Cure of Transverse Instabilities by Chromaticity Modulation

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1 Introduction

The modulation of chromaticity at synchrotron oscillation frequency is proposed to introduce static tune spread in a bunch.

The energy oscillation of synchrotron motion and chromaticity generate the modulation of the betatron tune of an particle. However the phase shift of betatron oscillation caused by this mudulation becomes zero after one synchrotron period and the maximum of the phase shift, which is called head-tail phase, is not sufficient to suppress instabilities in usual cases.

On the other hand, chromaticity modulated at synchrotron oscillation frequency, tune shift of betatron oscillation can be produced and this tune shift depends on the phase of synchrotron osillation.

Nonlinear field such as octupole field or field produced by trapped ions sometimes used to introduce tune spread in a bunch because these nonlinear field causes amplitude dependent tune shift. But for small emittance rings, the strong nonlinear fields must be required to introduce amplitude dependent tune shift because of its small beam size and it cause serious reduction of dynamic apertures. And trapped ions sometimes cause unwanted motion of a beam.

On the other hand, this scheme, modulation of chromaticity at synchrotron oscillation frequency , requires some fast varying sextupole magnets of moderate strength but these do not seriously reduce dynamic apertures [1].

2 Chromaticity Modulation

The chromaticity ξ which is modulated by the synchrotron frequency ω_s is described as,

$$\xi(t) = \xi_0 + \hat{\xi}_1 \cos \omega_s t \quad (1)$$

The relative energy deviation $\delta = (E-E_0)/E_0$ of aparticle is presented as

$$\varepsilon(t) = \hat{\varepsilon} \cos(\omega_s t + \phi) \ . \quad (2)$$

The variation of betatron tune of this particle is then given by

$$\begin{aligned} \nu(t) &= \xi(t)\varepsilon(t) \\ &= \left[\xi_0 + \hat{\xi}_1 \cos \omega_s t\right] \hat{\varepsilon} \cos(\omega_s t + \phi) \\ &= \xi_0 \hat{\varepsilon} \cos(\omega_s t + \phi) \\ &\quad + \frac{1}{2}\hat{\xi}_1 \hat{\varepsilon} \cos \phi + \frac{1}{2}\hat{\xi}_1 \hat{\varepsilon} \cos(2\omega_s t + \phi) \end{aligned} \quad (3)$$

The second term in equation (3), $\frac{1}{2}\hat{\xi}_1 \hat{\varepsilon} \cos \phi$, is a constant and is static tune shift which depends on the phase of the synchrotron oscillation, ϕ.

Suppose if the distribution of particles in the phase space of the synchrotron motion is assumed to be Gaussian;

$$f(\hat{\varepsilon}, \phi) d\hat{\varepsilon} d\phi = \frac{1}{2\pi\sigma_\varepsilon^2} e^{-\frac{\hat{\varepsilon}^2}{2\sigma_\varepsilon^2}} \hat{\varepsilon} d\hat{\varepsilon} d\phi \quad (4)$$

, the r.m.s. of this static tune spread becomes

$$\sigma_\nu = \sqrt{\iint \left(\frac{1}{2}\hat{\xi}_1 \hat{\varepsilon} \cos \phi\right)^2 f(\hat{\varepsilon}, \phi) d\hat{\varepsilon} d\phi} = \frac{1}{2}\hat{\xi}_1 \sigma_\varepsilon \quad (5)$$

. In the SPring-8 storage ring, σ_ε is 1×10^{-3} and the revolution frequency f_0 is 208kHz. If ξ_1 is set to 1, then we get $\sigma_\nu = 0.5\times10^{-3}$ and the Landau damping time τ_L by this tune spread is $\tau_L \sim 1/(2\pi f_0 \sigma_\nu) = 1/(2\pi\times 208\text{kHz}\times 0.5\times10^{-3}) = 2\text{ms}$. This damping time is faster than the radiation damping time, 8.3 ms.

Figure 1 is the result of the simulation of this damping by the code CISR[2]. The vertical axis is the betatron oscillation amplitude of the center of bunches defined as

$$\varepsilon = \frac{\sum_{n=1}^{N_b} \varepsilon_i}{N_b}, \ \varepsilon_i = \beta x_i'^2 + 2\alpha x_i x_i' + \gamma x_i^2 \quad (6)$$

where (x_i , x_i') and ε_i are the position, angle and emittance of the center of mass of the of i-th bunch, respectively. And N_b is the number of bunches.

Figure 2 is the result of the simulation by CISR[2] of transverse coupled-bunch instabilities driven by a transverse resonator impedance described in the caption. This shows that the chromaticity modulation scheme effectively damp the transverse coupled-bunch instabilities.

3 Conclusion

The chromaticity modulation at synchrotron frequency can be effective to create Landau damping and suppress transverse instabilities.

References

[1] H. Tanaka, SPring-8, Private communication

[2] T. Nakamura, "The Coupled-Bunch Instabilities Simulation Code CISR- Transverse Motion -," in this report.

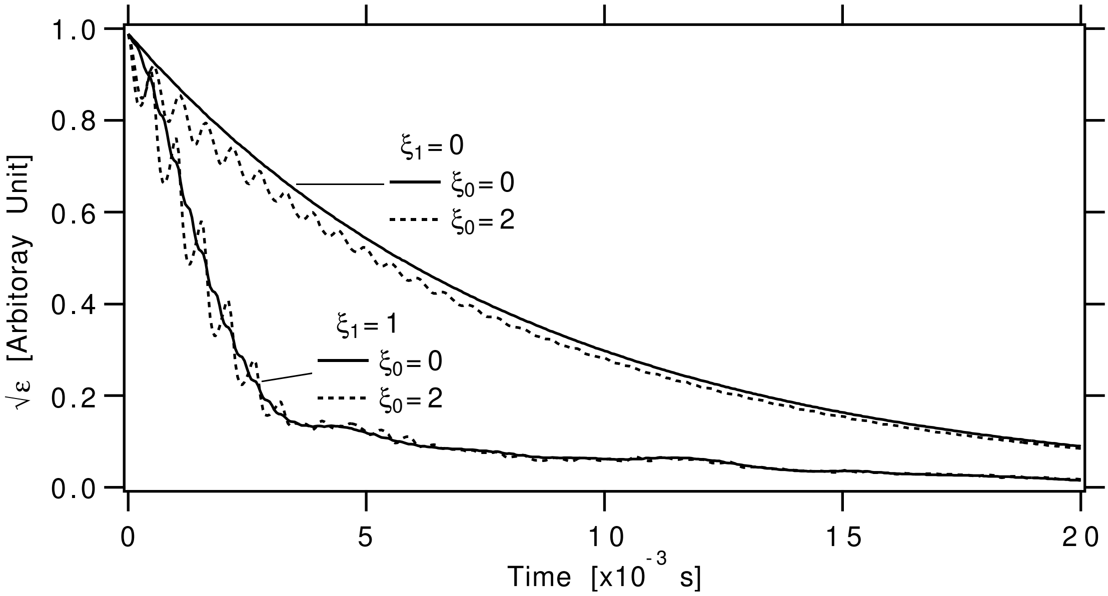

Fig. 1. Betatron oscillation amplitude of multi-bunch motion
with and without chromaticity modulation $\xi(t) = \xi_0 + \hat{\xi}_1 \cos \omega_s t$.
For $\xi_1 = 0$, the radiation damping only (τ=8.3ms).
For $\xi_1 = 0$, faster damping can be seen.

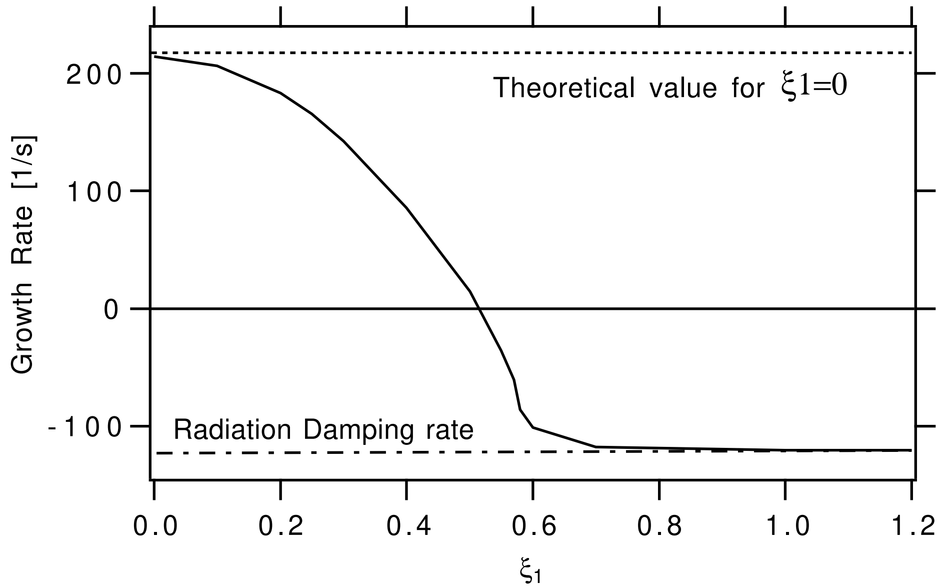

Fig. 2. Result of transverse coupled-instabilities simulation.
This include radiation damping(τ~8ms).
Parameters are those of the SPring-8 storage ring;
ξ_0 = 0, E=8GeV, I_{ave}=100mA $f_0=208\times10^3$Hz, ν_x=51.22, $f_0\nu_s$=1.8kHz, τ_x=8.3ms
and a cavity;R/Q=2000, Q=13000, f=865.33MHz, β_{cav}=10m,

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