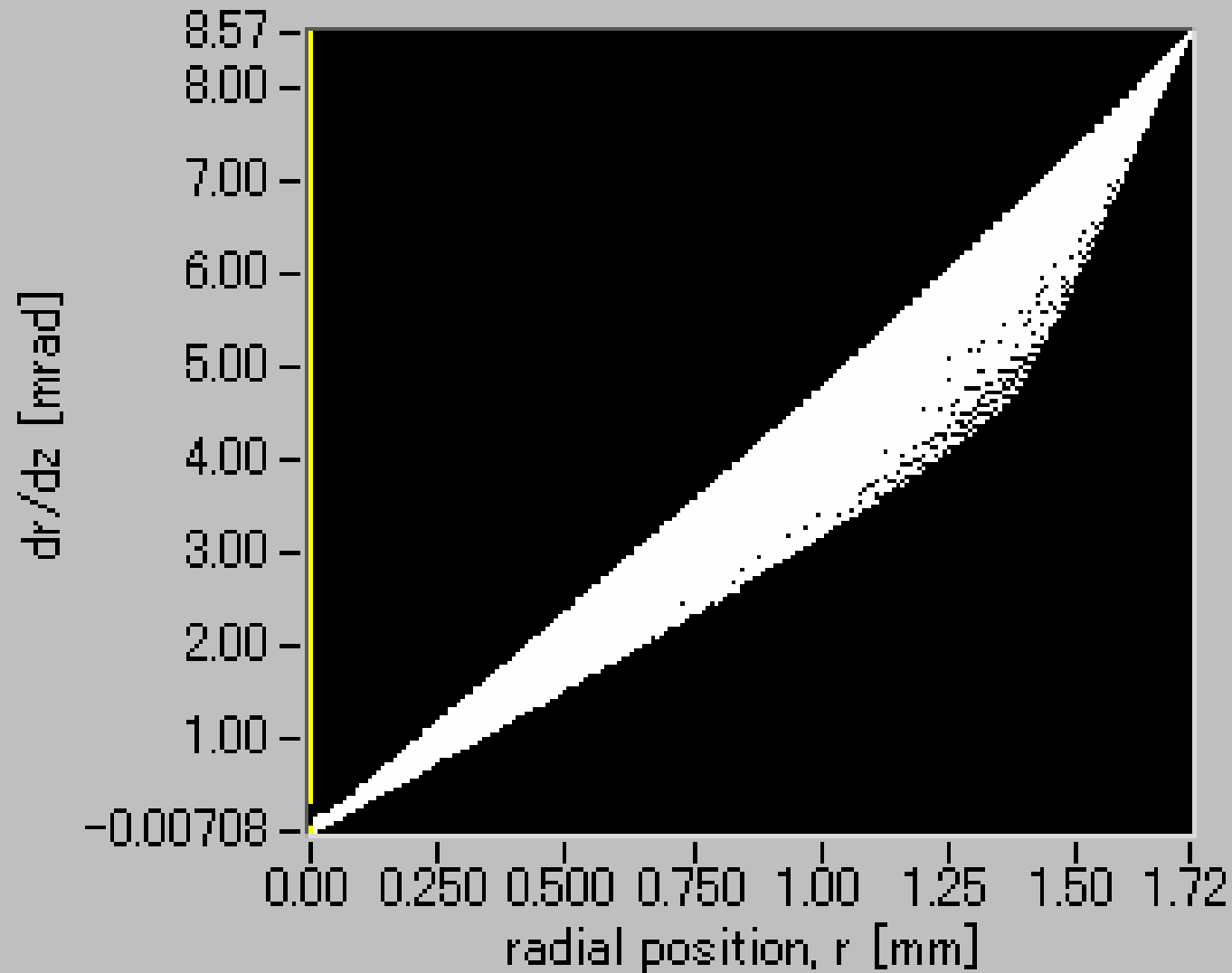
**20 psec**

**(0.1 nC)**

# 全粒子

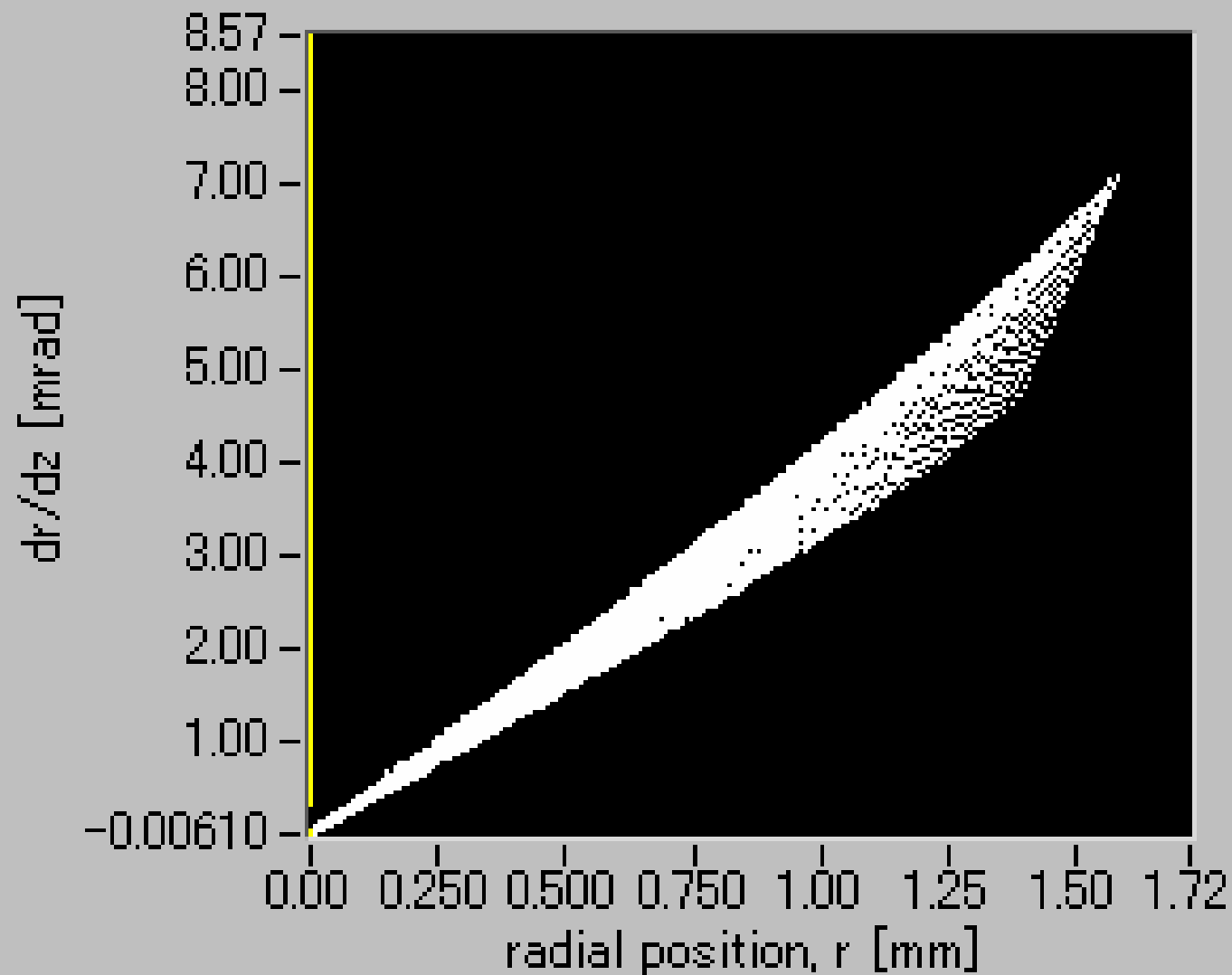
@  $z = 50$  mm

r-r' 散布図



# #1 / 8 (先頭) @ z = 50 mm

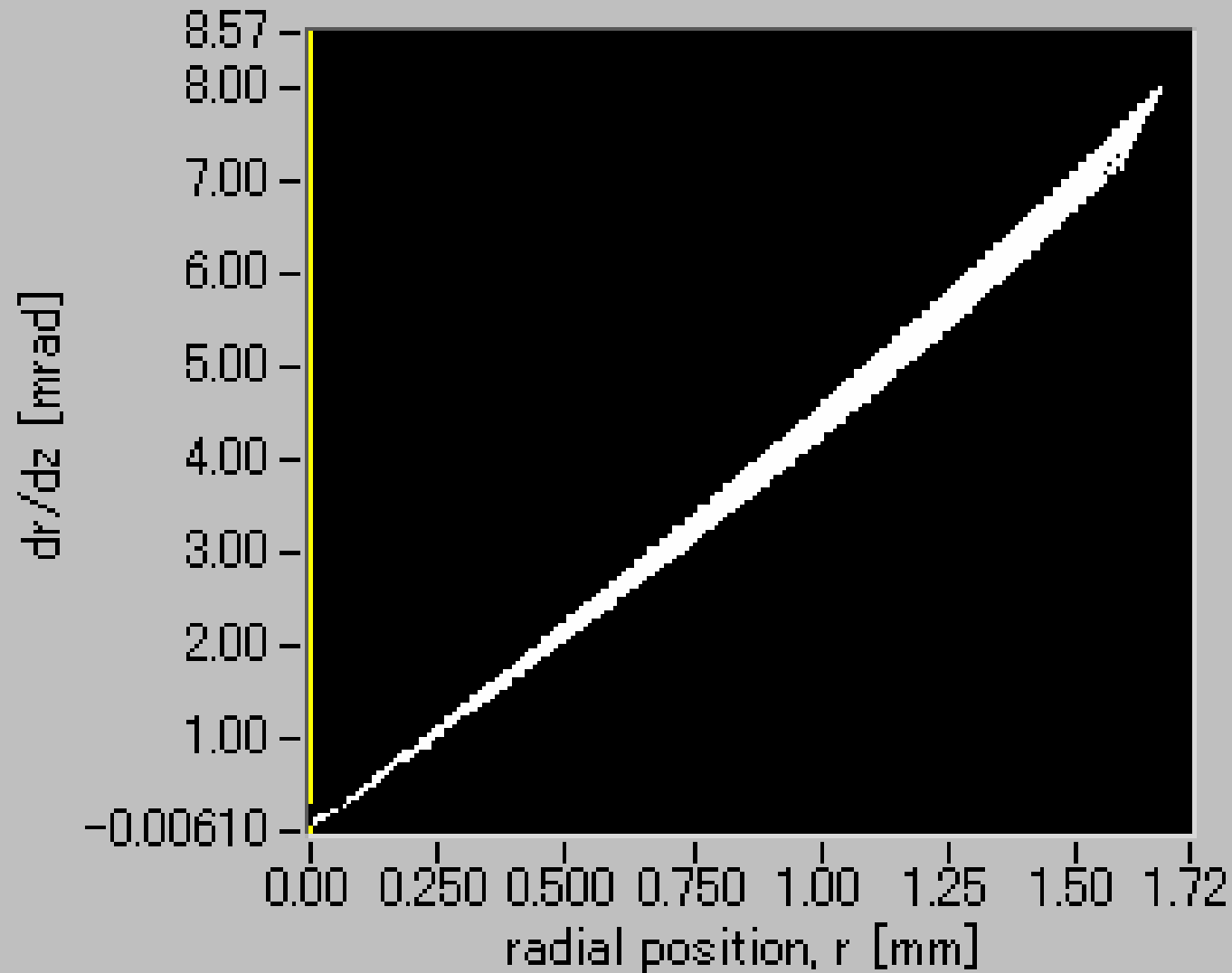
r-r' 散布図



#2 / 8

@ z = 50 mm

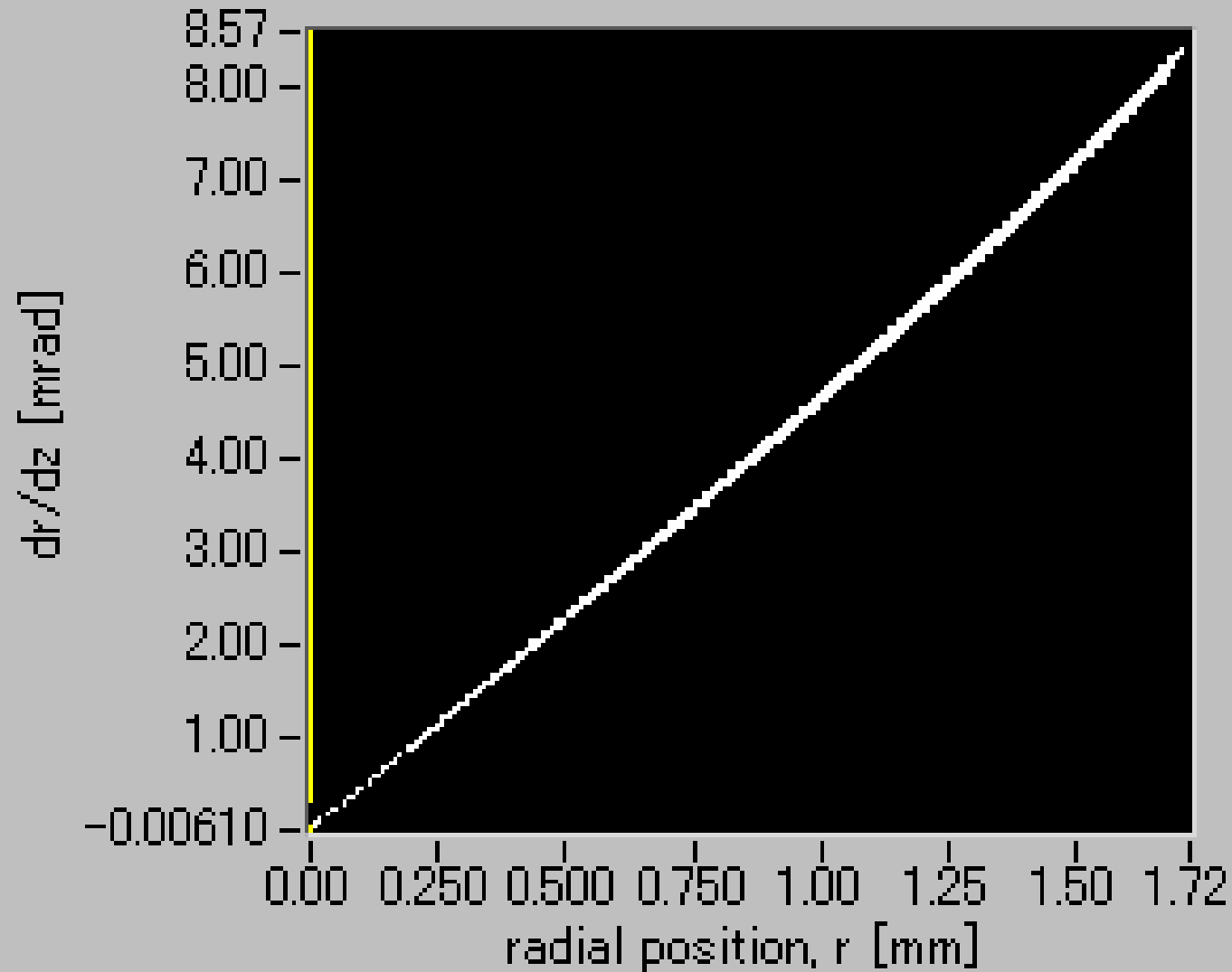
r-r' 散布図



#3 / 8

@ z = 50 mm

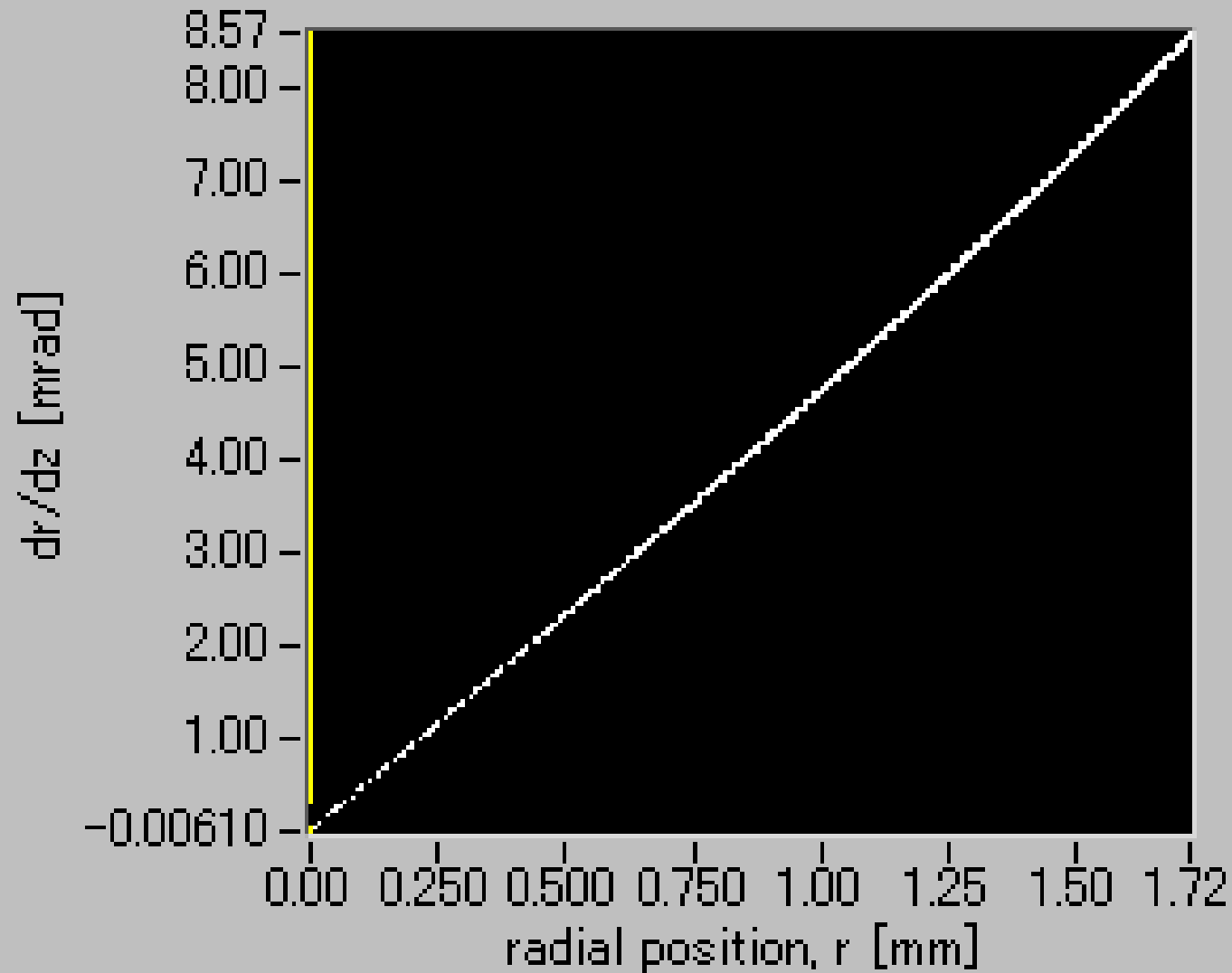
r-r' 散布図



#4 / 8

@ z = 50 mm

r-r' 散布図

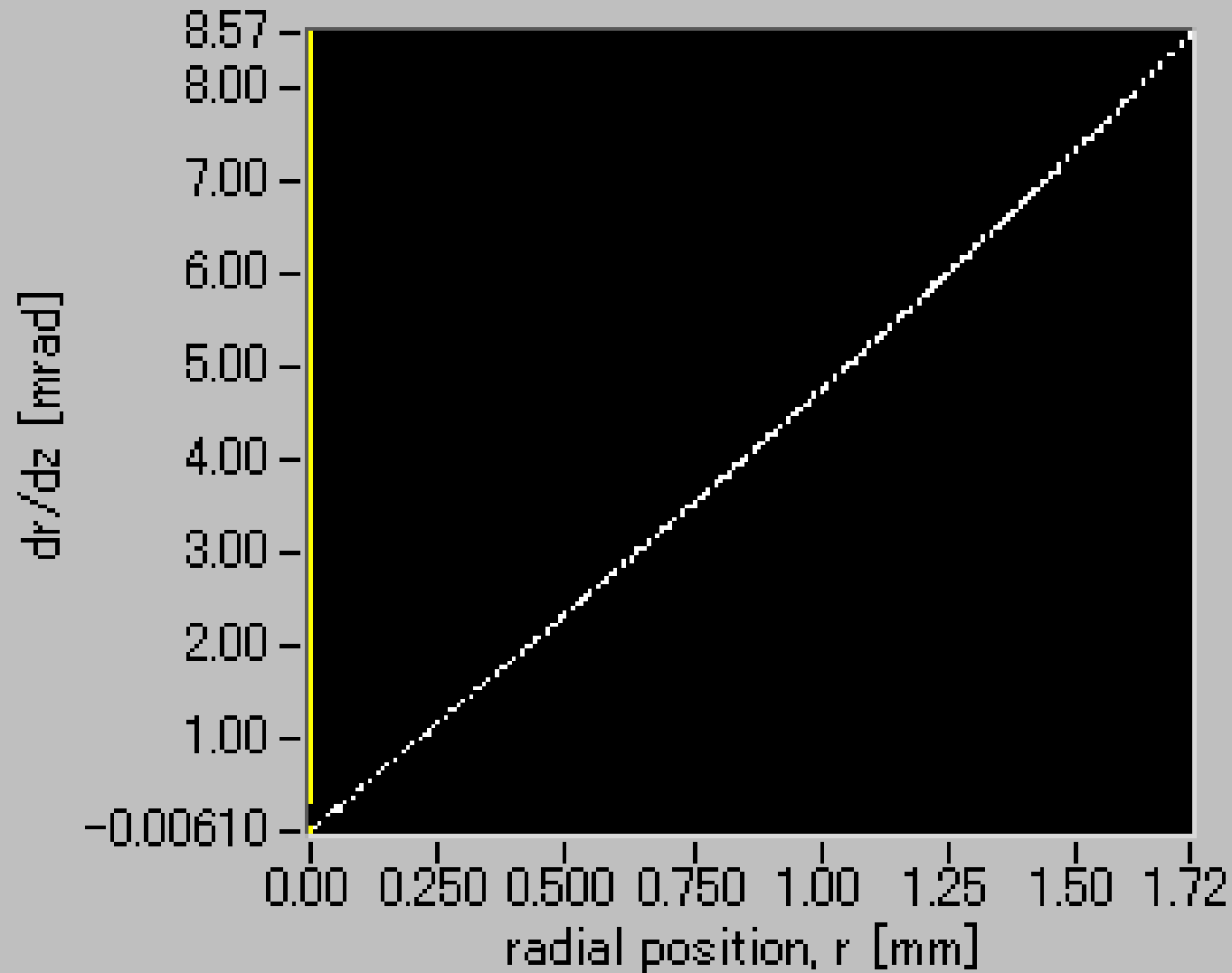




#5 / 8

@ z = 50 mm

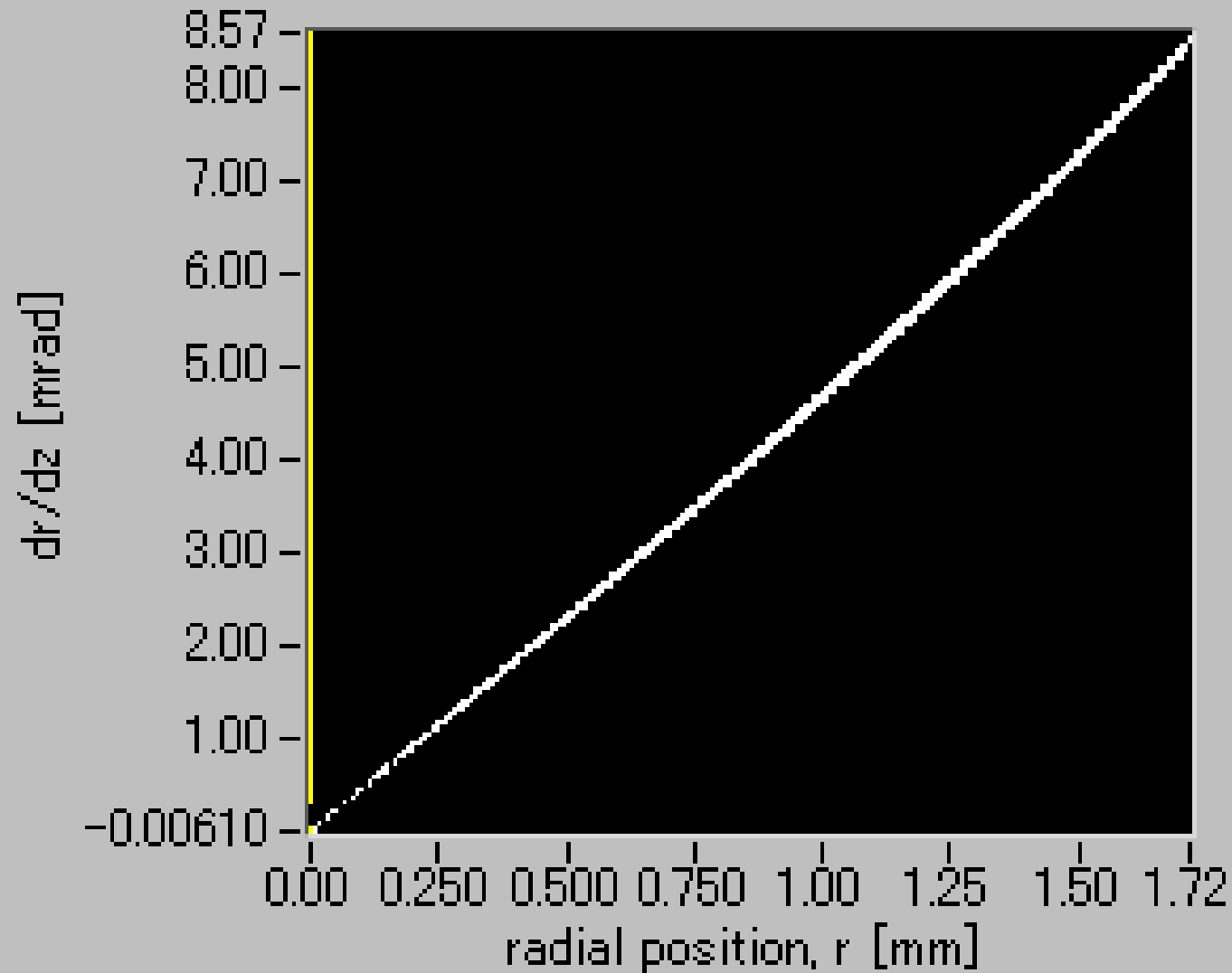
r-r' 散布図



#6 / 8

@ z = 50 mm

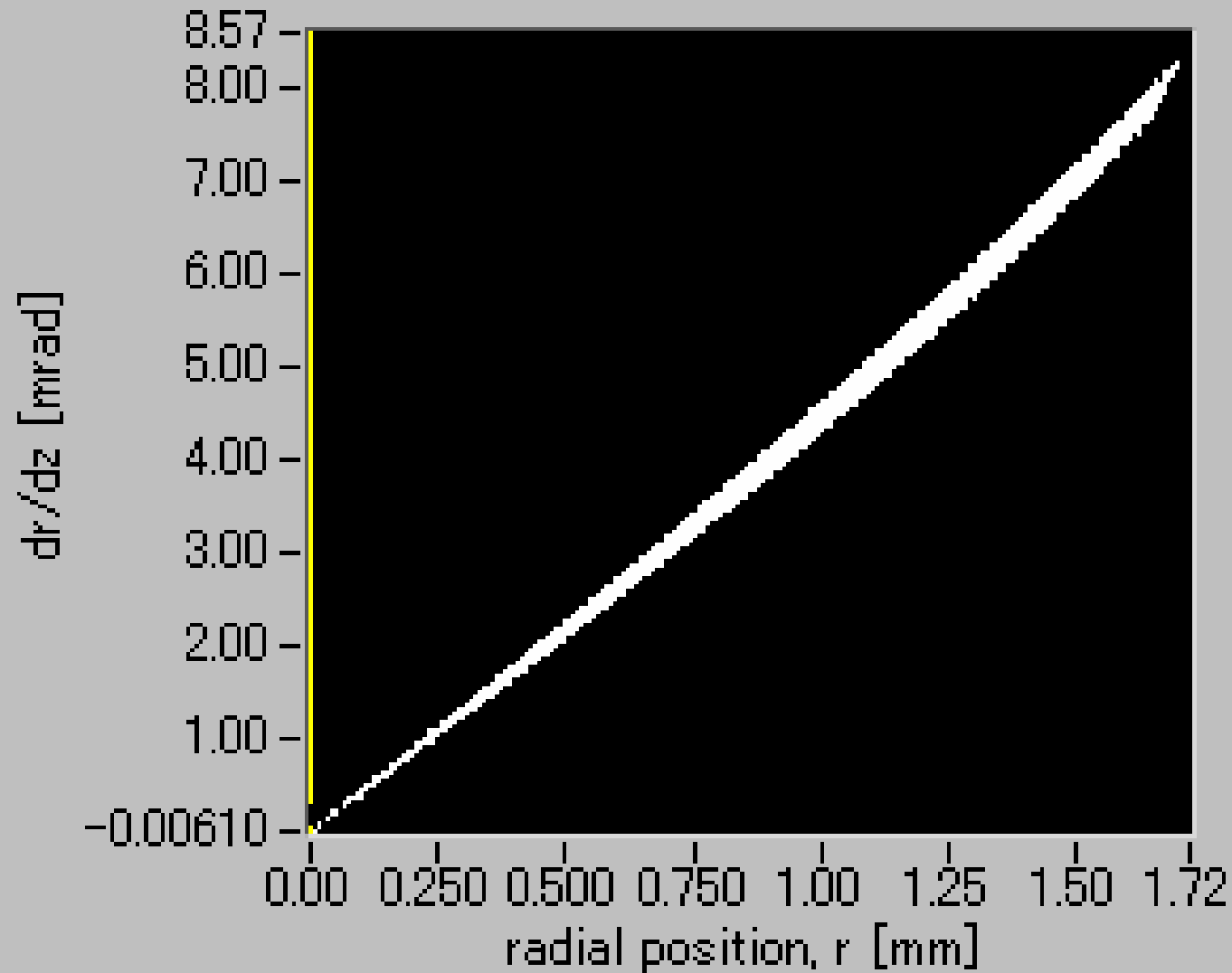
r-r' 散布図



#7 / 8

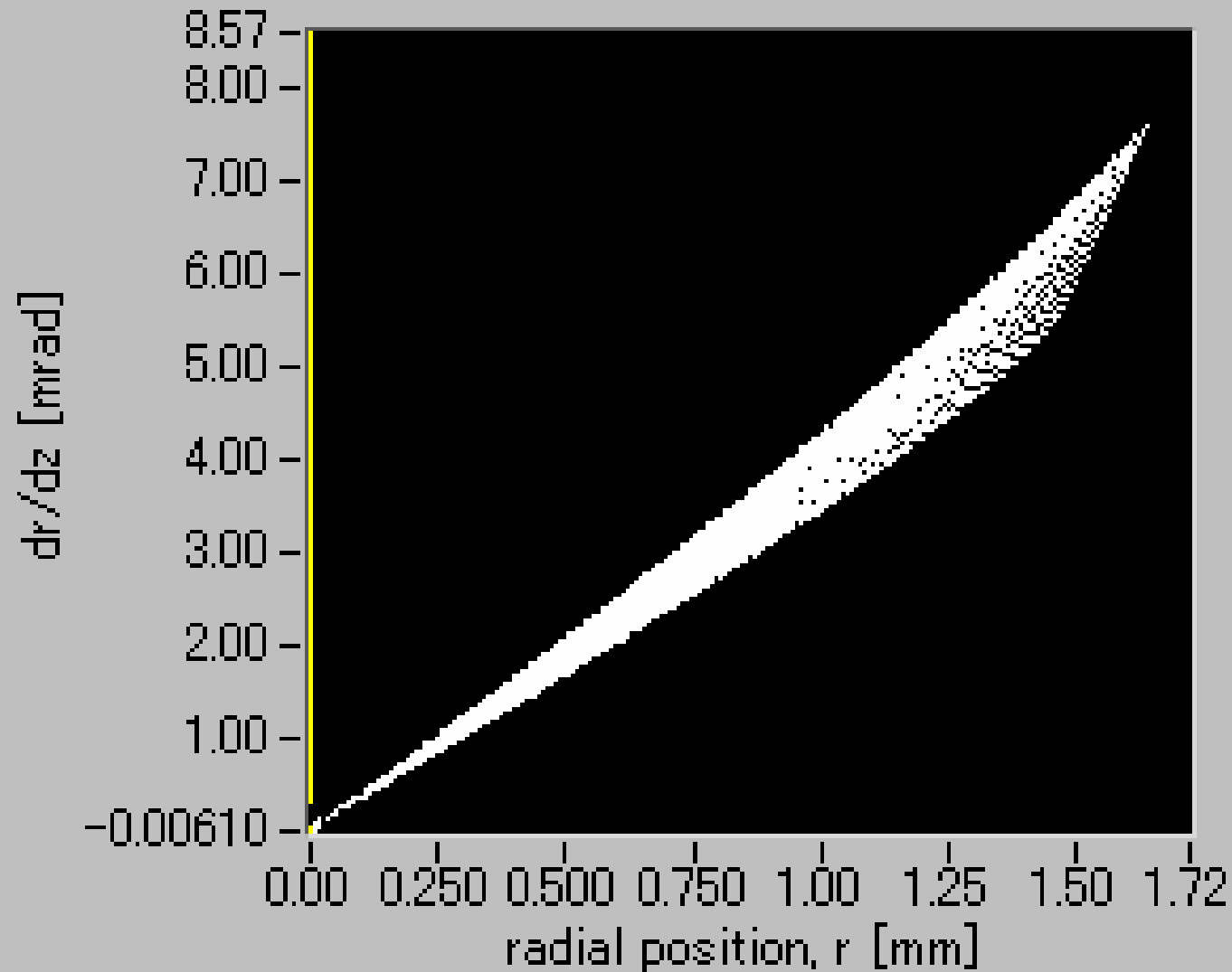
@ z = 50 mm

r-r' 散布図



# #8 / 8 (最後尾) @ $z = 50$ mm

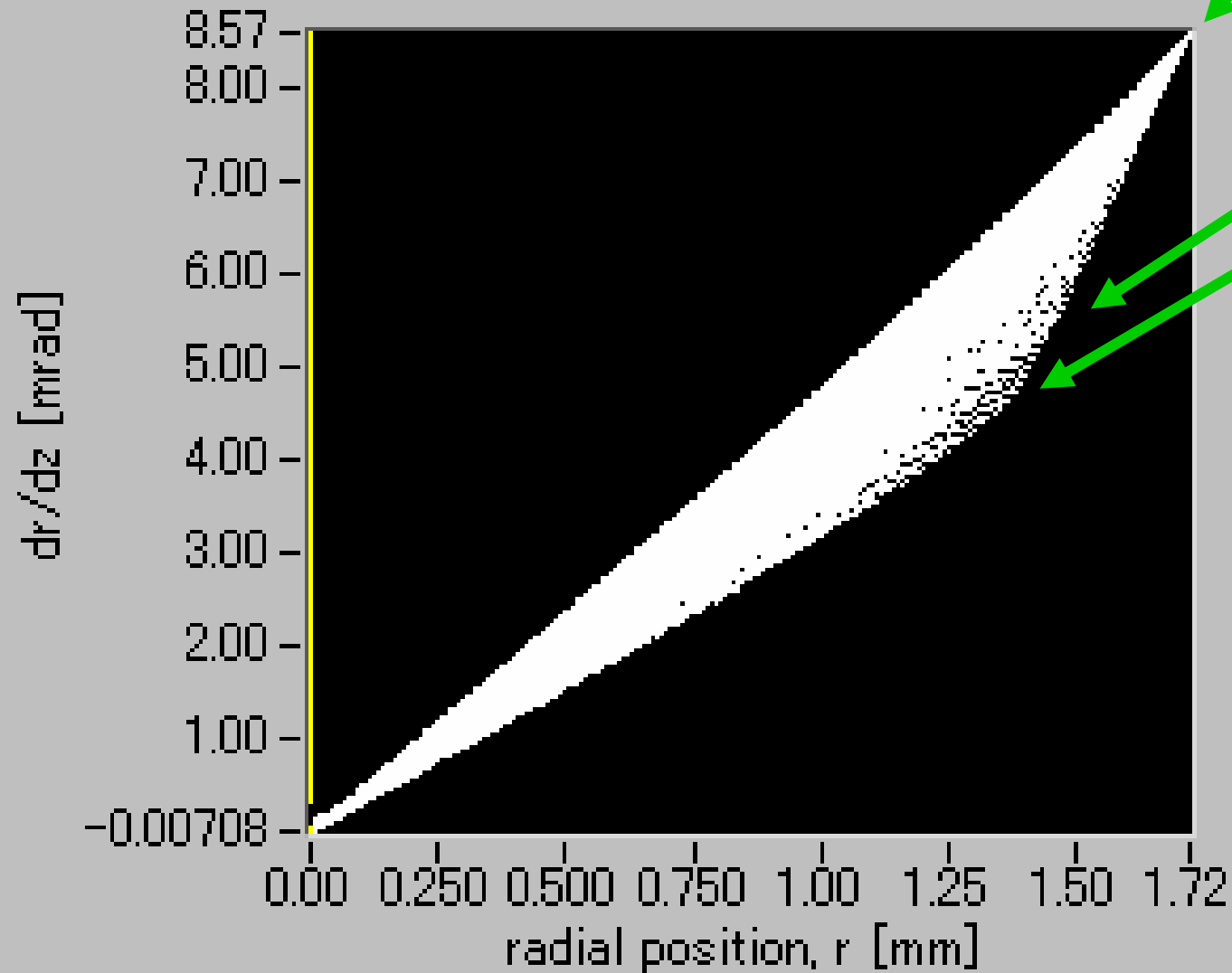
r-r' 散布図



# 全粒子

@ z = 50 mm

r-r' 散布図



中

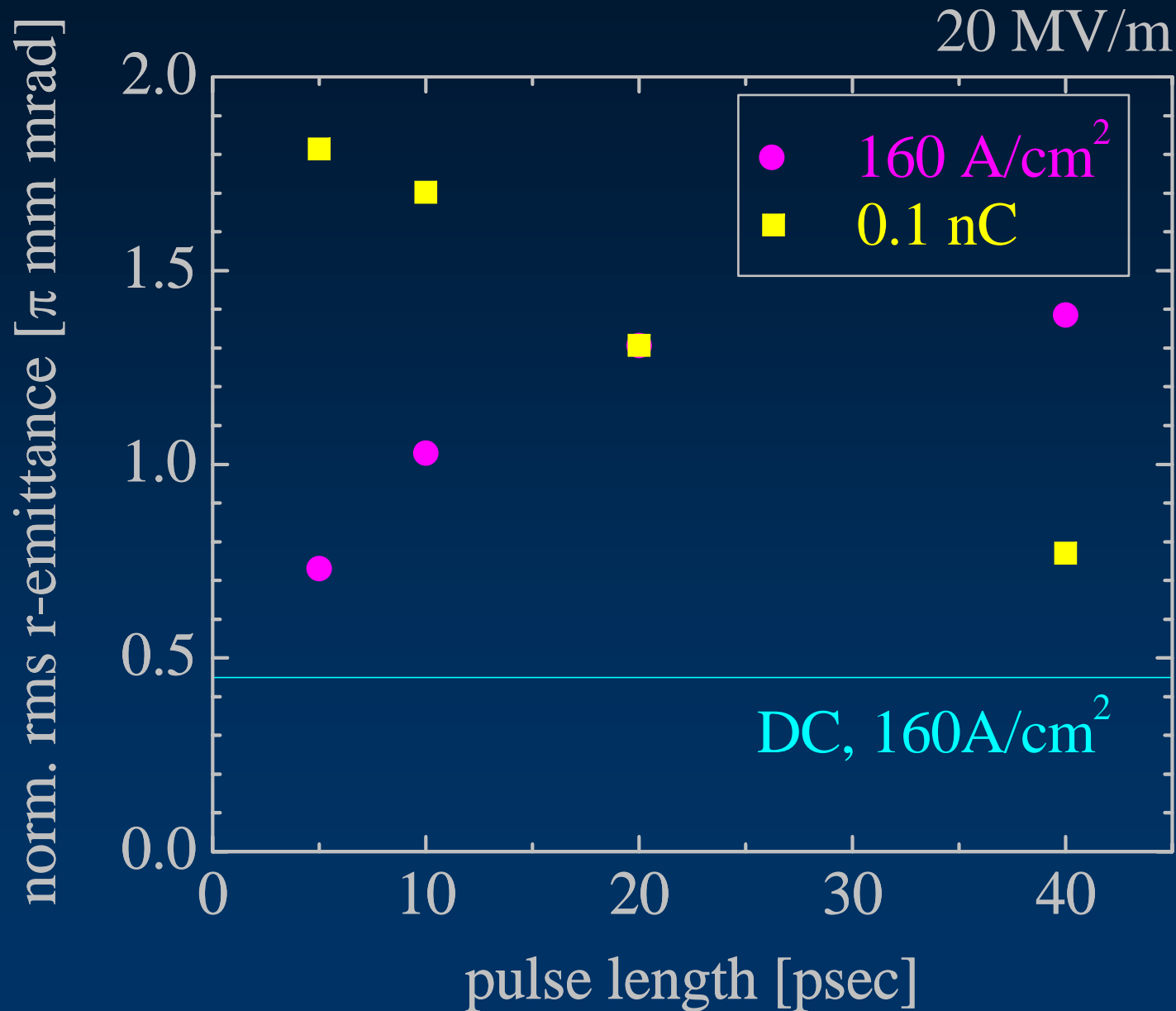
後

先

パルスの先頭と最後尾が受ける鏡像の効果は異なる

カソードを離れた後の空間電荷効果の影響の方が大きい

# norm. rms r-emittance @ z = 50 mm



# まとめ

- ✓ 陰極近傍 ( $z < 1 \text{ mm}$ :  $2 \text{ mm}\phi$ の場合) でエミッタンスが急増.  
(エミッタンス@ $z=1 \text{ mm}$ )      (電流密度) / (電界)
- ✓ 同時に, 横方向の電流密度分布が一様ではなくなる.  
 $z < 1$ でのエミッタンスを一時的に打ち消すこともある.
- ✓  $\epsilon_{\text{total}}^2 = \epsilon_{\text{th}}^2 + \epsilon_{\text{SC}}^2$  は見事に成り立つ.
- ✓ パルスだと, DCに比べて最大で約3倍, エミッタンス増  
(電流密度  $160 \text{ A/cm}^2$ , 電界  $20 \text{ MV/m}$ で)