The Coupled-Bunch Instabilities Simulation Code CISR - Transverse Motion -

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1 Introduction

The simulation code CISR for coupled-bunch instabilities caused by the resonator type impedance was developed. The treatment of transverse motion in CISR is described in this report and the longitudinal motion is described in ref. [1].

2 Evolution of Wake Voltage

The transverse motion described here is added to the longitudinal motion described in ref.[1]. The definition of parameters are the same as ref.[1].

The bunch time spacing is $T_b = T_0 / M$ where T_0 is the revolution frequency and M is the number of bunches. The suffix i, j shows that it is the parameter of the j-th particle in i-th bunch. For example, the charge of the j-th particle in i-th bunch is $q_{i,j}$. Its arrival time at the impedance source in n-th revolution is $\tau^n_{i,j}$, and its energy shift $\delta = \Delta E/E_0$ in n-th revolution is $\delta^n_{i,j}$

The transverse impedance of a resonator have the form[2],

$$Z^\perp = \frac{\omega^\perp}{\omega} \frac{R^\perp}{1 + i\, Q^\perp \left(\frac{\omega^\perp}{\omega} - \frac{\omega}{\omega^\perp} \right)}$$

and its wake voltage $V(t)$ produced by the charge q at position x is

$$V^\perp(t) = x\, q\, \frac{R^\perp}{Q^\perp}\, \omega^\perp \frac{\omega^\perp}{\overline{\omega^\perp}}\, e^{-\alpha t} \sin \overline{\omega^\perp}\, t \;= \mathrm{Re}\left[\tilde{V}^\perp e^{i \lambda^\perp t} \right]$$

where $\tilde{V}^\perp = -\, i\, x\, q\, \frac{R^\perp}{Q^\perp}\, \omega^\perp \frac{\omega^\perp}{\overline{\omega^\perp}}$, $\quad \lambda^\perp = i\, \alpha^\perp + \overline{\omega^\perp}$

$$\alpha^\perp = \frac{\omega^\perp}{2Q^\perp} \text{ and } \overline{\omega^\perp} = \sqrt{\omega^{\perp 2} - \alpha^{\perp 2}} .$$

The time t and t' are set as show in Fig.1 in ref.[1]. If the wake voltage between the (i-1)th and i-th bunch in n-th turn is assumed to be

$$V^{\perp n}_{i}(t) = \mathrm{Re}\left[\tilde{V}^{\perp n}_{i}\, e^{i \lambda^\perp t} \right] ,$$

the wake voltage after the passage of (i+1)th bunch can be expressed as

$$\begin{aligned} V^{\perp n}_{i+1}(t') &= \mathrm{Re}\left[\tilde{V}^{\perp n}_{i}\, e^{i \lambda^\perp (t' + T_b)} \right] \\ &\quad + \mathrm{Re}\left[\sum_{j=1}^{N_{p\ i+1}} V^{\perp '}_{i+1,\, j}\, e^{i \lambda^\perp \left(t' - t^n_{i+1,\, j}\right)} \right] \\ &= \mathrm{Re}\left[\tilde{V}^{\perp n}_{i+1}\, e^{i \lambda^\perp t'} \right] \end{aligned}$$

,where

$$\tilde{V}^{\perp n}_{i+1} = \tilde{V}^{\perp n}_{i}\, e^{i \lambda^\perp T_b} + \sum_{j=1}^{N_{p\ i+1}} \tilde{V}^{\perp '}_{i+1,\, j}\, e^{-i \lambda^\perp t^n_{i+1,\, j}}$$

$$\tilde{V}^{\perp '}_{i,\, j} = -\, i\, x_{i,\, j}\, q_{i,\, j}\, \frac{R^\perp}{Q^\perp}\, \omega^\perp \frac{\omega^\perp}{\overline{\omega^\perp}} .$$

3 Particle Motion

The equations of transverse motion of particles used in CISR is

$$\frac{d\dot\eta}{d\theta} = -\left(\nu_0 + \Delta\nu\right)^2 \eta + \nu_0{}^2\, \beta^{\frac{3}{2}} \frac{F}{E}$$

$$\frac{d\eta}{d\theta} = \dot\eta$$

$$\theta = \frac{1}{\nu_0} \int^s \frac{ds'}{\beta} .$$

With this variable, we just input CISR the Twiss parameters β and α at impedance and the phase difference between elements. If we use (x,x') instead of η, we have to input transfer matrix itself and sometimes it is tedious to know.

The transformation between x and η is

$$\begin{pmatrix} \eta \\ \dot\eta \end{pmatrix} = U^{-1} \begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \nu \frac{\alpha}{\sqrt{\beta}} & \nu\sqrt{\beta} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix} = U \begin{pmatrix} \eta \\ \dot\eta \end{pmatrix} = \begin{pmatrix} \sqrt{\beta} & 0 \\ -\frac{\alpha}{\sqrt{\beta}} & \frac{1}{\nu\sqrt{\beta}} \end{pmatrix} \begin{pmatrix} \eta \\ \dot\eta \end{pmatrix} .$$

4 Simulation

The transverse motion is added to the longitudinal motion described in ref.[1]. The simulation procedure of transverse motion is as follows.

4-1 Resonator wake field

The difference equations for a wake field of a resonator are now written by,

$$\tilde{V}^{\perp n}_{i+1} = \tilde{V}^{\perp n}_{i}\, e^{i \lambda^\perp T_b} + \sum_{j=1}^{N_{p\ i+1}} \tilde{V}^{\perp '}_{i+1,\, j}\, e^{-i \lambda^\perp t^n_{i+1,\, j}}$$

$$\tilde{V}^{\perp '}_{i,\, j} = -\, i\, x_{i,\, j}\, q_{i,\, j}\, \frac{R^\perp}{Q^\perp}\, \omega^\perp \frac{\omega^\perp}{\overline{\omega^\perp}} .$$

4-2 Single Kick

The difference equations for particles by single kick $\Delta\theta$ = FL/E at k-th element is

$$\begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix}_{i,j,k+} = \begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix}_{i,j,k-} + \begin{pmatrix} 0 \\ \nu \beta^{\frac{1}{2}} \frac{FL}{E} \end{pmatrix}^{k}_{i,j,k} .$$

For a transverse resonator impedance,

$$FL = \mathrm{Re}\left[e\tilde{V}^{\perp n}_{i} \, e^{i \lambda^{\perp} \left(T_b + t^{n}_{i+1,j} \right)} \right] .$$

4-3 Lattice

The transformation between k-th and (k+1)th elements is

$$\begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix}_{i,j,k+1} = \begin{pmatrix} \cos\nu\theta_{k+1,k} & \frac{1}{\nu}\sin\nu\theta_{k+1,k} \\ -\nu\sin\nu\theta_{k+1,k} & \cos\nu\theta_{k+1,k} \end{pmatrix} \begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix}_{i,j,k}$$

, $\nu\theta_{k+1,k}$ is the betatron phase advance between k-th and (k+1)th elements.

4-4 Acceleration

Radiation damping is occurs at acceleration and is treated as

$$x' \leftarrow \left(1 - \frac{eV_a}{E_0} \right) x'$$

where V_a is an acceleration voltage.

4-5 Lattice Element

The difference equations for k-th element which described by a transfer matrix M is

$$\begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix}_{i,j,k+1} = U_k^{-1} M_k U_k \begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix}_{i,j,k} .$$

An example of this is a RFQM,

$$M = \begin{pmatrix} 1 & 0 \\ -k\cos\omega_0 t & 1 \end{pmatrix}$$

where ω_0 is the revolution frequency.

5 Conclusion

The simulation code for coupled-bunch instabilities, CISR, was developed and simulation method of transverse motion of the particle in transverse coupled-bunch instabilities are described in this report.

Instead of transverse position x in real space, $\eta = x/\sqrt{\beta}$ is used to represent the transverse position of particles. With this, The imput to CISR is just the twiss parameters at the element and the phase difference between each elements. On the other hand, if x is used directly, transfer matrix is necessary.

References

[1] T. Nakamura, “The Coupled-Bunch Instabilities Simulation Code CISR - Longitudinal Motion -,” in this report.

[2] A. W. Chao, "Physics of Collective Beam Instabilities in High Energy Accelerators," 1993, John-Wiley & Sons Inc.