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Optimizing Orbit Correction Configuration - Method and Applications

- Motivation
- The Method
    - Generalized response matrices
    - Mathematical recipes
    - Performance criteria
    - Configuration optimization
    - Failure modes
    - Critical elements
- Application to LHC Transfer Lines
- Extension of Method & Application

Motivation

Orbit Correction System Configuration

- **Optimizing performance and cost at design level, avoiding**
  - **unpleasant surprises**
  - **squandered operational resource**
  - **costly retrofits**
- **Has been traditionally done through simulation - not desirable ‘‘or necessary.**
  - Time-consuming (coverage of parameter space increases exponentialy as dimensionality)

    → Comparative studies extremely so.
  - “Passively waiting for problems to happen”.

    → Cannot cleanly detect structural problems, provide intuitive pictures and find optimal solutions.
  - Awkward to answer certain questions by simulation.

Typical questions (want fast and unambiguous answers)

Are there blind spots, and how bad?

Are there singularities, and how bad?

What does a certain observed orbit say about orbit everywhere?

How well can an error distribution be corrected (inc. monitor errors)?

Do we have too many monitors or correctors?

Can the system handle a known tight spot?

What is the most critical monitor or corrector?

What causes a particular observed orbit problem?

Do the correctors have enough range?

What is the worst case (combination) at a given probability envelope?

→ There should be a quantitative method to rigorously seek out problems, demonstrate viability and optimize performance.

Generalized Optimization Program

- Form probability distributions of design and operational errors in injection, alignment, field and monitor offsets

  → Translate into distributions of their impact on various performance parameters at every location.

- Develop mathematical recipes for mapping, projection, intersection, tangency, extremum solutions

  → Quantitative measures of performance.

- Systematically identify and correct structural defects.

- Use analytical performance criteria

  → Unambiguous merit functions to improve configuration

  → Advantage over simulation.

- Other analytic evaluation/optimization methods

Components of the method

Generalized actuators & responders

Mathematical recipes

Secondary response matrices

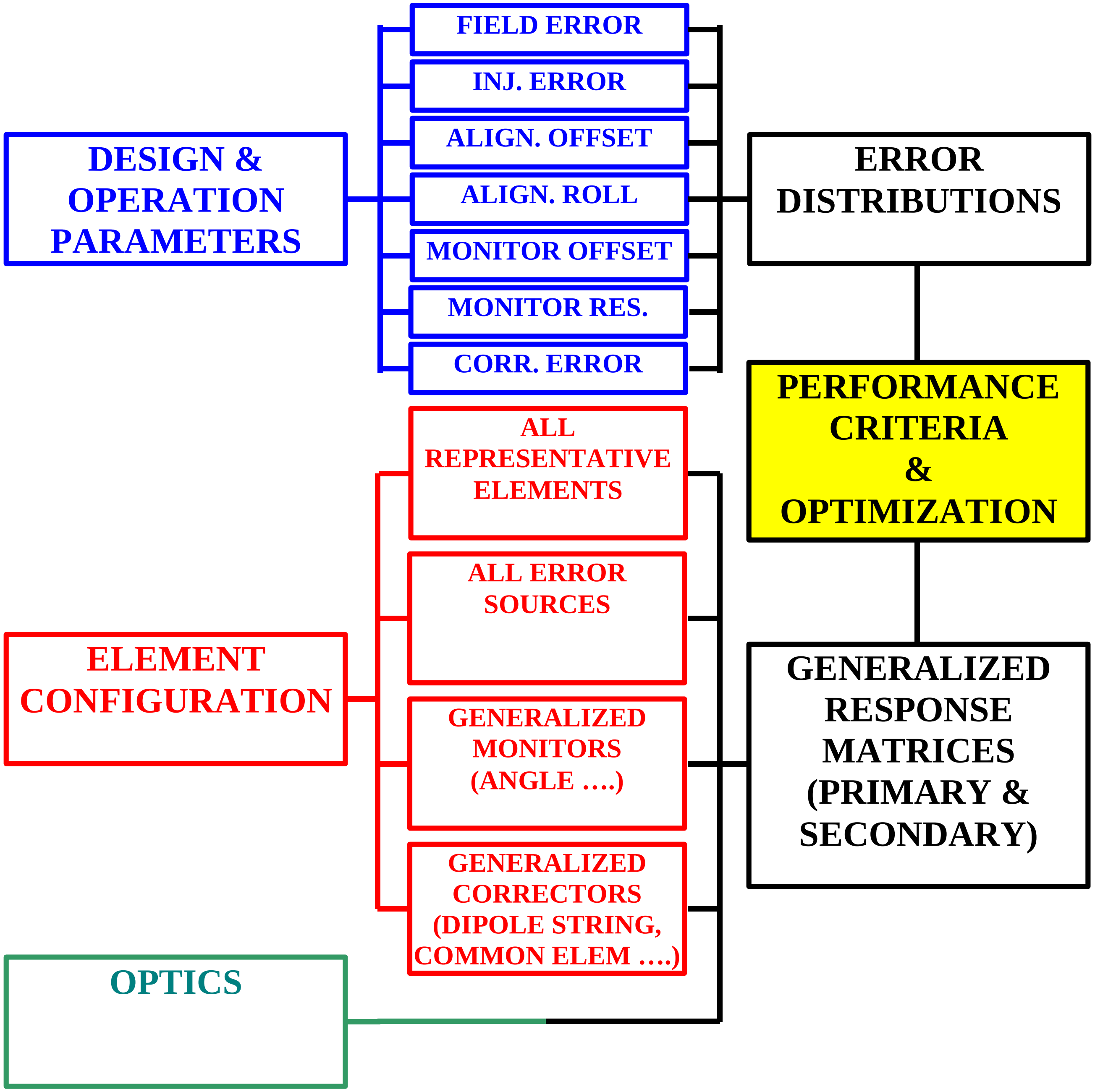
Performance criteria

Configuration optimization methods

- **Analytical for structural improvements**
- **Numerical for fine-tuning**

Failure modes

Critical elements


FIELD ERROR
INJ. ERROR
ALIGN. OFFSET
ALIGN. ROLL
MONITOR OFFSET
MONITOR RES.
CORR. ERROR
DESIGN & OPERATION PARAMETERS
ERROR DISTRIBUTIONS
PERFORMANCE CRITERIA & OPTIMIZATION
ALL REPRESENTATIVE ELEMENTS
ALL ERROR SOURCES
GENERALIZED MONITORS (ANGLE ….)
GENERALIZED CORRECTORS (DIPOLE STRING, COMMON ELEM ….)
ELEMENT CONFIGURATION
GENERALIZED RESPONSE MATRICES (PRIMARY & SECONDARY)
OPTICS

Generalized Actuators and Responders

Extending the traditional set to
Errors,
Other orbit degrees of freedom (angle, path length, dispersion, momentum compaction, ……)
Un-monitored locations

$$\begin{pmatrix} R^1 \\ R^2 \\ \vdots \\ R^n \end{pmatrix} = \begin{pmatrix} C^{11} & C^{12} & \cdots & C^{1m} \\ C^{21} & C^{22} & \ddots & C^{2m} \\ \vdots & \ddots & \ddots & \vdots \\ C^{n1} & C^{n2} & \cdots & C^{nm} \end{pmatrix} \cdot \begin{pmatrix} A^1 \\ A^2 \\ \vdots \\ A^m \end{pmatrix}$$

→ Will build secondary matrices based on these primary matrices

Generalised Response Matrix	(Generalised) Actuator	(Generalised) Responder	Response Coefficients
$\mathbf{M}^{CM}$	$\mathbf{A}^{C}$: correctors, dipoles, dipole strings	$\mathbf{R}^{M}$: position & angle at monitors	$M^{11}, M^{12}, M^{21}, M^{22}$ and linear combinations
$\mathbf{M}^{EM}$	$\mathbf{A}^{E}$: alignment type errors (injection, misalignment, field, ….)	$\mathbf{R}^{M}$: position & angle at monitors	$M^{11}, M^{12}, M^{21}, M^{22}$
$\mathbf{M}^{CA}$	$\mathbf{A}^{C}$: correctors, dipoles, dipole strings	$\mathbf{R}^{A}$: position & angle at all representative elements	$M^{11}, M^{12}, M^{21}, M^{22}$ and linear combinations
$\mathbf{M}^{EA}$	$\mathbf{A}^{E}$: alignment type errors (injection, misalignment, field, …)	$\mathbf{R}^{A}$: position & angle at all representative elements	$M^{11}, M^{12}, M^{21}, M^{22}$
$\mathbf{M}^{AM}$	$\mathbf{A}^{A}$: angle at all representative elements	$\mathbf{R}^{M}$: position & angle at monitors	M^{12}, M^{22}
$\mathbf{M}^{MM}$	$\mathbf{A}^{M}$: monitor offset error	$\mathbf{R}^{M}$: apparent orbit error at monitor	δij

Mathematical Recipes

Establish efficient, robust algorithms for mapping probability distributions between various vector spaces, projection, boundary, intersection, inscription, extremum etc..

Although all calculations look quadratic or of higher order, nonlinear optimization algorithm is not an option. They must be cast in linear form to be practical as dimensionality runs to 100's or 1000's.

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Allowed calculations:

- **Matrix addition, multiplication, permutation, transpose, sub-matrices and direct sum;**
- **Inversion of non-degenerate square matrices;**
- **Matrix pseudo-inverse;**
- **Null space vectors;**
- **Eigenvalues and eigenvectors;**
- **Singular value decomposition (SVD).**
-

Efficient, robust recipe is critical!

Mathematical Recipes - continued

Null Space

Projection Operators

- Critically or over-constrained response matrix M^{CM}
- Under-constrained response matrix M^{CM}
- Rank-deficient M^{CM}

Orthonormal Transformation

Decomposition of Matrix into Orthogonal Parts

Projection of Ellipsoids

Properties of the Error Distribution

Normalizing the Ellipsoid

Projection of Ellipsoids onto Lower and Higher Dimensions

- Under-constrained map (higher to lower dimension)
- Critically-constrained map
- Over-constrained map (lower to higher dimension)

Inverse Projection of Point(s) onto an Ellipsoid

- Under-constrained map (higher to lower dimension)
- Critically-constrained map
- Over-constrained map (lower to higher dimension)

Tangent Points between an Ellipsoid and an Arbitrary Hyper-Plane

Extreme Values of Arbitrary Operators on a Constrained Surface

Hessian and the Curvature of an Ellipsoid

Mathematical Recipes - continued

Multi-Dimensional Gaussian Distribution

Probability Density Distribution in Mapped Space

Probability Density Distribution of Extrema and Length

- Distribution of Absolute Maximum:
- Distribution of Length:
- Cutoff on Distribution of Length:

Singularity Related Issues

Singular Value Decomposition, Condition Number

Gram Determinant

Principal Axes

Exception Handling for Rank Deficiency and Near Singularity

Examples

Projection of ellipsoids onto lower and higher dimensions

- Under-constrained map (higher to lower dimension)
- Critically-constrained map
- Over-constrained map (lower to higher dimension)

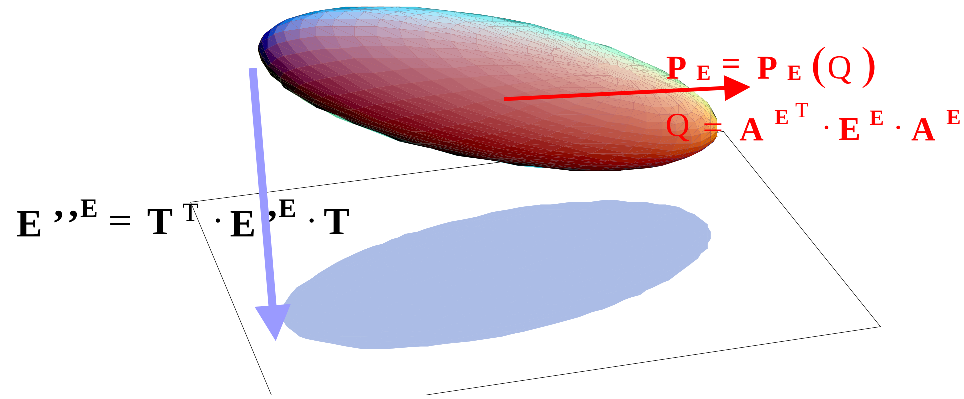
Equations defining point set on the ellipsoid projected onto the image boundary

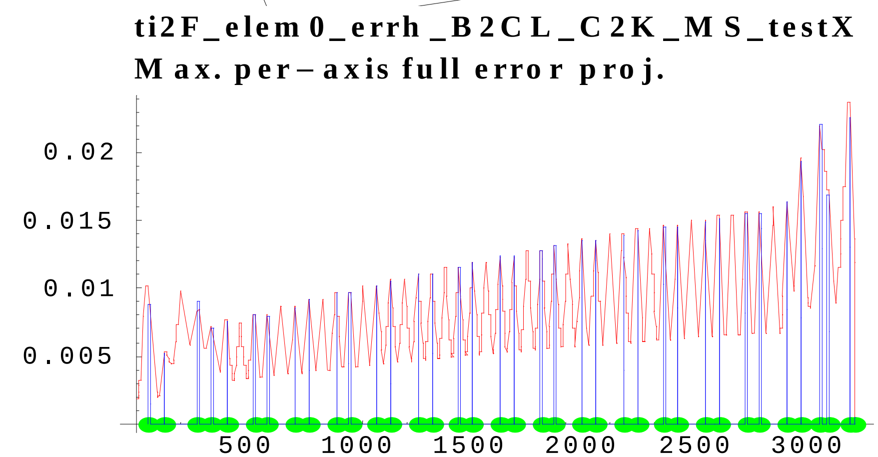
$$\nabla^{i}_{N} S = \mathbf{N}^{i} \bullet \nabla S = 0 = \sum_{kmn} M^{ik}_{null} \frac{\partial A^{E}_{m} A^{E}_{n} E^{E}_{mn}}{\partial A^{E}_{k}},$$

$$\mathbf{i} = Nr+1, Nr+2, \ldots Nc, \quad \mathbf{N} = \mathbf{M}_{null}.$$

Under-Constrained:

> Example: Projection of the error ellipsoid $\mathbf{E}^{E}$ by $\mathbf{M}^{EM}$ (under-constrained) gives the footprint of errors on all monitors.





Projection of ellipsoids onto lower and higher dimensions

Over-Constrained:

Example: Projection of a normalized ellipsoid in the unobservable error subspace by

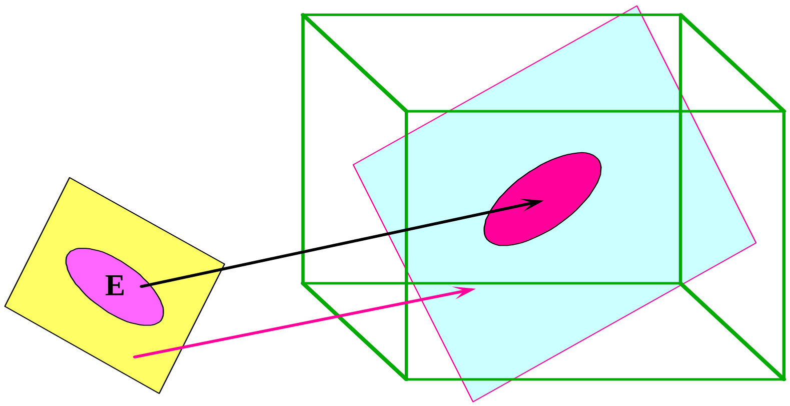
$$M_{\mathbf{M}}^{\text{unobs}} = \mathbf{M}^{\mathbf{EA}} \cdot \left(\mathbf{M}^{\mathbf{EM}}\right)_{\text{out}}^{\mathbf{M}^{\mathbf{EM}}}$$

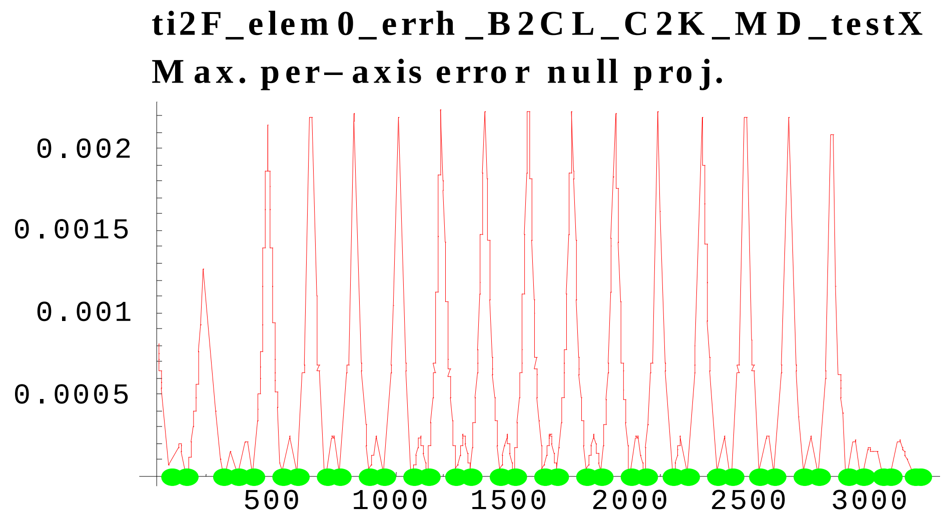
onto the all-element space $\mathbf{R}^{\mathbf{A}}$ (over-constrained) allows the determination of impact of unobservable error at all elements.

Map ellipsoid into subspace spanned by image of M.

$$\mathbf{M}^{T} \rightarrow \mathbf{M}^{T} \cdot \mathbf{T}^{\mathbf{M}^{T}}$$

$$\mathbf{E}^{\mathbf{E}} \rightarrow \mathbf{M}^{\mathbf{p\text{-}1}^{T}} \cdot \mathbf{E}^{\mathbf{E}} \cdot \mathbf{M}^{\mathbf{p\text{-}1}}.$$





Inverse Projection of Point(s) onto an Ellipsoid

Find the point(s) **Z** on the original ellipsoid mapped into a point **X** (e.g., the point with the longest length or the largest component along an axis) in the image.

- Under-constrained map (higher to lower dimension)
- Critically-constrained map
- Over-constrained map (lower to higher dimension)

Worst case residual orbit
→ What error combination caused it?

Example: Inverse-projection through $\mathbf{M}^{\mathrm{UOE}}_{\mathbf{EMA}}$ from a 3σ maximum orbit in the all-element orbit space onto the combined alignment+monitor error ellipsoid reveals offending composition of errors.

Inverse Image onto an Ellipsoid from Under-Constrained Map

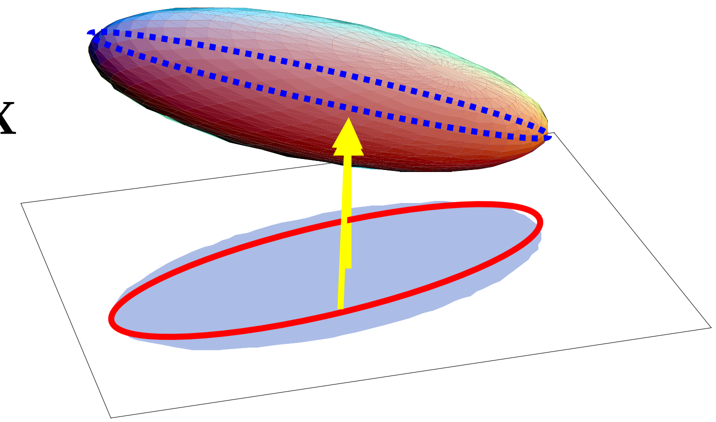
$$M \bullet (Z - Y) = 0,$$

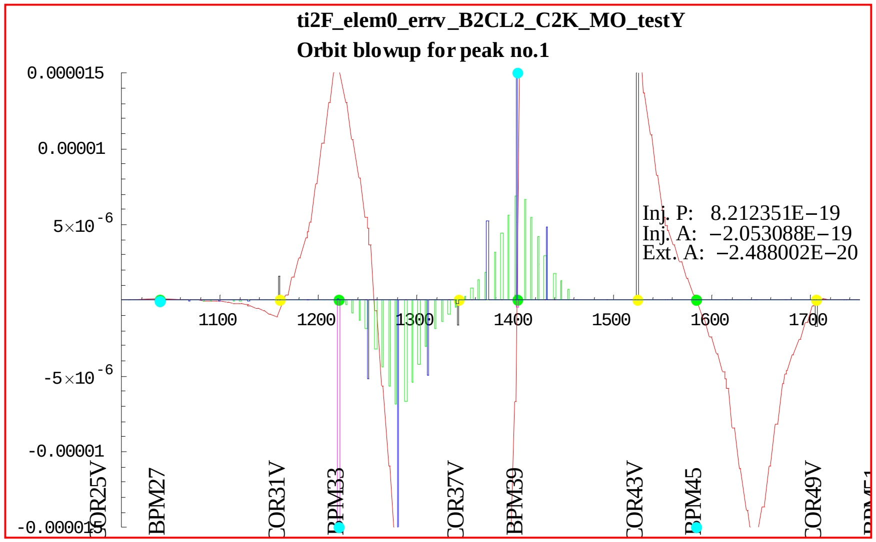
$$\sum_k M^{ik}\cdot\left(Z^k - Y^k\right) = 0, \quad i = 1, 2, \ldots.Nr.$$

$$\mathbf{N} = \mathbf{M}_{\mathbf{null}},$$

$$\nabla^{\mathbf{i}}_{\mathbf{N}} S\Big|_{\mathbf{Z}} = 2\mathbf{N}^{\mathbf{i}}\cdot E\cdot Z = 0, \quad i = Nr+1, Nr+2, \ldots.Nc.$$

$$\mathbf{Z} = \mathbf{M}^{\dagger}\cdot\mathbf{X}$$





Tangent Points between an Ellipsoid and an Arbitrary Hyper-Plane

How many error σ's can each corrector handle?

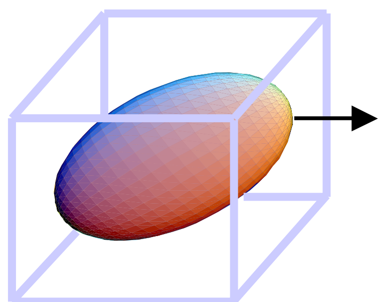
Example: Projection of the error ellipsoid $\mathbf{E}^E$ onto the corrector space, via $M_{\mathbf{CME}}^{\text{resp}} = \mathbf{M}_{\mathbf{CM}}^{\dagger} \cdot \mathbf{M}^{\mathbf{EM}}$, is tangent to the hyper-cube defined by corrector range at the maximally correctable errors in units of global error distribution σ.

Tangent Points between an Ellipsoid and an Arbitrary Hyper-Plane

$$\sum_k \mathbf{M}_i^k \cdot \mathbf{X}^k = \mathbf{P}^i(\mathbf{X}) = \pm \mathbf{V}_i,$$

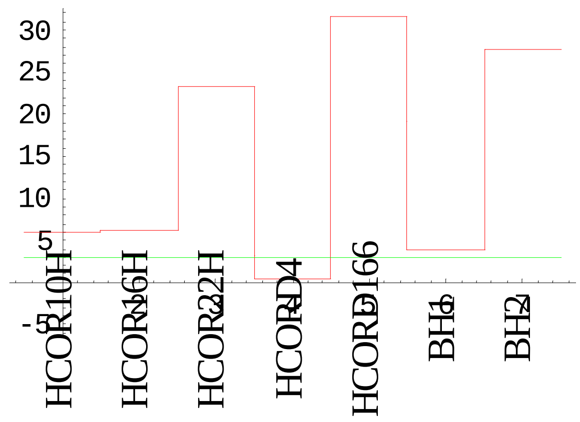
$$\nabla \mathbf{P}^i = \sum_k \hat{\mathbf{x}}^k \mathbf{M}_i^k, \quad i = 1,2,....Np, \quad k = 1,2,....N,$$

$$2\mathbf{E} \cdot \mathbf{X} = \lambda_i \mathbf{M}_i$$



cngs_elem(… _CD_testX

Corrector range in units of projected sigma



Extreme Values of Arbitrary Operator Π on a Constrained Surface

Examples of the operator Π:

Distance from origin:	$\mathbf{\Pi = I}$
Projection onto the null space of M^{EM}:	$\mathbf{\Pi = \Pi_{EM}^{\perp}}$
Component along the i-th axis:	$\mathbf{\Pi = \hat{x}_i \cdot \hat{x}_i^T}$
Component along the i-th axis inside null space of M^{EM}:	$\mathbf{\Pi = \Pi_{EM}^{\perp} \cdot \hat{x}_i \cdot \hat{x}_i^T}$
Component along a vector V:	$\mathbf{\Pi = V \cdot V^T}$
Component along a vector V in higher dimension:	$\mathbf{\Pi = K \cdot V \cdot V^T \cdot K^T}$

Example: Maximum length of

$$M_{\mathbf{M}}^{unobs} = \mathbf{M}^{\mathbf{EA}} \cdot \left(\mathbf{M}^{\mathbf{EM}}\right)_{out}^{\mathbf{M}^{\mathbf{EM}}}$$

, constrained by the image of ellipsoid in the un-observable error subspace, along each axis of the all-element space $\mathbf{R}^{\mathbf{A}}$ gives the 3σ extent of impact of unobservable error at all locations.

Extreme Values of Arbitrary Operators on a Constrained Surface

$$\Pi \cdot \mathbf{X} = \mathbf{Y}$$

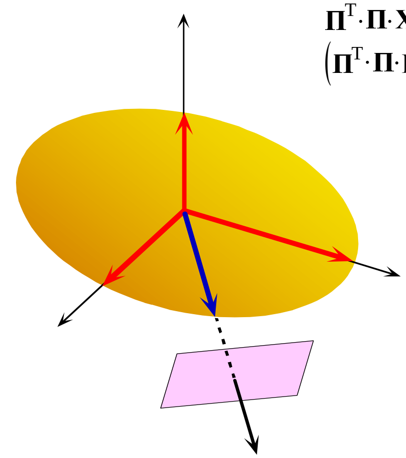
$$\mathbf{L}_\Pi = Y^T \cdot Y = X^T \cdot \Pi^T \cdot \Pi \cdot X$$

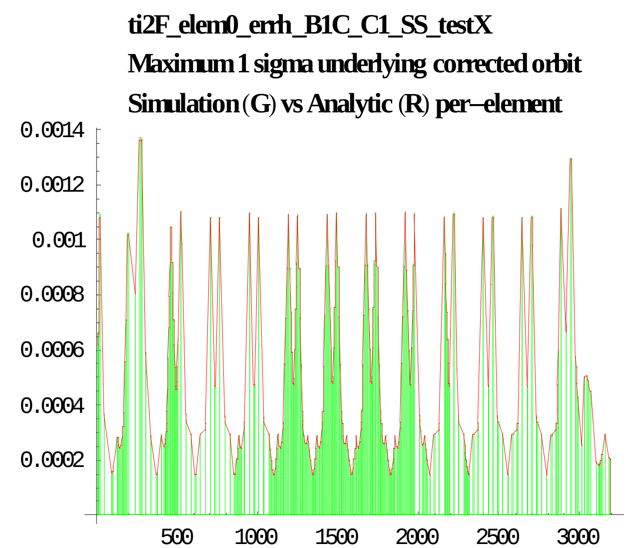
$$\nabla L_\Pi = \sum_{\mathbf{ijkm}} \hat{\mathbf{x}}_\mathbf{i} \frac{\partial \Pi_{\mathbf{jk}} X_\mathbf{k} \Pi_{\mathbf{jm}} X_\mathbf{m}}{\partial X_\mathbf{i}} = 2 \sum_{\mathbf{ijkm}} \hat{\mathbf{x}}_\mathbf{i} \delta_{\mathbf{ik}} \Pi_{\mathbf{jk}} \Pi_{\mathbf{jm}} X_\mathbf{m}$$

$$= 2 \sum_{\mathbf{ijm}} \Pi_{\mathbf{ji}} \Pi_{\mathbf{jm}} X_\mathbf{m} \hat{\mathbf{x}}_\mathbf{i} = 2 \Pi^\mathbf{T} \cdot \Pi \cdot \mathbf{X}$$

$$\mathbf{\Pi}^T \cdot \mathbf{\Pi} \cdot \mathbf{X} = \lambda \mathbf{E} \cdot \mathbf{X},$$

$$\left(\mathbf{\Pi}^T \cdot \mathbf{\Pi} \cdot \mathbf{E}^{-1} - \lambda\right) \cdot \mathbf{E} \cdot \mathbf{X} = \mathbf{0}$$



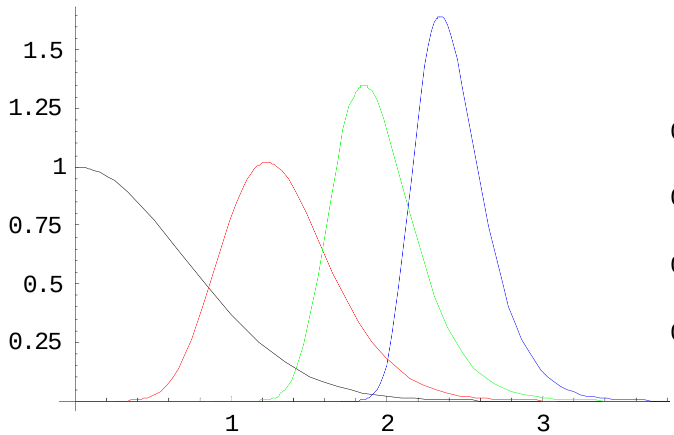


Multi-Dimensional Gaussian Distribution

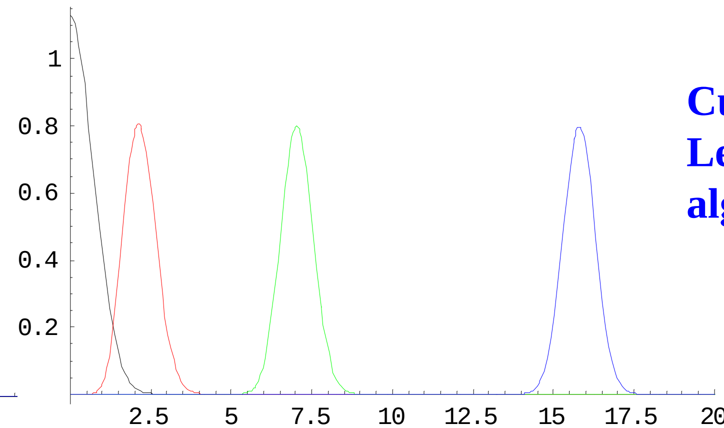
Probability Density Distribution in Mapped Space

- Most algorithms are developed without assuming Gaussian distribution.
- Much simpler mapping and inverse mapping properties are obtainable for Gaussian distributions.

Probability Density Distribution of Extrema and Length



N = 1, 10, 100 and 1000; abscissa: multiple of σ



N = 1, 10, 100 and 500; abscissa: multiple of length

Cutoff on Distribution of Length → Efficient algorithm developed

Singularity Related Issues

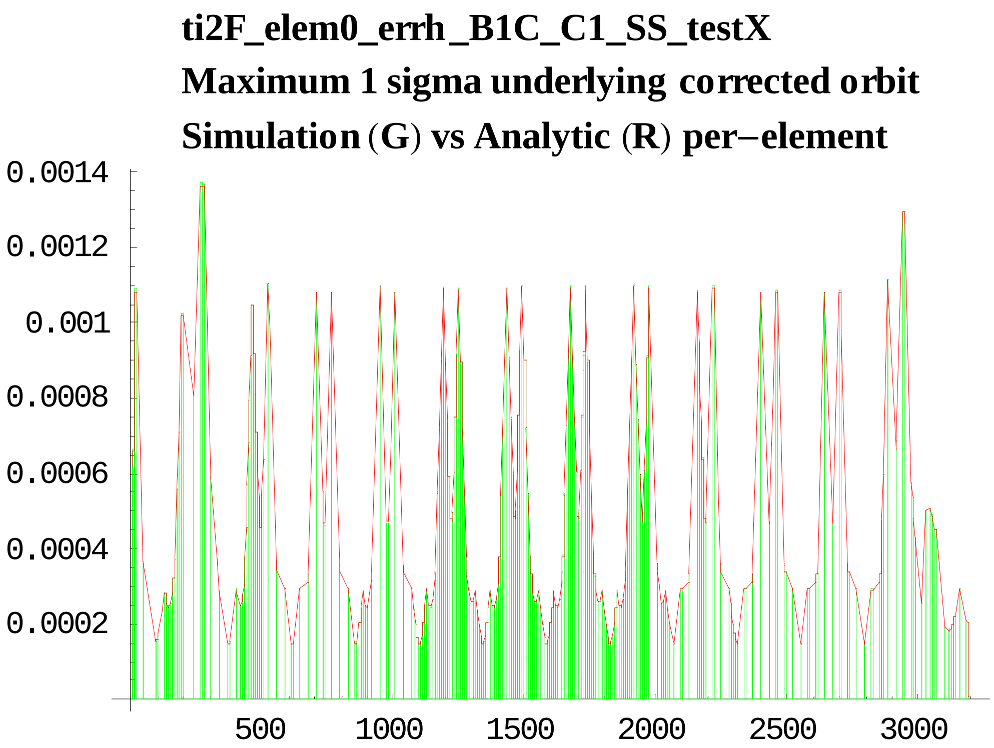
Singular Value Decomposition, Condition Number

Gram Determinant

Principal Axes

Exception Handling for Rank Deficiency and Near Singularity

Comparison between analytic formula & simulation of 20000 runs



Assembling Secondary Response Matrices

Recipes are useful only if we can construct response matrices capturing various physical processes

Observability and Impact of Unobservable Errors

Unobservable and observable effects at all elements

$$M_{\mathbf{M}}^{\text{unobs}} = \mathbf{M}^{\mathbf{EA}} \cdot \left(\mathbf{M}^{\mathbf{EM}}\right)_{\text{out}}^{\mathbf{M}^{\mathbf{EM}}},$$

$$M_{\mathbf{M}}^{\text{obs}} = \mathbf{M}^{\mathbf{EA}} \cdot \left(\left(\mathbf{M}^{\mathbf{EM}}\right)_{\text{in}}^{\mathbf{M}^{\mathbf{EM}}} \right)^{-1} M^{\mathbf{MM}}.$$

Alternative view with alignment and monitor errors on equal footing

$$M_{\mathbf{M}}^{\text{DS-unobs}} = \mathbf{K}_{\mathbf{A}} \cdot \Pi_{\mathbf{K}_{\mathbf{M}}}^{\perp},$$

$$\mathbf{K}_{\mathbf{A}} = \left(\mathbf{M}^{\mathbf{EA}} \oplus \mathbf{Z}^{\mathbf{MM}}\right),$$

$$\mathbf{Z}^{\mathbf{MM}} = Z_{ij}^{\text{MM}} = 0,\ i=1,2,....N_{A},\ j=1,2,....N_{M},$$

$$\mathbf{K}_{\mathbf{M}} = \left(\mathbf{M}^{\mathbf{EM}} \oplus \mathbf{M}^{\mathbf{MM}}\right).$$

Corrector Response to Errors and Limit on Correcting Power

Error to corrector a la SVD

$$M_{\mathbf{CME}}^{\text{resp}} = \mathbf{M}_{\mathbf{CM}}^{\dagger} \cdot \mathbf{M}^{\mathbf{EM}}.$$

Assuming unlimited monitoring power

$$M_{\mathbf{CAE}}^{\text{resp}} = \mathbf{M}_{\mathbf{CA}}^{\dagger} \cdot \mathbf{M}^{\mathbf{EA}}.$$

Correction of Monitored Orbit

Uncorrectable orbit at monitors and all-elements

$$M_{\mathbf{CM}}^{\text{uncorr}} = \Pi_{\mathbf{CM}}^{\perp} \cdot \mathbf{M}^{\mathbf{EM}}$$

$$M_{\mathbf{CA}}^{\text{uncorr}} = \Pi_{\mathbf{CA}}^{\perp} \cdot \mathbf{M}^{\mathbf{EA}}$$

Correctable orbit at monitors

$$M_{\mathbf{CM}}^{\text{corr}} = \Pi_{\mathbf{CM}}^{||} \cdot \mathbf{M}^{\mathbf{MM}}.$$

Orbit error at all elements induced by monitor offsets

$$M_{\mathbf{MCA}}^{\text{ind}} = \mathbf{M}^{\mathbf{CA}} \cdot \mathbf{M}_{\mathbf{CM}}^{\dagger} \cdot \mathbf{M}^{\mathbf{MM}}$$

Assembling Secondary Response Matrices - continued

Residue of Orbit Correction

Residue orbit after steering at all elements

$$\mathrm{M}_{\mathbf{CMA}}^{\mathrm{resid}} = \mathbf{M}^{\mathbf{EA}} - \mathbf{M}^{\mathbf{CA}} \cdot \mathrm{M}_{\mathbf{CM}}^{\mathrm{resp}} = \mathbf{M}^{\mathbf{EA}} - \mathbf{M}^{\mathbf{CA}} \cdot \mathbf{M}_{\mathbf{CM}}^{\dagger} \cdot \mathbf{M}^{\mathbf{EM}},$$
$$\mathrm{M}_{\mathbf{CAA}}^{\mathrm{resid}} = \mathbf{M}^{\mathbf{EA}} - \mathbf{M}^{\mathbf{CA}} \cdot \mathrm{M}_{\mathbf{CA}}^{\mathrm{resp}} = \mathbf{M}^{\mathbf{EA}} - \mathbf{M}^{\mathbf{CA}} \cdot \mathbf{M}_{\mathbf{CA}}^{\dagger} \cdot \mathbf{M}^{\mathbf{EA}}.$$

Residue orbit after steering at monitors

$$\mathrm{M}_{\mathbf{CMM}}^{\mathrm{resid}} = \mathbf{M}^{\mathbf{EM}} - \mathbf{M}^{\mathbf{CM}} \cdot \mathrm{M}_{\mathbf{CM}}^{\mathrm{resp}} = \mathbf{M}^{\mathbf{EM}} - \mathbf{M}^{\mathbf{CM}} \cdot \mathbf{M}_{\mathbf{CM}}^{\dagger} \cdot \mathbf{M}^{\mathbf{EM}}.$$

Implication on Errors and Correctors from Residual Orbit

$$\mathrm{M}_{\mathbf{EC}}^{\mathrm{rms\text{-}orb}} = \left(\mathbf{M}'^{\mathbf{EA}} \cdot \mathbf{P}_{\mathrm{E}} + \mathbf{M}^{\mathbf{CA}} \cdot \mathbf{P}_{\mathrm{C}}\right) \cdot \mathbf{M}^{\mathbf{MM}},$$

$$\mathbf{P} = \left(\mathbf{M}'^{\mathbf{EM}} \oplus \mathbf{M}^{\mathbf{CM}}\right)^{\dagger} = \begin{pmatrix} \mathrm{P}^{11} & \cdots & \mathrm{P}^{1,\mathrm{N_M}} \\ \vdots & \ddots & \vdots \\ \mathrm{P}^{\mathrm{N_E},1} & \cdots & \mathrm{P}^{\mathrm{N_E},\mathrm{N_M}} \\ \hline \mathrm{P}^{\mathrm{N_E}+1,1} & \cdots & \mathrm{P}^{\mathrm{N_E}+1,\mathrm{N_M}} \\ \vdots & \ddots & \vdots \\ \mathrm{P}^{\mathrm{N_E}+\mathrm{N_C},1} & \cdots & \mathrm{P}^{\mathrm{N_E}+\mathrm{N_C},\mathrm{N_M}} \end{pmatrix} = \begin{pmatrix} \mathbf{P}_{\mathrm{E}} \\ \mathbf{P}_{\mathrm{C}} \end{pmatrix}.$$

- Numbers of errors, monitors and correctors : N_E, N_M, and N_C.
- $\mathbf{M'^{EA}}$ & $\mathbf{M'^{EM}}$ not normalized to make all σ's unity.
- Enhancement by $\sqrt{N_M}$ to reflect use of RMS of residual orbit.
- Convert orbit to contributing error and scale down to proper error σ.

Assembling Secondary Response Matrices - continued

Actual Underlying Orbit Including Effects of Monitor Offset

Combined error actuator $\mathbf{A}^{\mathbf{EM}}$ of alignment-type and monitor errors

$$\mathbf{A}^{\mathbf{EM}} = \mathbf{A}^{\mathbf{E}} \oplus \mathbf{A}^{\mathbf{M}} = \left(A_1^E, \cdots A_{N_E}^E, A_1^M, \cdots A_{N_M}^M\right)^T$$

Underlying Orbit Error At monitors

$$\mathbf{R}^{\mathbf{UM}} = \prod_{\mathbf{CM}}^{\perp} \cdot \mathbf{M}^{\mathbf{EM}} \cdot \mathbf{A}^{\mathbf{E}} - \prod_{\mathbf{CM}}^{\parallel\,\parallel} \cdot \mathbf{M}^{\mathbf{MM}} \cdot \mathbf{A}^{\mathbf{M}},$$

$$\mathbf{M}_{\mathbf{EMM}}^{\mathrm{UOE}} = \left(\prod_{\mathbf{CM}}^{\perp} \cdot \mathbf{M}^{\mathbf{EM}}\right) \oplus \left(-\prod_{\mathbf{CM}}^{\parallel\,\parallel} \cdot \mathbf{M}^{\mathbf{MM}}\right).$$

Underlying Orbit Error At all elements

$$\mathbf{M}_{\mathbf{EMA}}^{\mathrm{UOE}} = \left(\mathbf{M}^{\mathbf{EA}} \oplus \mathbf{Z}^{\mathbf{MM}}\right) - \mathbf{M}^{\mathbf{CA}} \cdot \mathbf{M}_{\mathbf{CM}}^{\dagger} \cdot \left(\mathbf{M}^{\mathbf{EM}} \oplus \mathbf{M}^{\mathbf{MM}}\right),$$

$$\mathbf{R}^{\mathbf{UA}} = \mathbf{M}_{\mathbf{EMA}}^{\mathrm{UOE}} \cdot \mathbf{A}^{\mathbf{EM}},$$

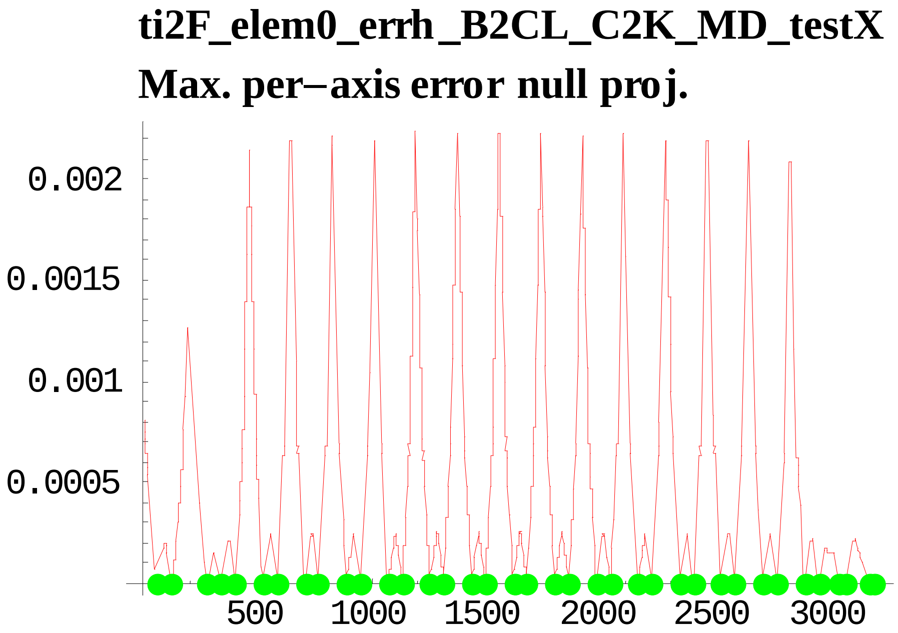
$$\mathbf{Z}^{\mathbf{MM}} = Z_{ij}^{MM} = 0,\ i=1,2,\ldots.N_A,\ j=1,2,\ldots.N_M.$$

Summary of Secondary Response Matrices

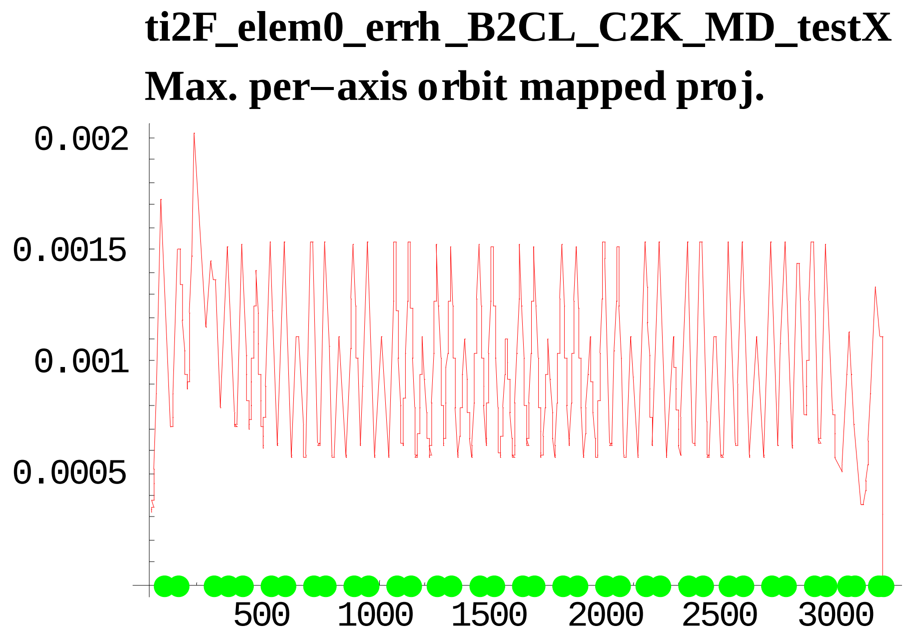
Response Matrix	Domain	Image Space	Physical Significance
$\mathbf{M}_{\mathbf{M}}^{\text{unobs}}$	$\mathbf{A}^{\mathbf{E}}$	$\mathbf{R}^{\mathbf{A}}$	Unobservable error
$\mathbf{M}_{\mathbf{M}}^{\text{obs}}$	$\mathbf{A}^{\mathbf{M}}$	$\mathbf{R}^{\mathbf{A}}$	Effect of observed orbit
$\mathrm{M}_{\mathbf{M}}^{\text{DS-unobs}}$	$\mathbf{A}^{\mathbf{EM}}$	$\mathbf{R}^{\mathbf{A}}$	Underlying orbit at all elem. under given monitor orbit
$\mathbf{M}_{\mathbf{CME}}^{\text{resp}}$	$\mathbf{A}^{\mathbf{E}}$	$\mathbf{A}^{\mathbf{C}}$	Corrector strength needed for error
$\mathbf{M}_{\mathbf{CAE}}^{\text{resp}}$	$\mathbf{A}^{\mathbf{E}}$	$\mathbf{A}^{\mathbf{C}}$	Corrector strength needed (unlimited monitor power)
$\mathbf{M}_{\mathbf{CM}}^{\text{uncorr}}$	$\mathbf{A}^{\mathbf{E}}$	$\mathbf{A}^{\mathbf{E}}$	Uncorrectable error
$\mathbf{M}_{\mathbf{CA}}^{\text{uncorr}}$	$\mathbf{A}^{\mathbf{E}}$	$\mathbf{A}^{\mathbf{E}}$	Uncorrectable error (unlimited monitor power)
$\mathbf{M}_{\mathbf{CM}}^{\text{corr}}$	$\mathbf{A}^{\mathbf{M}}$	$\mathbf{A}^{\mathbf{M}}$	Correctable monitored orbit or monitor error
$\mathbf{M}_{\mathbf{MCA}}^{\text{ind}}$	$\mathbf{A}^{\mathbf{E}}$	$\mathbf{R}^{\mathbf{A}}$	Monitor error induced orbit at all elements
$\mathbf{M}_{\mathbf{CMA}}^{\text{resid}}$	$\mathbf{A}^{\mathbf{E}}$	$\mathbf{R}^{\mathbf{A}}$	Residual orbit at all elements after correction at monitors
$\mathbf{M}_{\mathbf{CAA}}^{\text{resid}}$	$\mathbf{A}^{\mathbf{E}}$	$\mathbf{R}^{\mathbf{A}}$	Residual orbit at all elements (unlimited monitor power)
$\mathbf{M}_{\mathbf{CMM}}^{\text{resid}}$	$\mathbf{A}^{\mathbf{E}}$	$\mathbf{R}^{\mathbf{M}}$	Residual orbit at monitors after correction
$\mathbf{M}_{\mathbf{EC}}^{\text{rms-orb}}$	$\mathbf{A}^{\mathbf{M}}$	$\mathbf{R}^{\mathbf{A}}$	Effect implied by observed orbit on error and corrector
$\mathbf{M}_{\mathbf{EMM}}^{\text{UOE}}$	$\mathbf{A}^{\mathbf{EM}}$	$\mathbf{R}^{\mathbf{UM}}$	Real underlying orbit error at monitors
$\mathbf{M}_{\mathbf{EMA}}^{\text{UOE}}$	$\mathbf{A}^{\mathbf{EM}}$	$\mathbf{R}^{\mathbf{UA}}$	Real underlying orbit error at all elements

Performance Criteria

Maximally unobservable orbit error at all elements in [m] along the beam line in [m]; dots: position of monitors



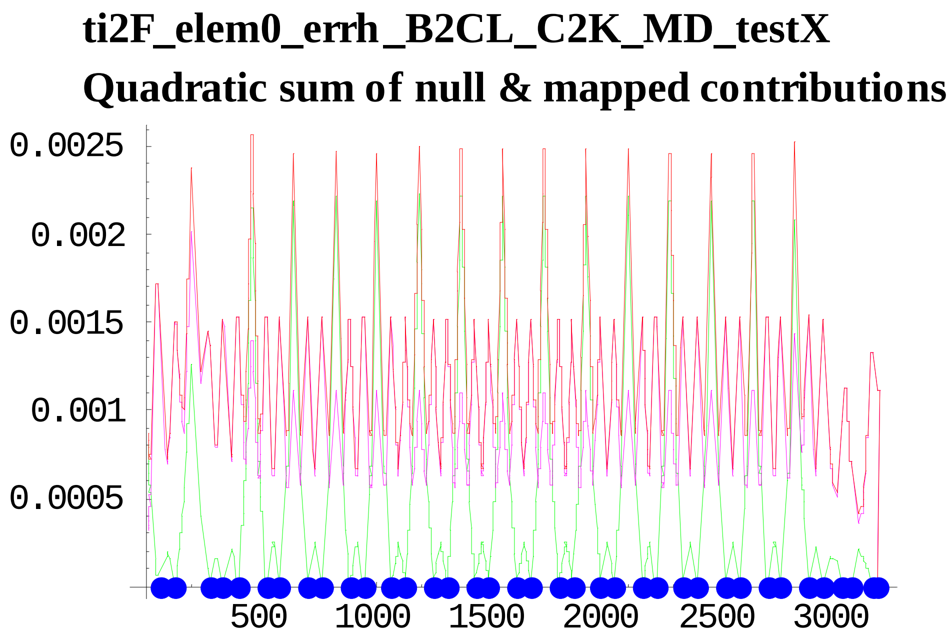
Maximal orbit error in [m] implied by unit RMS of observed residual orbit along the beam line in [m]; dots: position of monitors.

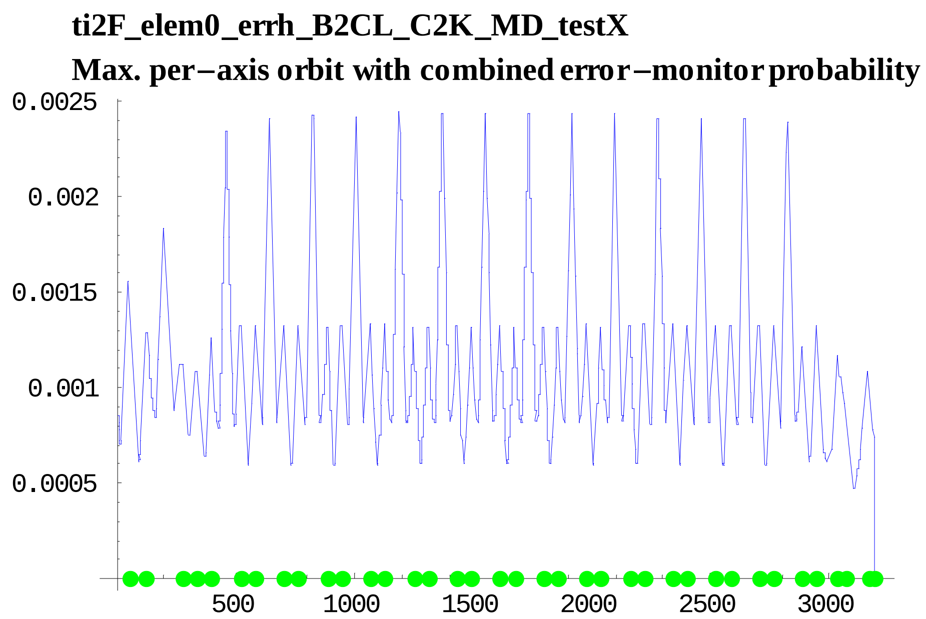


Performance Criteria - continued

Maximal orbit error in [m] along the beam line in [m] (quadratic sum of previous two graphs scaled by 3σ error); dots: position of monitors.

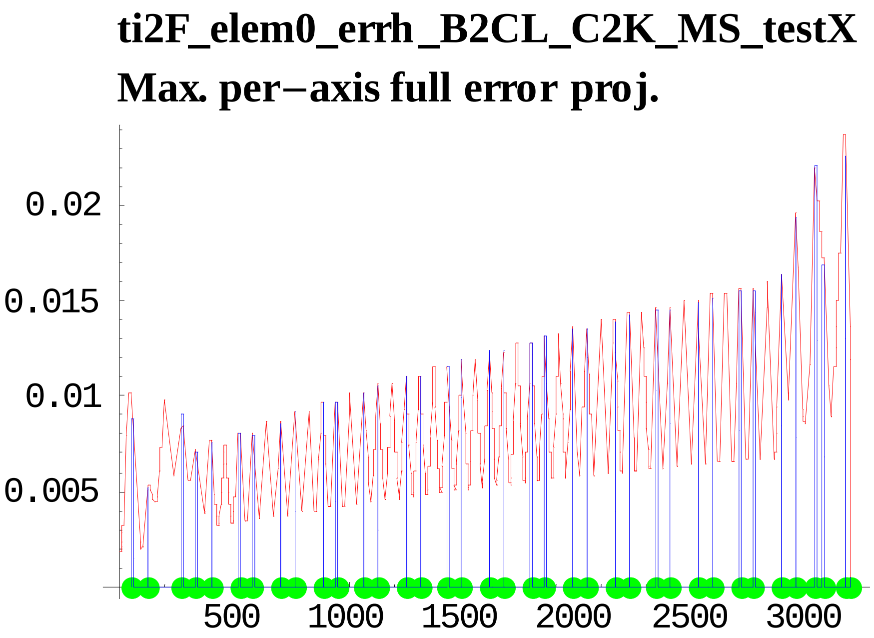
Maximal orbit error (alternative view: equal probabilistic footing) in [m] along the beam line in [m]; dots: position of monitors.





Performance Criteria - continued

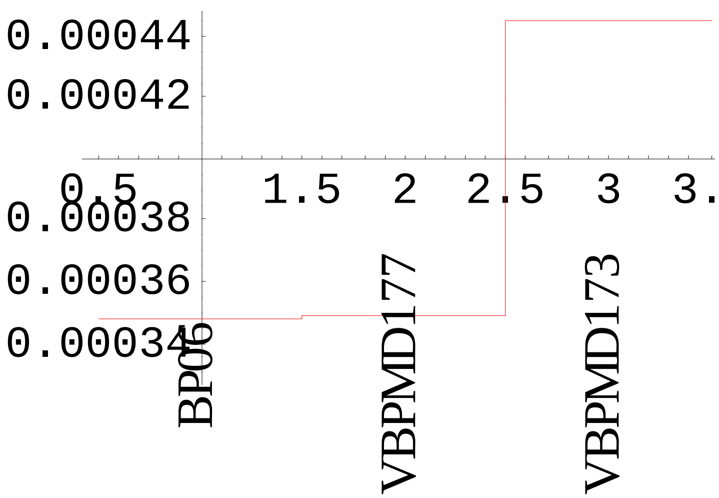
Projected 3σ error onto orbit space
in [m] along the beam line in [m];
dots: position of monitors



Performance Criteria - continued

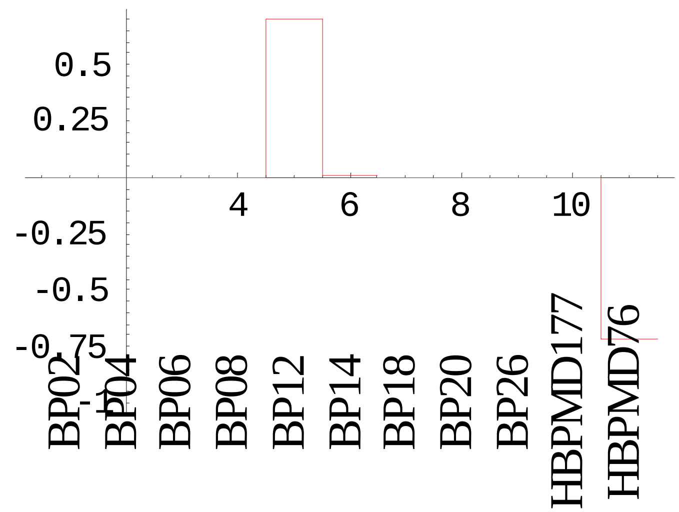
Example of ordered removal of redundant monitors based on singular values.

cngs_elem0_errv_NGH _MCHA _MS _
Singular values corresp. to removed m



Example of orbit pattern with very small singular values, with total magnitude normalised to 1, along the beam line in [m].
→ Singular combination monitors only BPM offset itself.

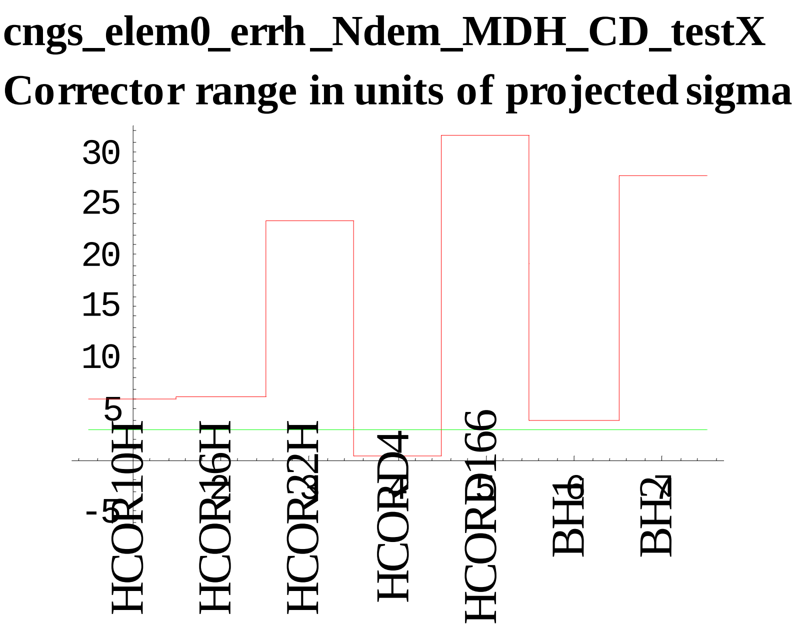
cngs_elem0_errh_Ndem_MDH_MS_testX
Singular orbit pattern
Leading contributors:
HBPMD76 BP12 BP14 BP06

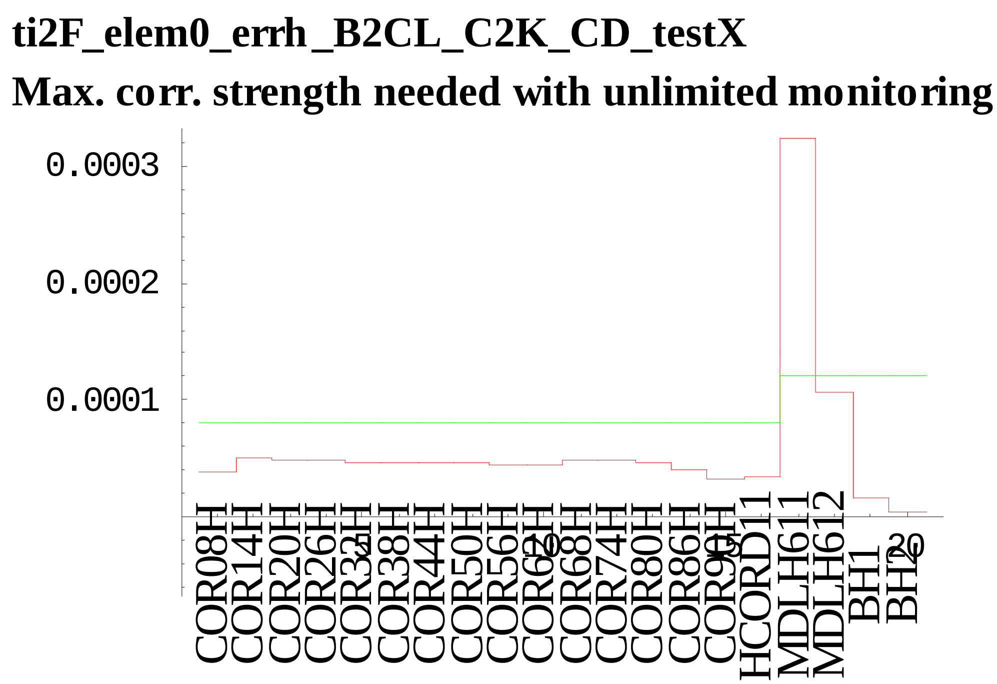


Performance Criteria - continued

Correction range of correctors in [σ] of the global error distribution.

Corrector strengths in [rad] when the global error magnitude corresponds to 3 σ, assuming unlimited monitoring power; green line: physical corrector limits.

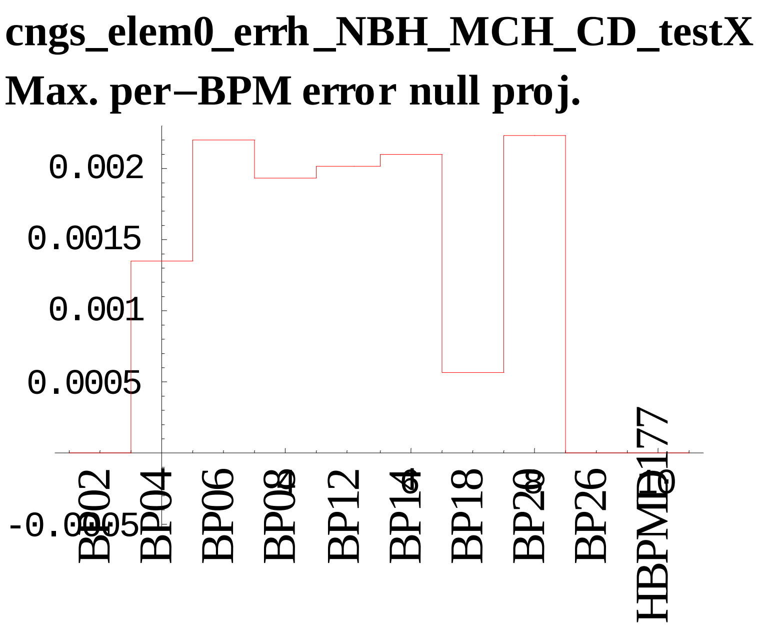


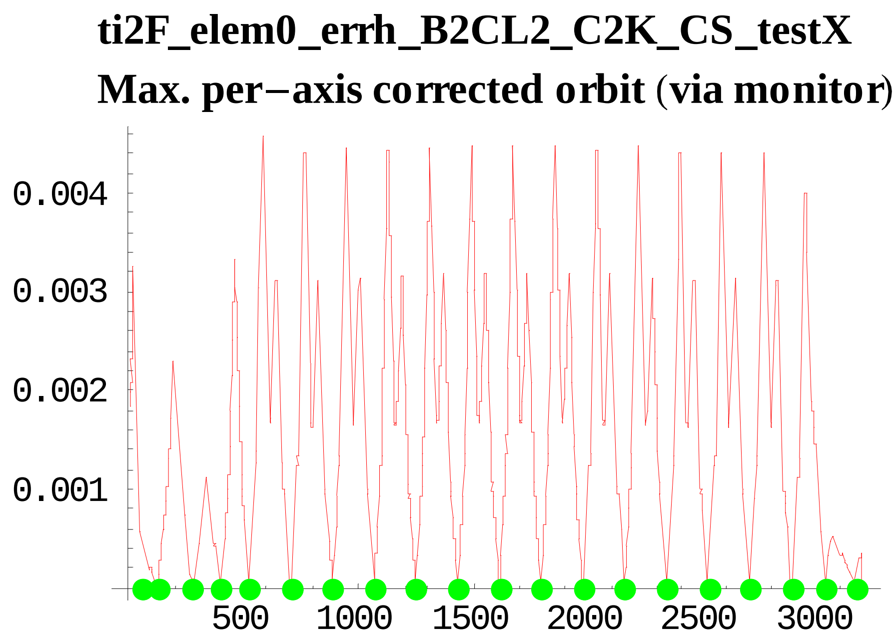


Performance Criteria - continued

Uncorrectable orbit at all monitors using existing monitor configuration.

Uncorrectable orbit at all elements in [m] along the beam line in [m].



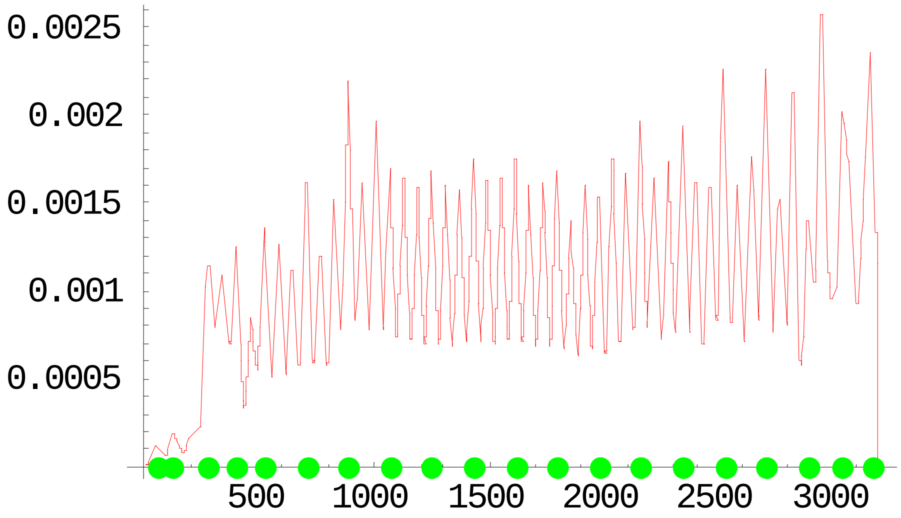


Performance Criteria - continued

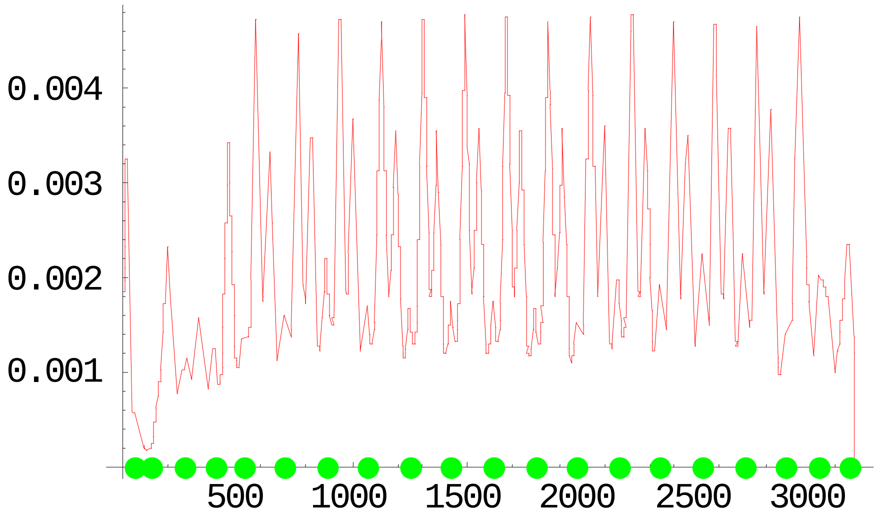
Orbit error implied by observed orbit at the monitors, scaled by 3σ error, in [m] along the beam line in [m]; dots: location of beam position monitors.

Quadratic sum of the two previous orbit envelopes in [m] along the beam line in [m]; dots: location of beam position monitors

ti2F_elem0_errh_B2CL2_C2K_CS_testX
Scaled per−axis from rms monitor resdual



ti2F_elem0_errh_B2CL2_C2K_CS_testX
Quad. sum of uncorr. orbit & rms monitor resdual

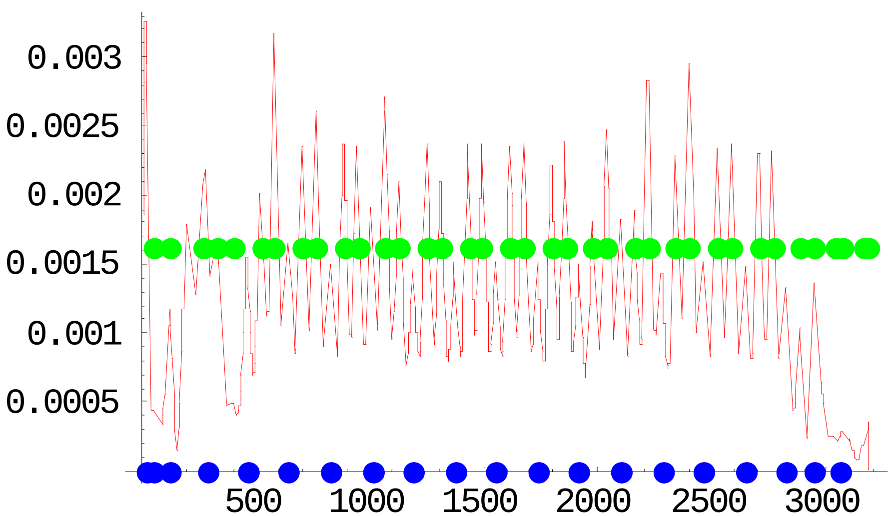


Performance Criteria - continued

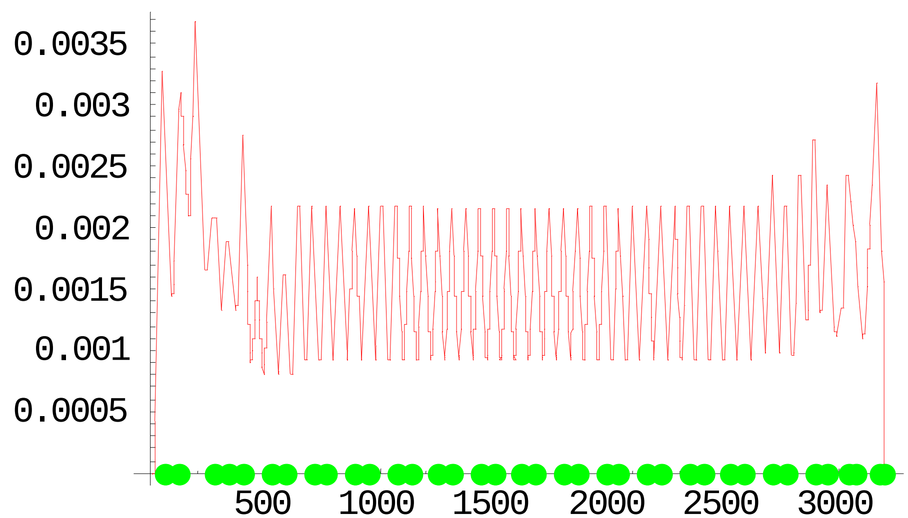
Uncorrectable orbit at all elements in [m], assuming unlimited monitoring power along the beam line in [m]; black → Fundamental measure of corrector configuration

Monitor offset induced orbit error in [m] along the beam line in [m]; dots: location of beam position monitors.

ti2F_elem0_errh_B2CL_C2K_CD_testX
Max. null space orbit with unlimited monitoring po



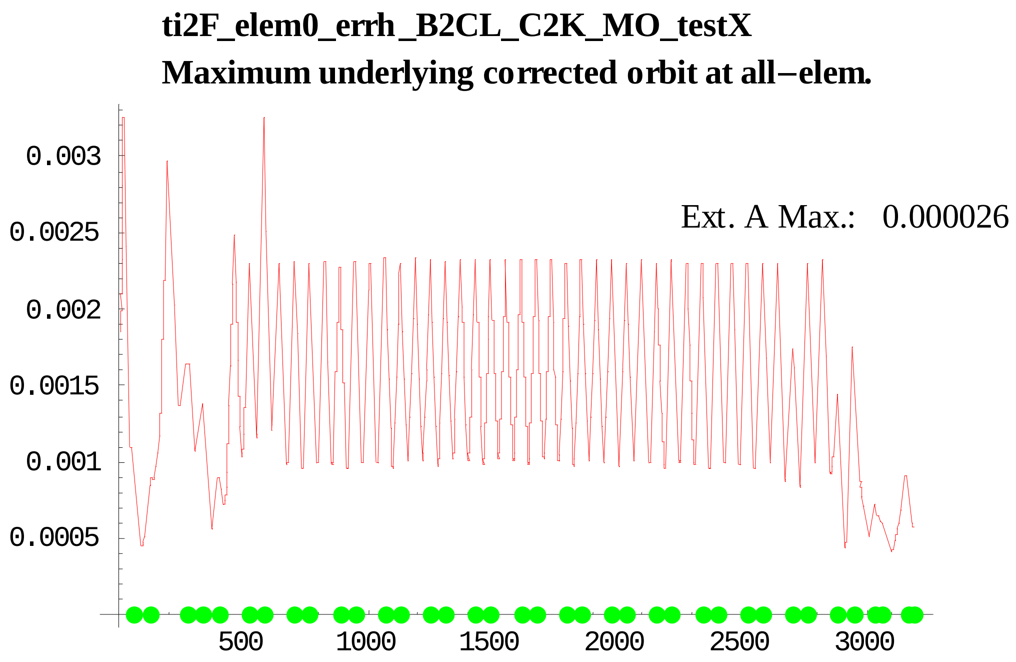
ti2F_elem0_errh_B2CL_C2K_CS_testX
Max. per−axis monitor error induced orbit



Performance Criteria - continued

Actual underlying orbit error after correction in [m] along the beam line in [m]; dots: location of beam position monitors.

Includes all alignment-type errors and monitor errors
→ Includes possibility of BPM lying

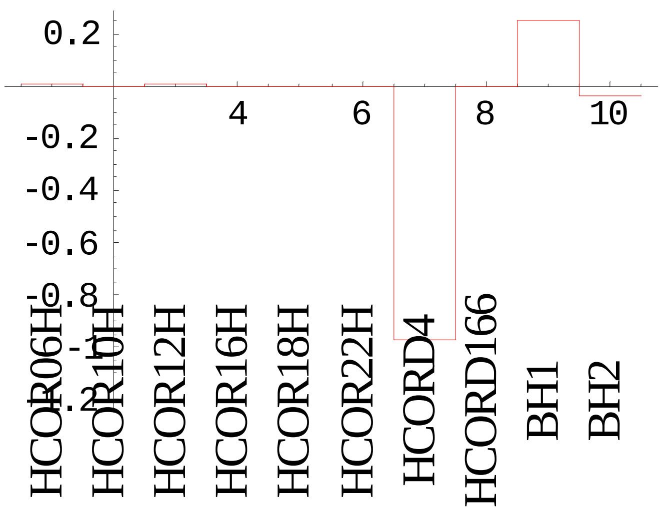


Performance Criteria - continued

Corrector surplus
→ Excessive correction

Example of near degenerate corrector with total magnitude normalized to 1 along the beam line in [m].

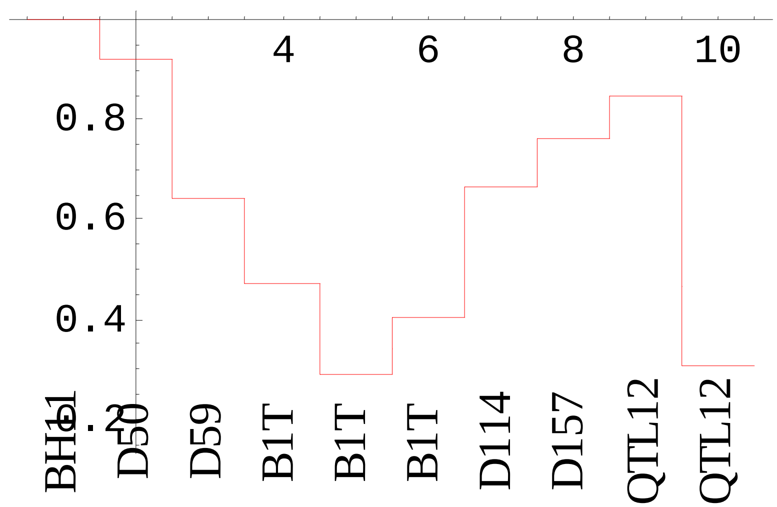
cngs_elem0_errh_NBH_MCHA_CS_testX
Singular corrector pattern



Corrector deficit

Example of new correctors sorted by projection with principal axis of error ellipsoid mapped onto monitor space.

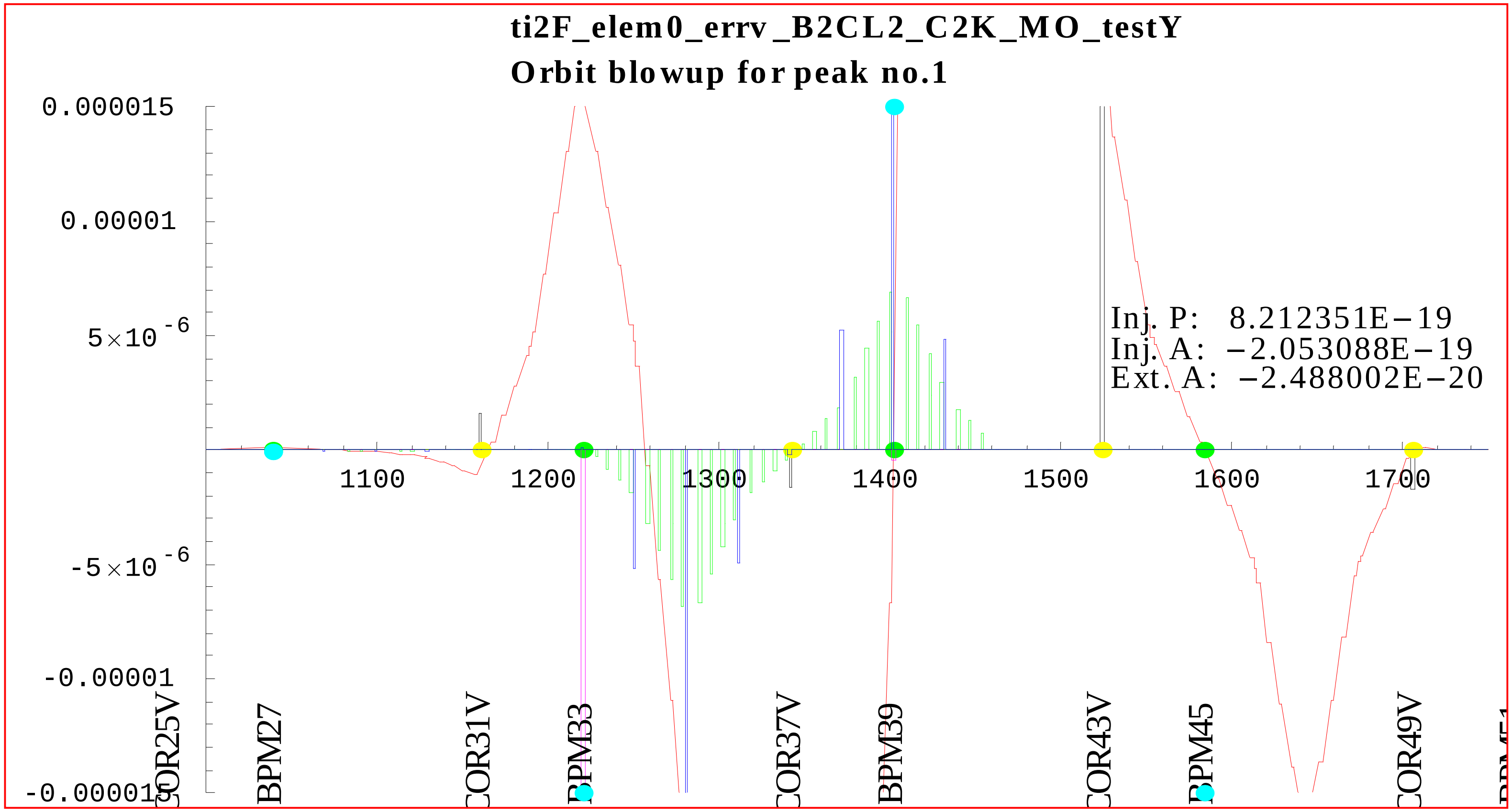
cngs_elem0_errv_NGH_MCHA_CD_testY
Sorted new corrector proj.(PA based, truncated)



Studying Failure Modes

→What caused a particular orbit peak, or corrector maxing out?

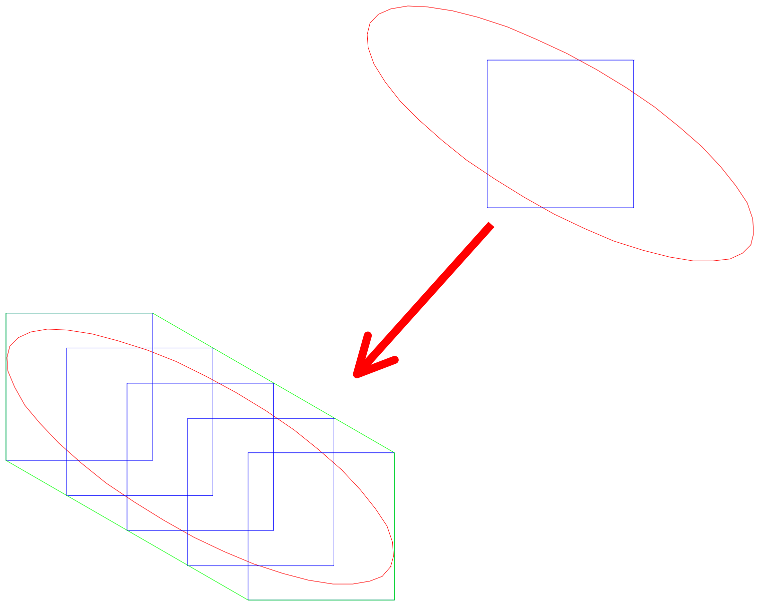
→ Decomposing an offending peak into underlying error combination



green dot: BPM, **cyan** dot: initial orbit (/10, clipped), **yellow** dot: corrector, **red** line: underlying error after correction (/10), **black** bar: corrector, **magenta** bar: BPM offset (/10), **red** bar: dipole field kick (*10), **green** bar: dipole roll kick (*10), **blue** bar: quad offset kick.

Configuration Optimization

Ability to improve on an existing configuration



Structural Defect Detection and Correction

Well-Established Recipes Based on Analytic Methods

- Adding monitors by unobservable error-induced orbit
- Adding correctors by uncorrectable residual orbit
- Adding correctors by principal axes of error ellipsoid mapped onto monitor space
- Removing monitors by null space/SVD analysis
- Removing correctors by null space/SVD analysis

Numerical Fine-Tuning

Effect of configuration parameter change may be algebraically intractable.

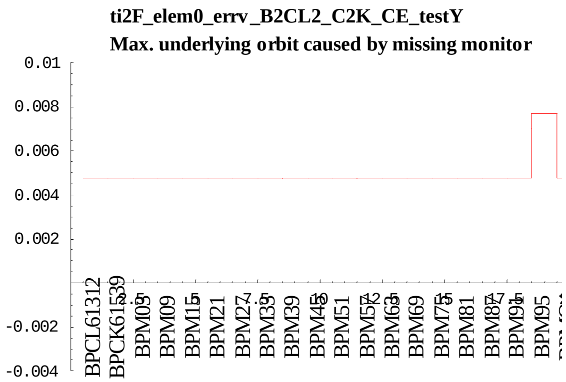
e.g., Locations of correctors may have big impact on the orbit envelope, but interplay between competing factors prevents an analytic optimization.

→ Scanning of the parameter space with one **analytic** performance criterion as merit function.
→ Fast: performance criteria are analytic.
→ Unambiguous answers not possible with simulation.

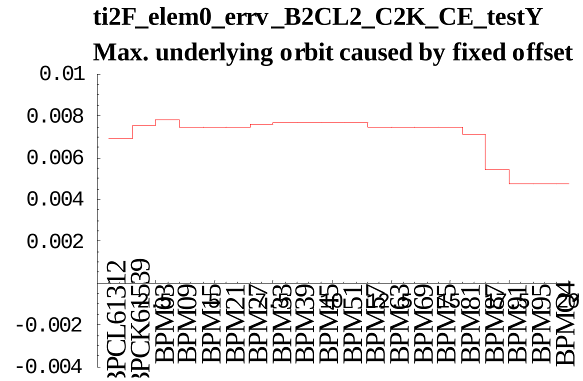
Critical Elements

- Again relying on **analytic** performance criteria, impact of element failures can be efficiently evaluated with unambiguous outcome.
- Elements can be ordered according to criticality, or their impact on the performance criteria in various failure modes.
- Many identified failures can be fixed by the optimization recipes.

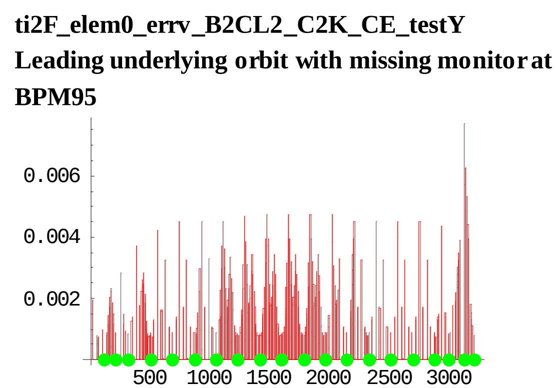
Missing monitor



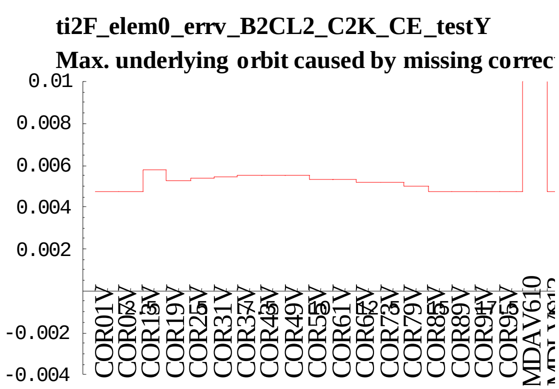
Large monitor offset



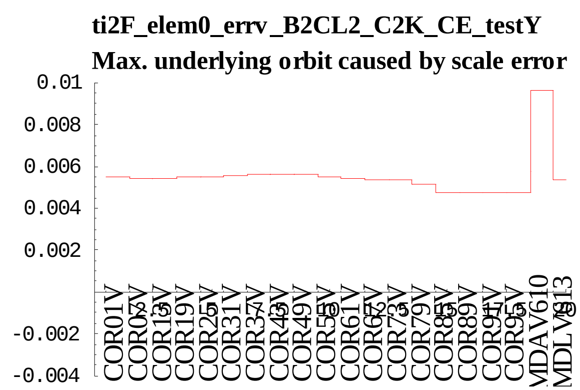
Performance criterion used:
Underlying corrected 3σ orbit for a particular missing monitor.



Missing corrector



Large corrector scaling error



Application to LHC Transfer Lines

Collaborators: V.Mertens, B. Goddard, M. Meddahi

- Combined length of about **5.6 km** using over 700 room-temperature magnets.
- Mainly periodic arcs with 90° per cell FODO lattice, 4 bending magnets per half-cell, and a half-cell length of 30.3 meters. Combined number of periods ~**90**.
- Matching sections connecting transfer lines to SPS and to LHC

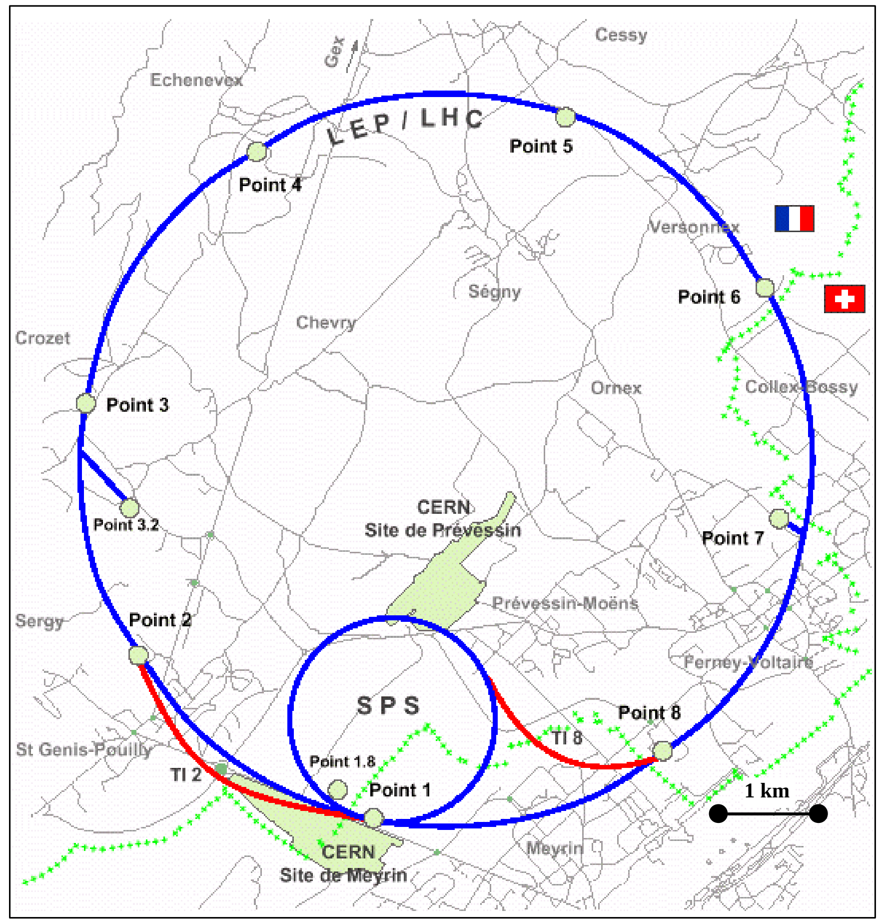
Challenges:

- Maximal vertical trajectory excursion allowed **: ±4.5 mm**.
- Cost optimization: All BPM's are **single-plane**.

Evolution:

- **F**irst study (1997) led to the adoption of "2-in-4" configuration, a one-to-one scheme.
- Analytic method developed (2000), ideal for improving matching sections.
- Allowed fast evaluation of scenarios in periodic sections, even discovered possibility for major improvement.

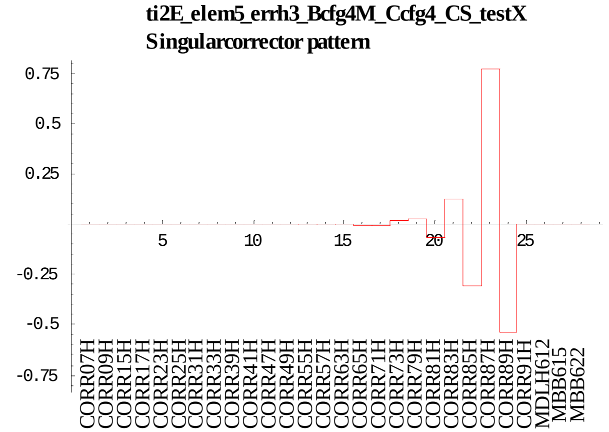
Configuration still in evolution



Development of Analytic Program

- All design errors translated into probability distributions
-
- All generalized response matrices constructed from MAD output and different combinations of
  - Representative set of elements
  - Errors
    - Injection (P & A)
    - Dipole filed error
    - Dipole roll
    - Quad offset
    - Monitor offset
  - Monitors
    - Position (monitored & un-monitored)
    - End angle
  - Correctors
    - Regular corrector
    - Dipoles (with length effect)
    - Dipole strings
- Various one-to-one schemes with different periodicity evaluated using analytical performance criteria.
- Evolution into the over-constrained regime in periodic sections
- Optimization recipes (analytical & numerical) applied in non-periodic sections to bring envelope everywhere within spec.
- Cross-calibration with simulation.
- Close inspection of performance in localized areas – Numerical optimization
- Critical elements in various failure modes

Even for the periodic lattice, improvements can happen
(although maybe not through simulation)

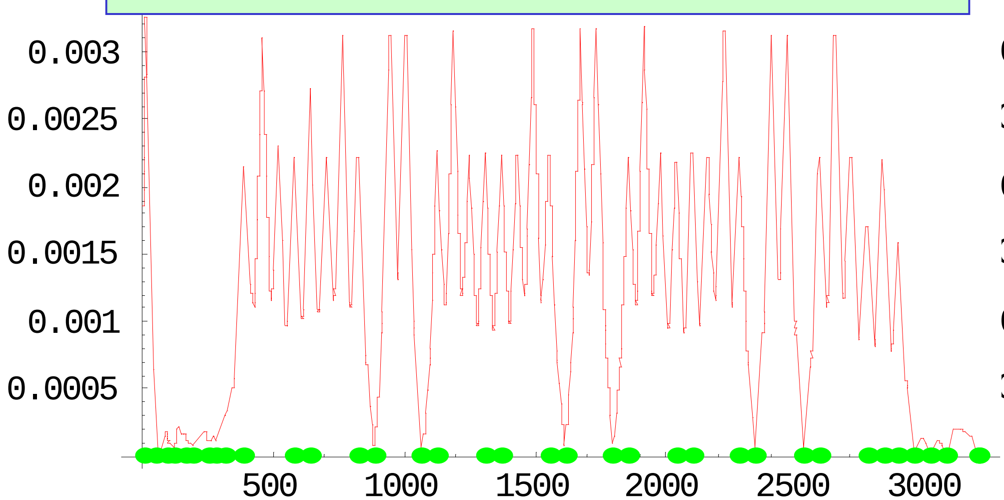


Slight corrector singularity observed in 2-in-4 scheme → May need corrector reduction

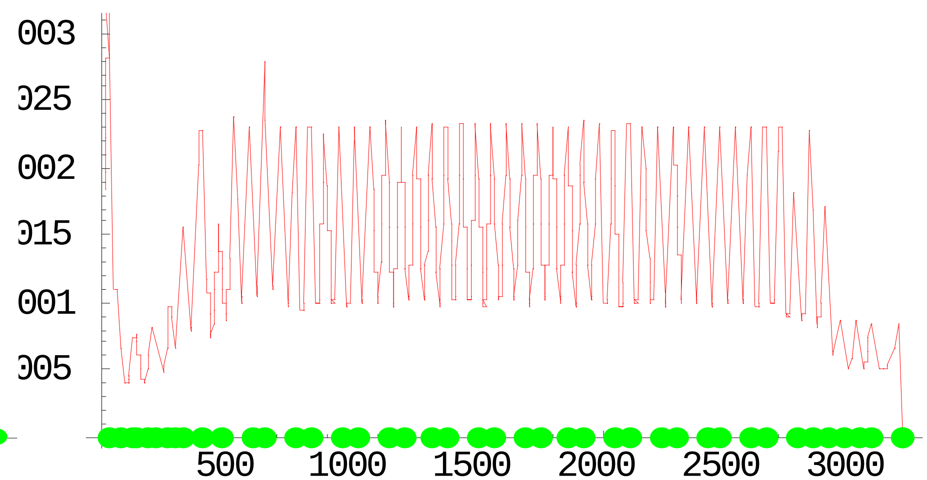
Immediate option for corrector reduction
→ 1-in-3

→ Balance under-monitored locations against over-monitored ones

Outcome:
Corrector 1-in-3, Monitor 2-in-3
More balanced and much smaller envelope.
(And cheaper)



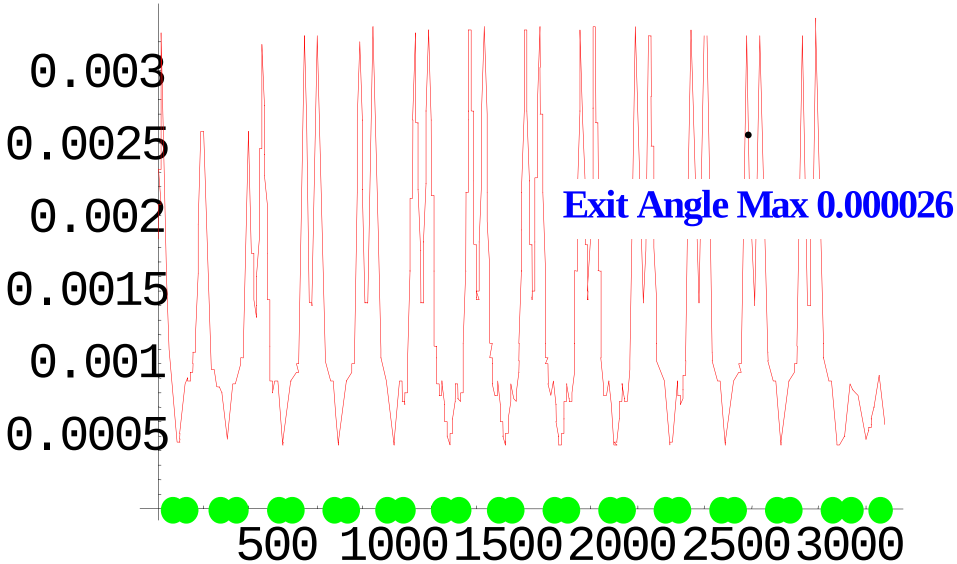
(Display does not include monitor offset errors)



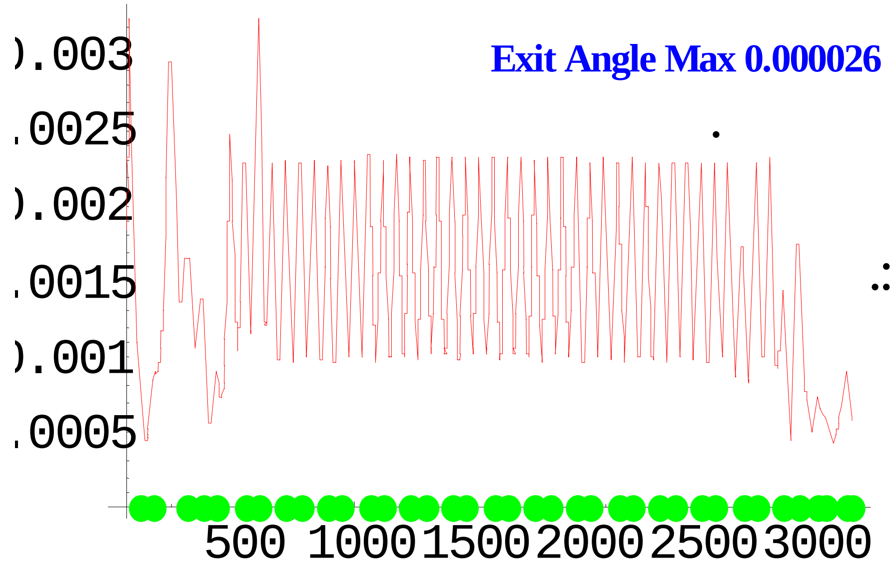
(Display includes monitor offset errors)

“Official” Scenarios

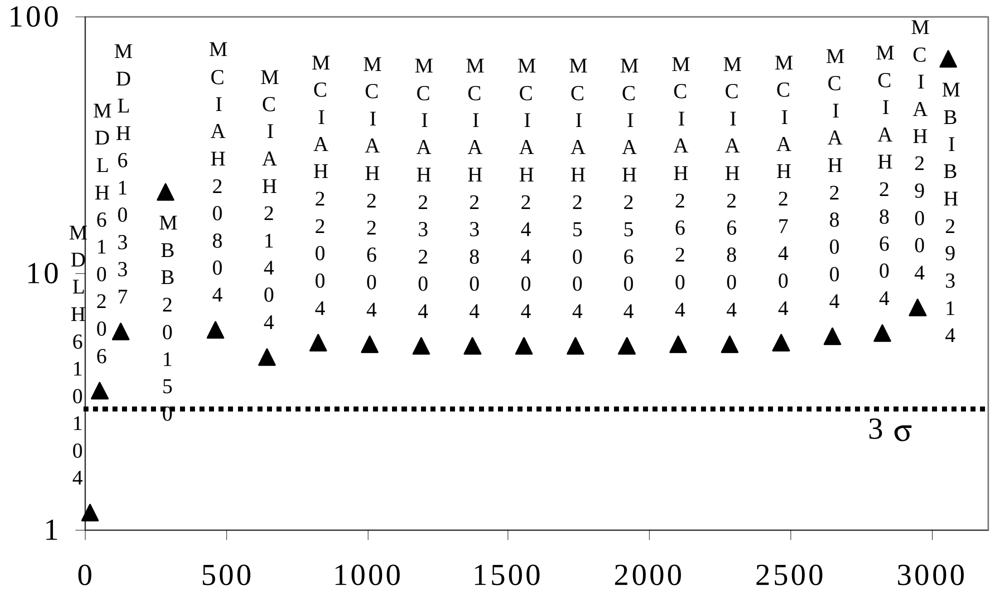
TI 2: 2-in-4 for monitors, 2-in-4 for correctors, horizontal plane



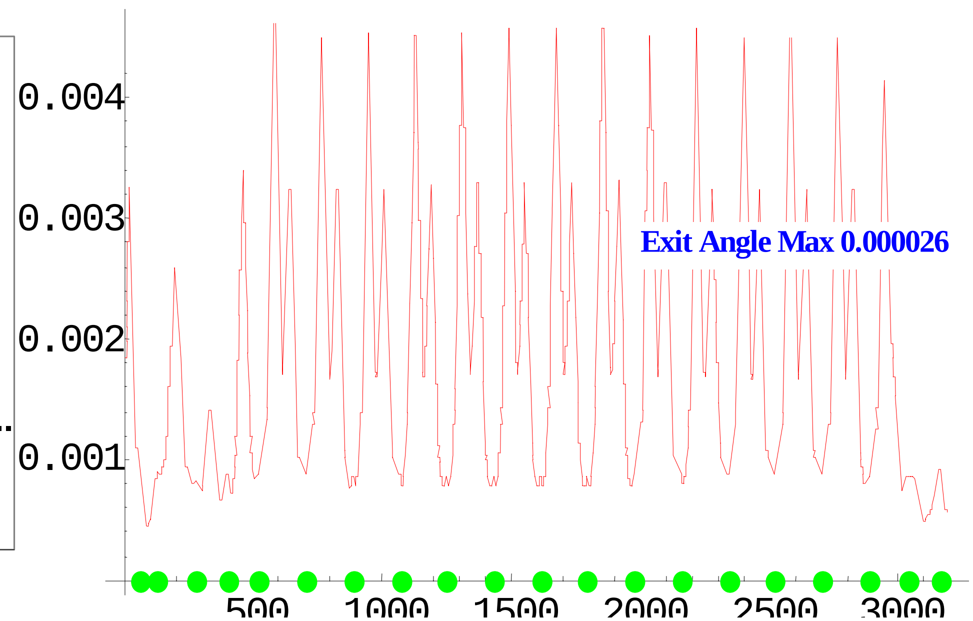
TI 2: 2-in-3 for monitors, 1-in-3 for correctors, horizontal plane



TI 2: 2-in-3 for monitors, 1-in-3 for correctors, horizontal plane



TI 2: 1-in-3 for monitors, 1-in-3 for correctors, horizontal plane



Critical Elements

Disabled Monitor		Fixed Monitor Offset of 3 mm		Disabled Corrector		Fixed Corrector Scale Error of 50 %	
2-in-4 Monitor and Corrector (Horizontal)							
BPCL610211	9.65*	BPMIH20404	7.10	MDLH610104	10.18¶	MDLH610206	8.99¶
BPCL610340	7.17*	BPMIH28404	6.37	MDLH610206	6.96¶	MDLH610104	8.82¶
BPMIH29304	6.86*	BPMIH25204	6.22	MCIAH21004	4.47	MDLH610337	4.37
BPMYBQ5L2	5.20‡	BPMIH22004	6.21	MCIAH28204	4.01	MCIAH28204	4.21
BPMIH29004	4.84	BPMIH25804	6.21	MCIAH25004	3.96	MCIAH25004	4.15
2-in-4 Monitor and Corrector (Vertical)							
BPMIV29504	6.40*	BPCK610539	7.52	MDAV610013	17.65¶	MDAV610013	9.62¶
BPCL610312	4.63*	BPMIV20504	6.45	MCIAV24904	4.14	MCIAV24904	4.32
BPMIV25304	3.46	BPMIV25104	6.37	MCIAV24304	4.10	MCIAV24304	4.28
BPMIV24504	3.45	BPMIV23704	6.30	MCIAV24104	4.09	MCIAV24104	4.27
BPMIV25904	3.45	BPMIV24504	6.30	MCIAV23504	4.07	MCIAV23504	4.26
2-in-3 Monitor, 1-in-3 Corrector (Horizontal)							
BPCL610340	105.61*	BPCL610340	5.81	MDLH610104	10.18¶	MDLH610206	9.12¶
BPMYBQ5L2	10.53‡	BPMIH20204	4.49	MDLH610206	7.24¶	MDLH610104	8.94¶
BPCL610211	9.65*	BPMIH28204	4.15	MCIAH21404	4.69	MDLH610337	4.88
BPMIH28804	5.84	BPMIH20404	4.12	MCIAH23804	3.94	MCIAH21404	4.14
BPMIH22804	4.59	BPMIH21204	4.08	MCIAH24404	3.91	MCIAH22604	4.01
2-in-3 Monitor, 1-in-3 Corrector (Vertical)							
BPCK610539	19.06*	BPCK610539	7.74	MDAV610013	17.65¶	MDAV610013	9.81¶
BPMIV28704	11.74†	BPCL610312	4.82	MCIAV21304	4.51	MDLV610304	5.04
BPMIV29504	6.14†	BPMIV28104	4.02	MCIAV24304	4.08	MCIAV24304	4.17
BPCL610312	5.05*	BPMIV20304	3.89	MCIAV24904	4.07	MCIAV23704	4.16
BPMIV24504	4.76	BPMIV21104	3.88	MCIAV23704	4.07	MCIAV23104	4.13
1-in-3 Monitor and Corrector (Horizontal)							
BPCL610211	9.65*	BPMIH25804	7.51	MDLH610104	10.18¶	MDLH610206	9.86¶
BPCL610340	7.24*	BPMIH21604	7.49	MDLH610206	6.97¶	MDLH610104	8.96¶
BPMYBQ5L2	7.11‡	BPMIH24604	7.48	MCIAH21404	5.56	MCIAH25604	5.44
BPMIH28204	5.76*	BPMIH23404	7.48	MCIAH25604	5.30	MCIAH24404	5.43
BPMIH28804	5.60	BPMIH24004	7.48	MCIAH24404	5.30	MCIAH23804	5.43
1-in-3 Monitor and Corrector (Vertical)							
BPMIV29504	7.70*	BPMIV20304	7.82	MDAV610013	18.35¶	MDAV610013	9.62¶
BPMIV23904	4.76*	BPMIV25104	7.67	MCIAV21304	5.77	MCIAV24304	5.65
BPMIV24504	4.76*	BPMIV23304	7.67	MCIAV24304	5.52	MCIAV24904	5.64
BPMIV25104	4.76	BPMIV24504	7.67	MCIAV24904	5.51	MCIAV23704	5.63
BPMIV22704	4.75	BPMIV23904	7.67	MCIAV23704	5.50	MCIAV23104	5.58

*** Artefact caused by near singularity easily correctable by disabling correctors. See main text.**

~~† Artefact caused by phase anomaly, easily correctable by disabling correctors. See main text.~~

‡ Artefact caused by loss of anchoring point downstream. See main text.

¶ Artefact caused by insufficient leverage for correcting injection error. See main text.

Extension of Method & Other Applications

Method:

- Algorithmic improvements
- General formulation for periodic lines
- Extension to other types of systems
  - Multiple lines with common elements
  - Finite recirculation
  - Acceleration, coupled lines, ……
  - Closed orbit for storage rings
- Zero-th order configuration → Populate a beam line with initial orbit correction configuration → least redundant while satisfying all performance specs.

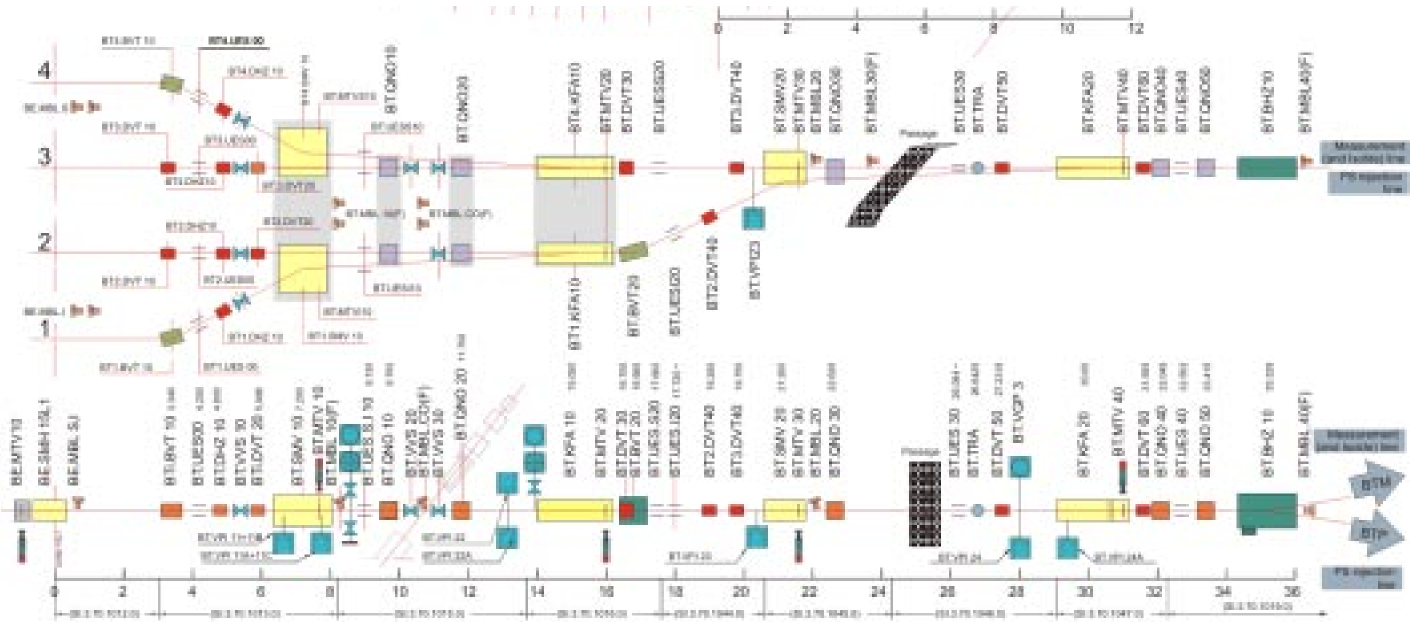
Other Applications:

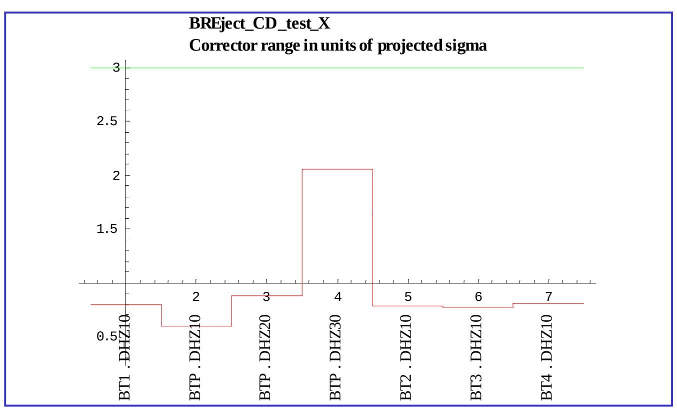
- CNGS (CERN-Neutrino-to-Gran-Sasso, exit angle criterion)
- PS Booster Ejection (4-line extraction, 2-stage recombination)
- TT2-TT10 (PS to SPS)
- CLIC Test Facility EPA (5-turn recirculation, single injection)
- LCLS (half quads eliminated)
- CEBAF 12 GeV Upgrade

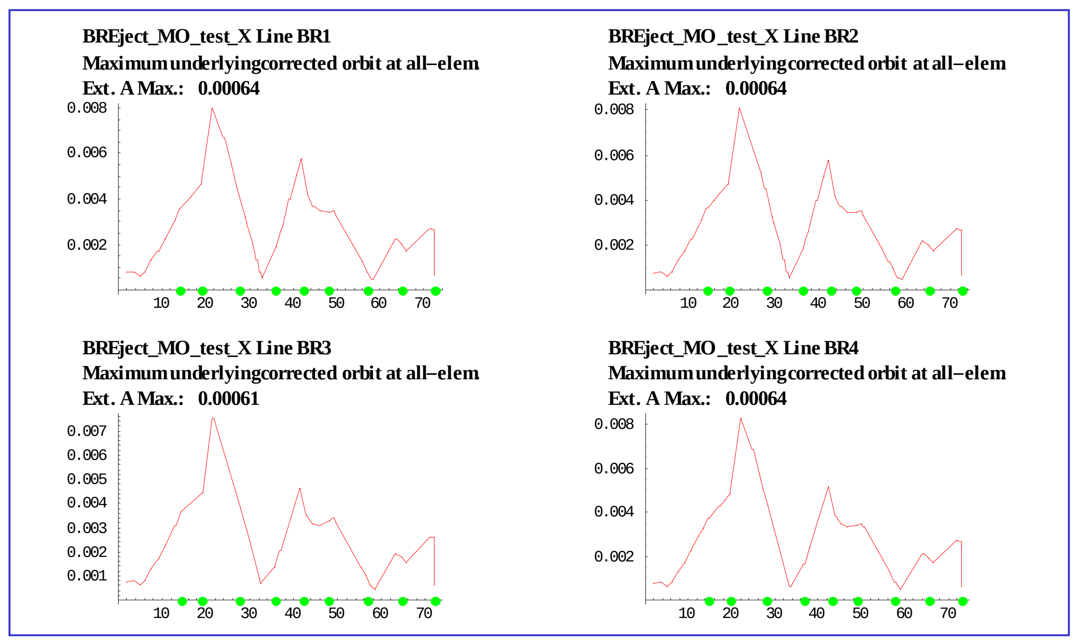
Other

CERN PS Booster Multi-Line Extraction System

Input from M. Lindroos, A. Jansson

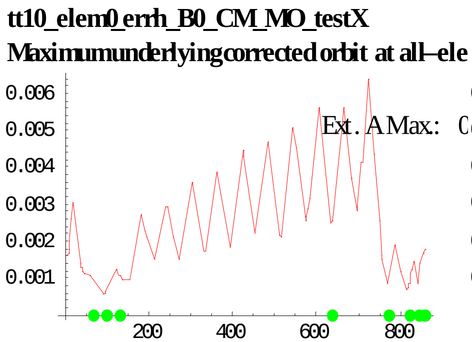


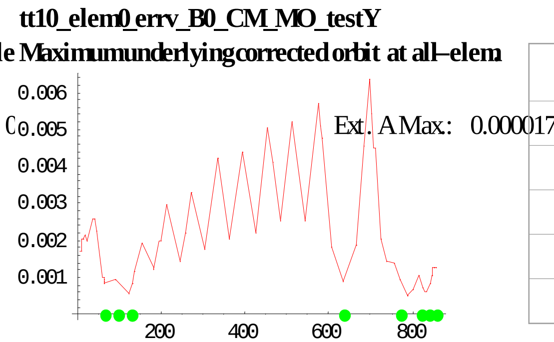




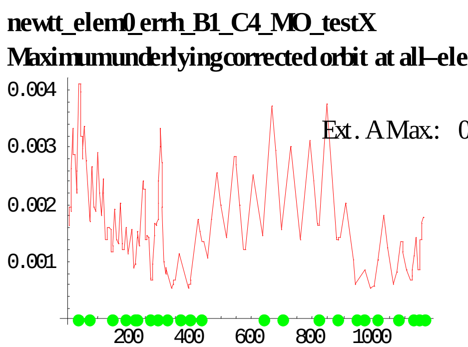
CERN PS-SPS Line (TT2-TT10) Upgrade – Improving an Existing System

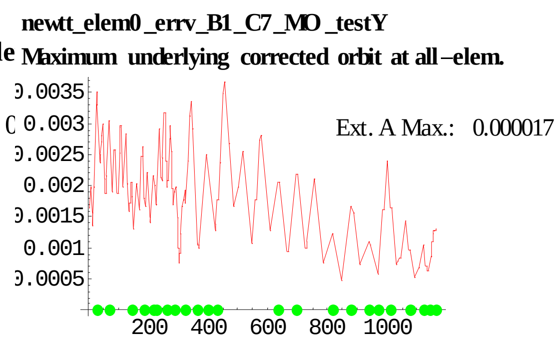
Input from G. Arduini, M. Giovanozzi

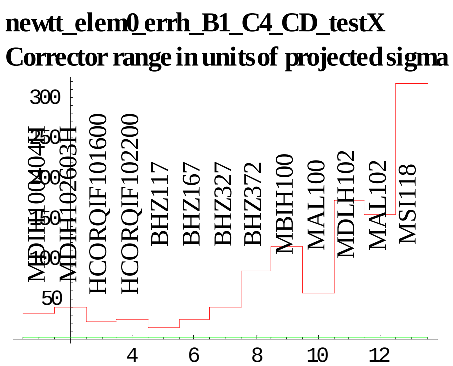


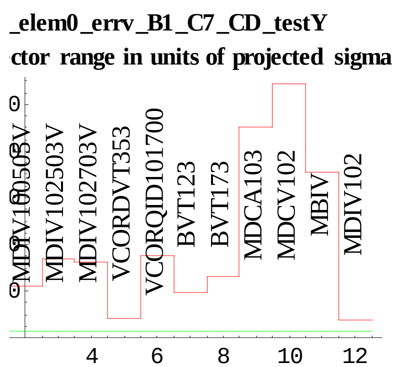


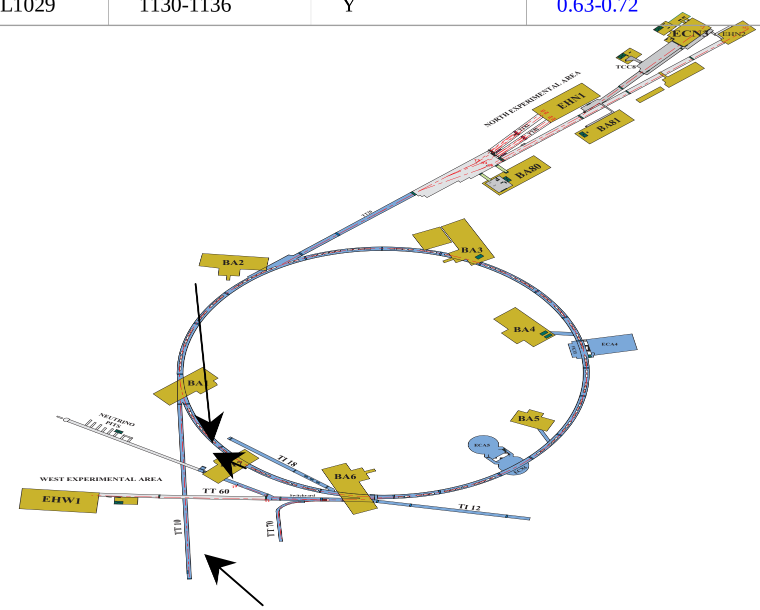
Location	S-Coordinate (m) from TT2	Plane of Aperture Limitation	3σ orbit error envelope in the tight plane (mm)
MBIH100	304-306	Y	0.77-0.91
MAL1001	315-322	Y	1.66-1.92
MBIV100	342-349	X	0.53-0.69
MBIV102	882-888	X	1.40-1.40
MAL1029	1130-1136	Y	0.63-0.72









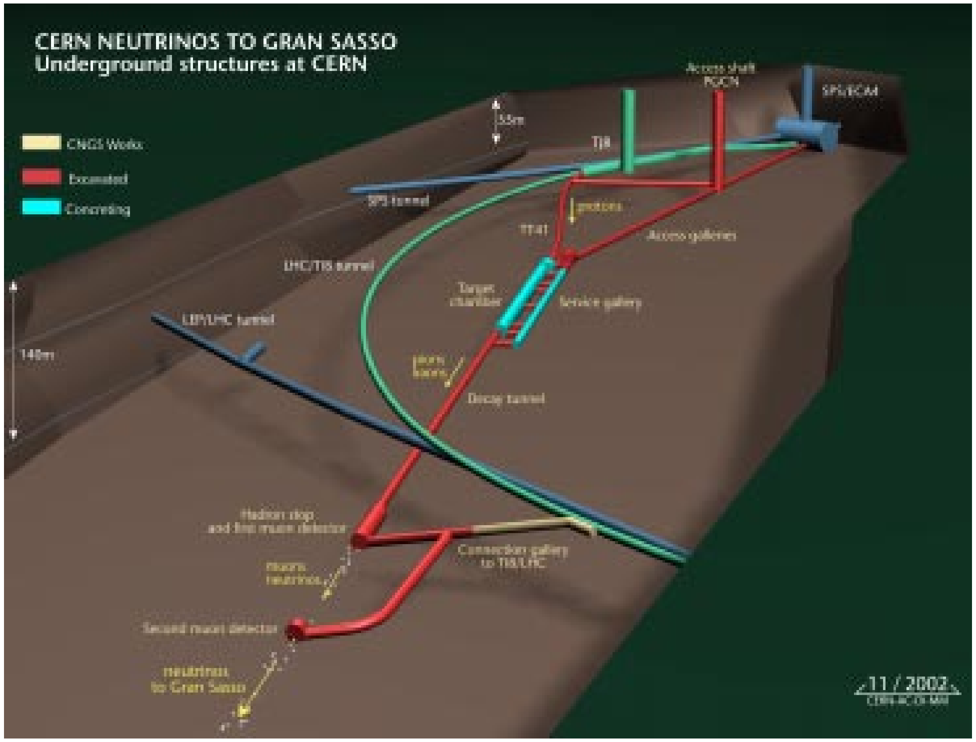


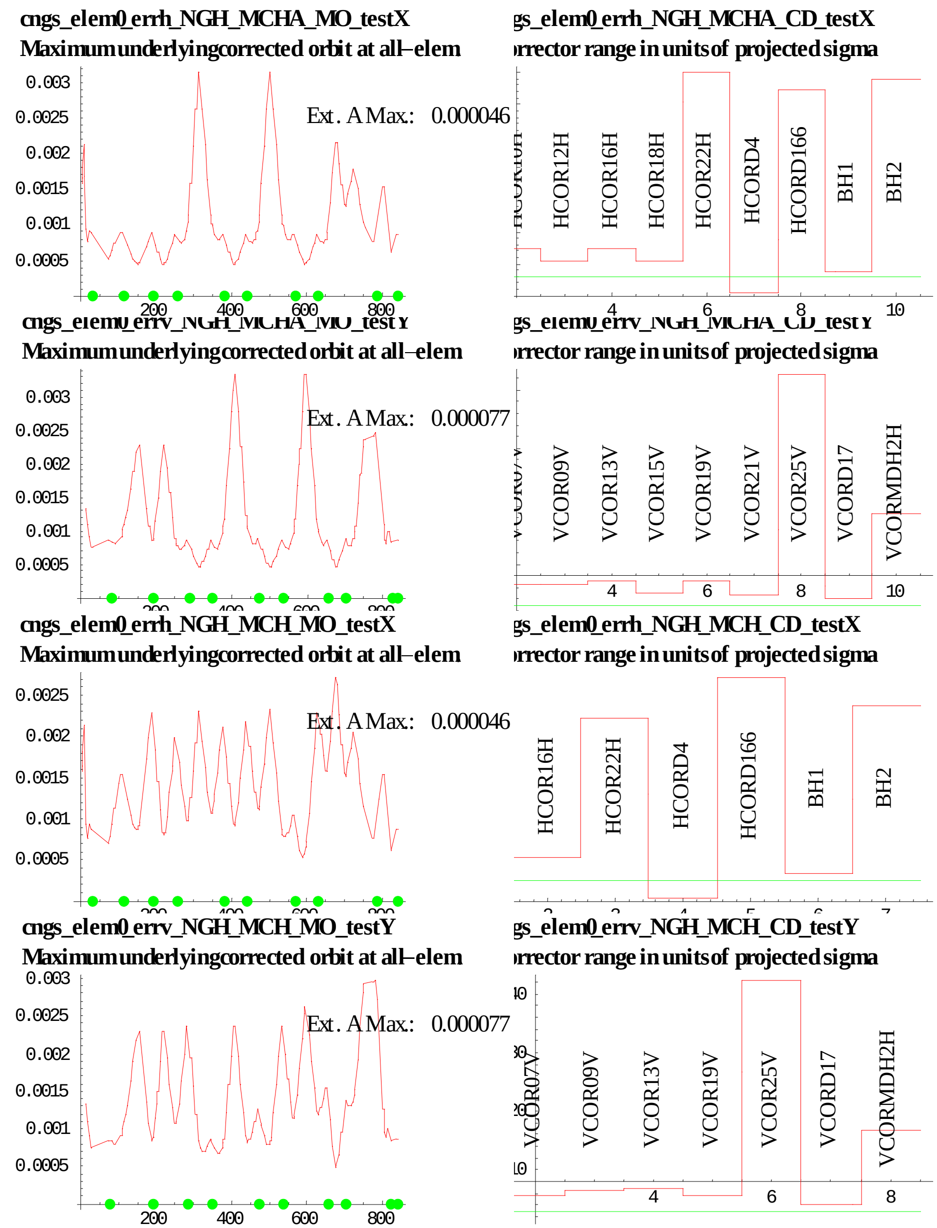
TT2/TT10
Additional Beam Instrumentation
2000-2001

Updated 7/7/98

CERN Neutrino to Gran Sasso Project

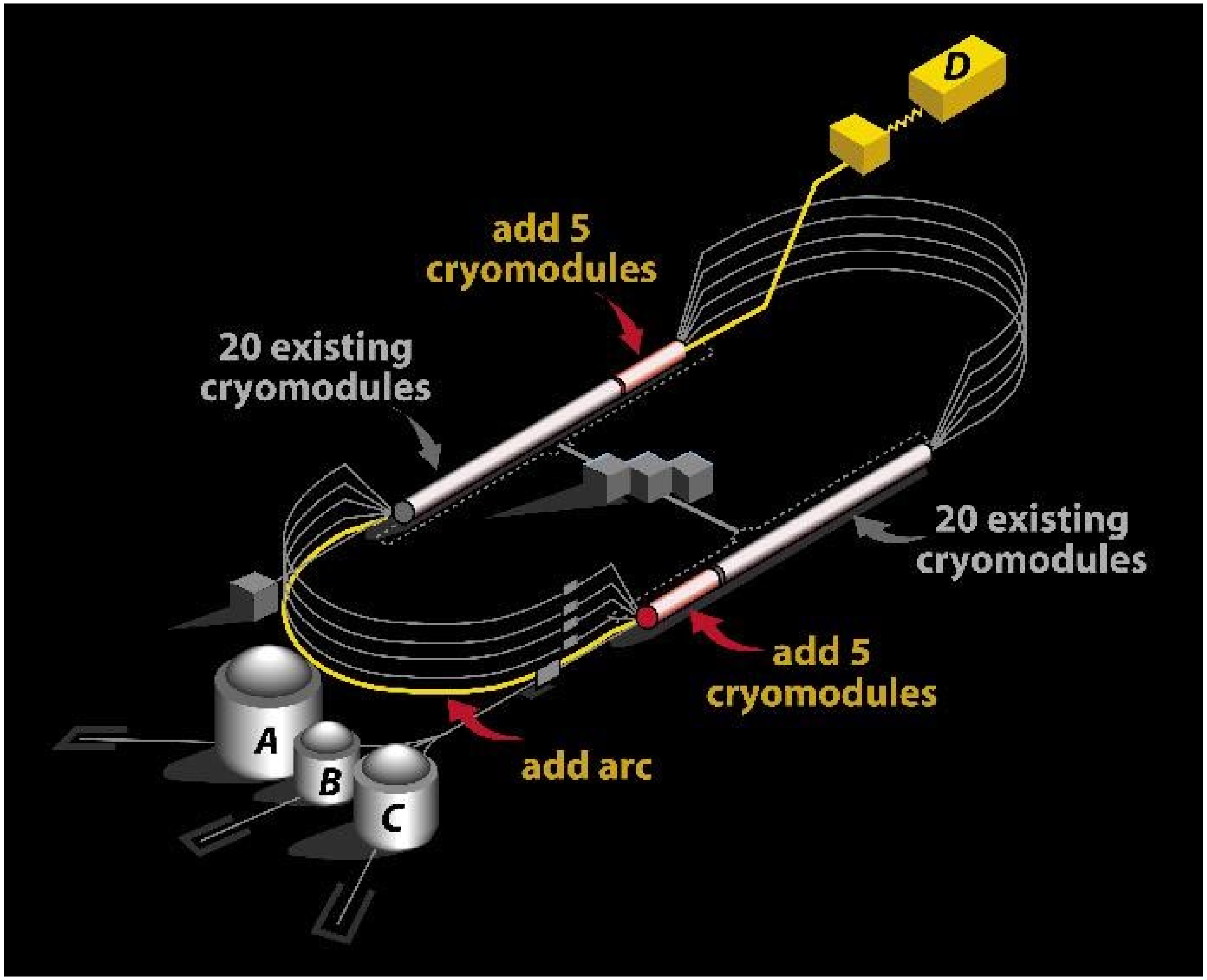
Input from M. Meddahi





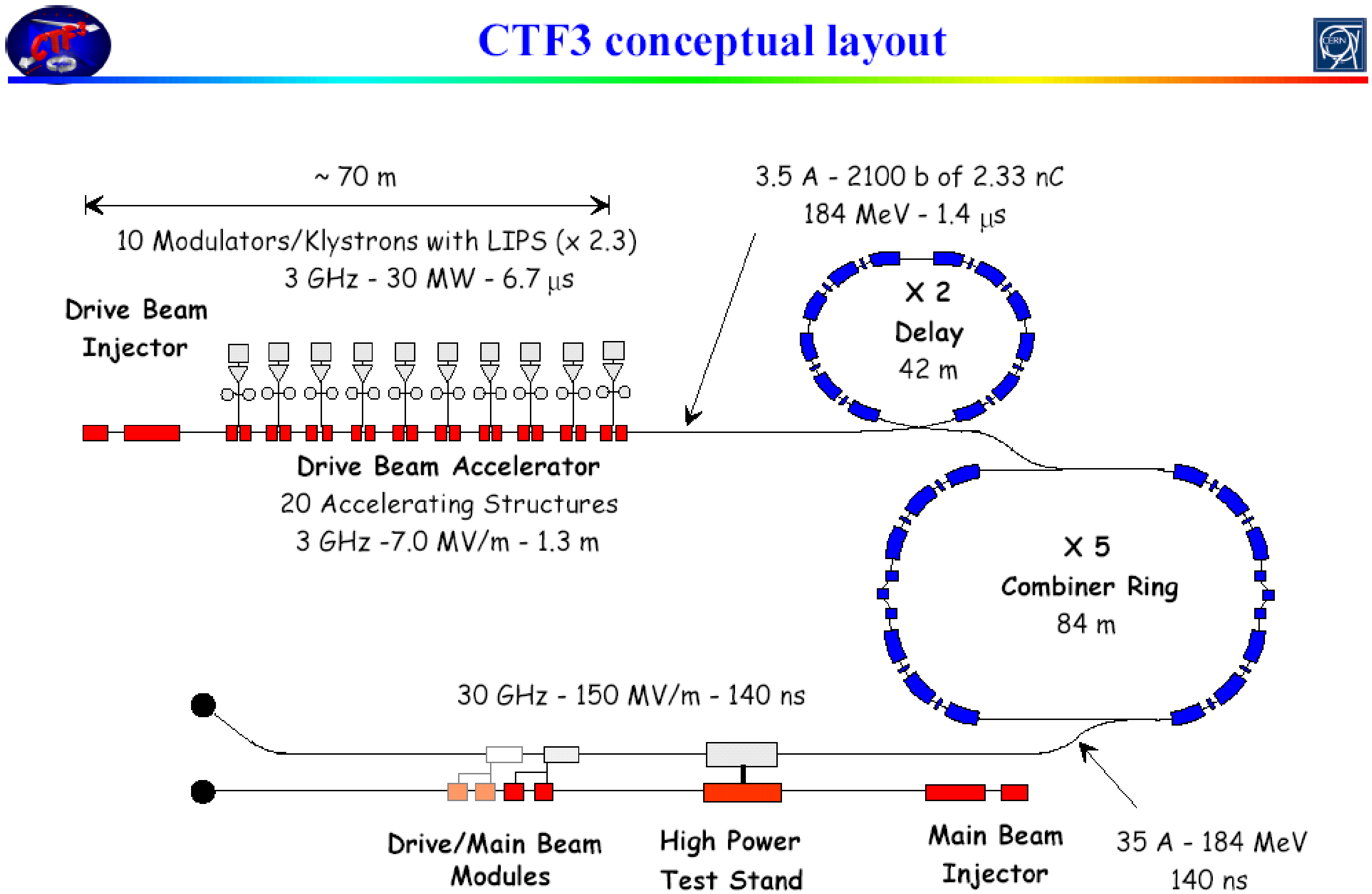
	Steering method	Orbit / Exit angle envelopes (3σ)	Number of BPM / CORR	Correction Range
Baseline scheme	1-to-1	**X**: ≤ 2.2 mm 2 peaks @ 3.1 mm **Target: <1 mm/0.05 mrad** **Y**: ≤ 2.3 mm 2 peaks @ 3.2 mm Target: <1 mm/0.08 mrad	10 / 10 10 / 10	All correctors can handle 3σ error except 1 horizontal & 1 vertical, both due to orbit error coming in from SPS.
Alternate scheme	**Over-constrained**	**X**: ≤ 2.3 mm 1 peak @ 2.7 mm **Target: <1 mm/0.05 mrad** **Y**: ≤ 2.4 mm 2 peaks @ 2.9 mm Target: <1 mm/0.08 mrad	10 / 7 10 / 8	All correctors can handle 3σ error except 1 horizontal & 1 vertical, both due to orbit error coming in from SPS.

JLAB CEBAF 12 GeV Upgrade



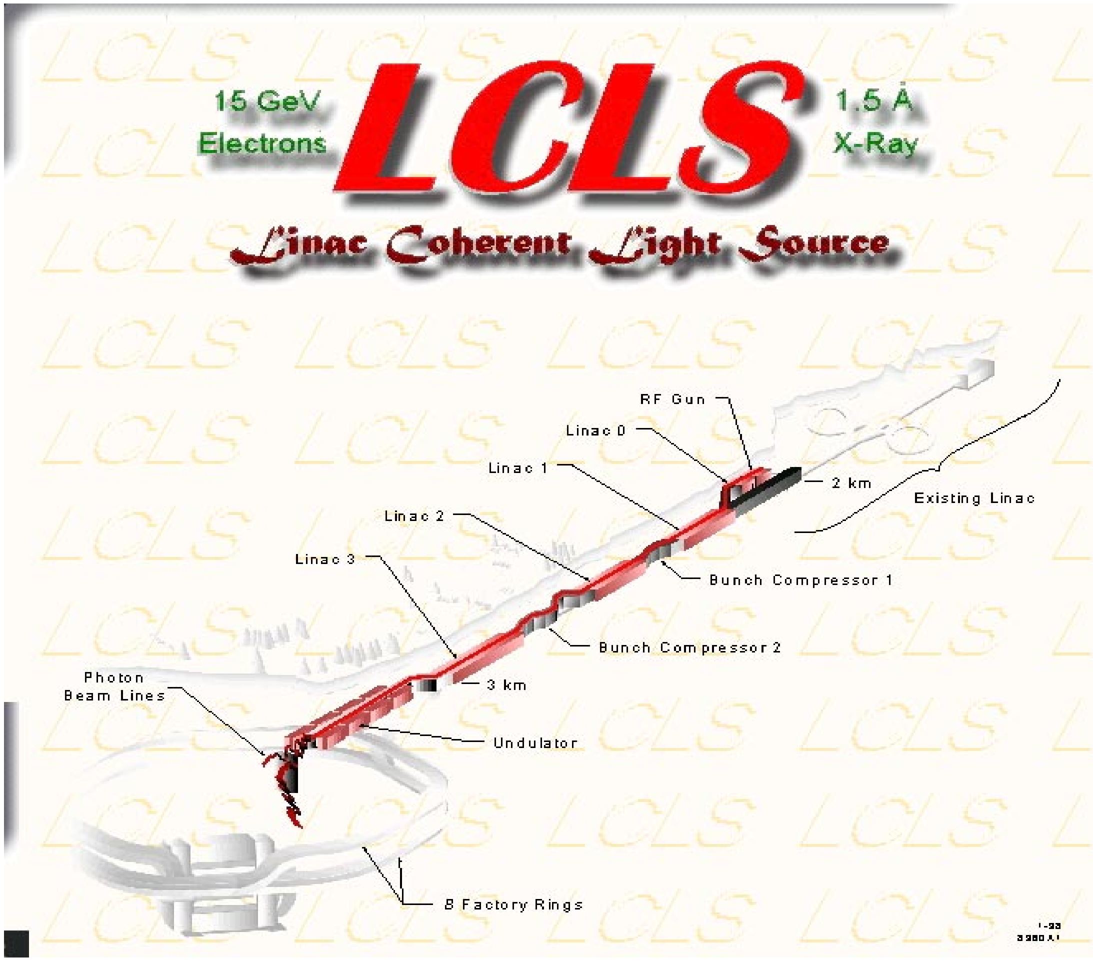
CERN CLIC Test Facility EPA Ring

Input from F. Tecker, P. Royer; Under Study



SLAC LCLS Project

Input from P. Emma, M. Woodley ; Under Study



Conclusion

Complete program developed for evaluating orbit correction system performance, identifying defects and corrective measure.

Application to LHC transfer lines resulted in efficient and quantitative evaluation, as well as optimized configurations in 2 basic scenarios.

Method is being extended to different machine configurations, with current and future applications in mind.

What does this say about orbit stability?

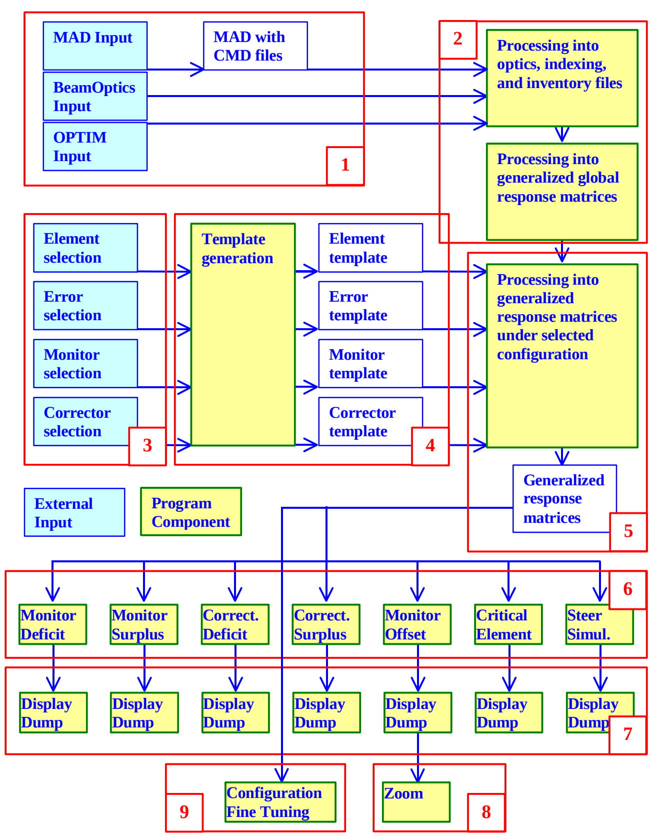
- It is complementary to the "bandwidth" side of the problem. Can't avoid configuration-induced problems regardless of bandwidth of monitors & correctors
- The probabilistic approach used here translates well into a time-based formulation of stability.

3 σ of error distribution over time $\rightarrow$ 3 σ of orbit jitter over time

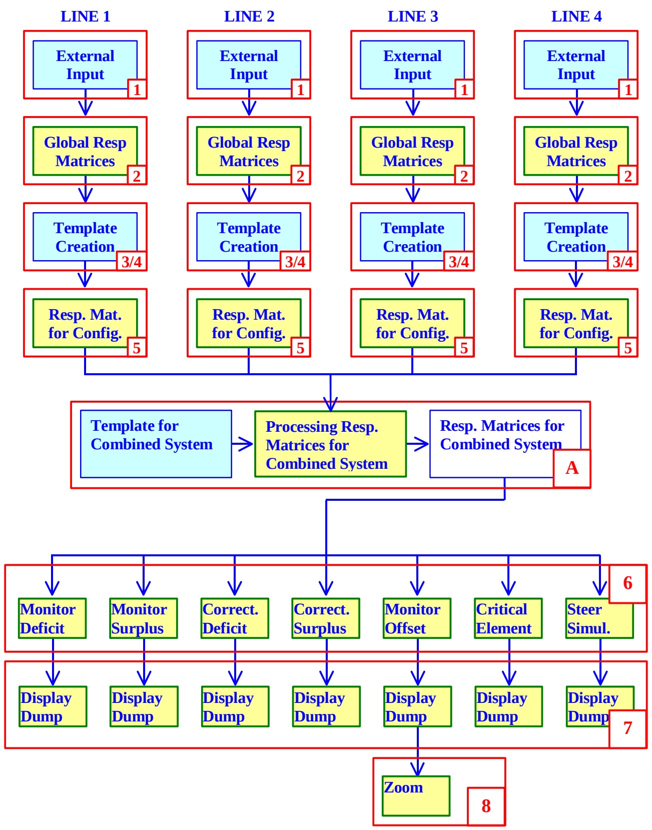
-

A self-contained Mathematica package has been developed, taking optics input from MAD, Optim, and BeamOptics

The package will be integrated into the BeamOptics environment and become a public-accessible program.



Flow Chart of the Program



Flow Chart for Multi-Line Analysis