高輝度電子銃シミュレーション研究会
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時間領域境界要素法（TDBEM）による粒子加速器航跡場の数値解析法の概要と課題

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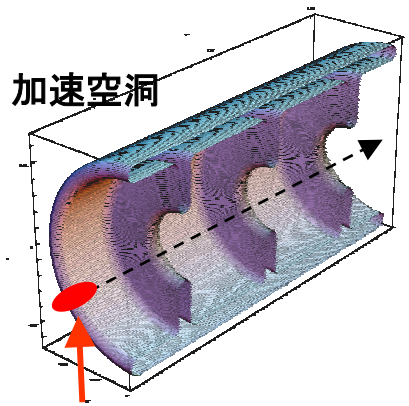
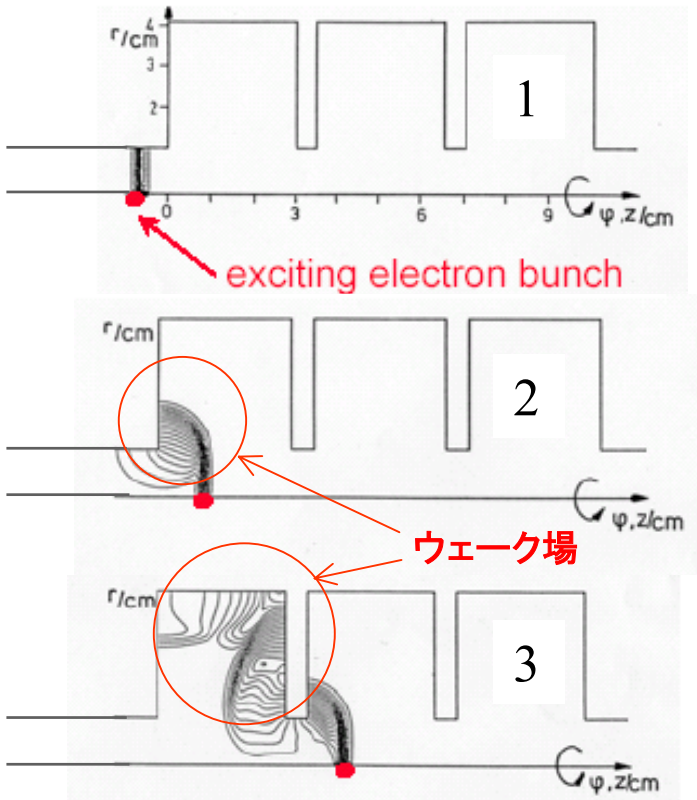
１．航跡場解析の背景

１．１　航跡場解析の概要





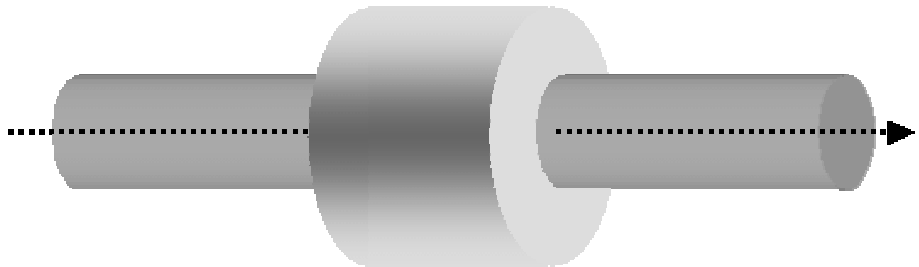
ウェーク場の影響

- エネルギーロス
- 質の劣化
- 強度の制限

ウェーク場の数値解析法

- 解析的（近似形状）
- FDTD/FIT法（差分法）
- 時間領域境界要素法

１．２　従来の航跡場の数値解析法

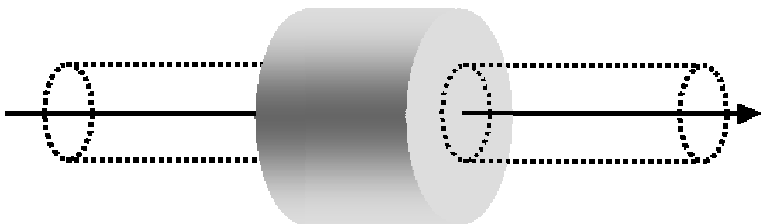


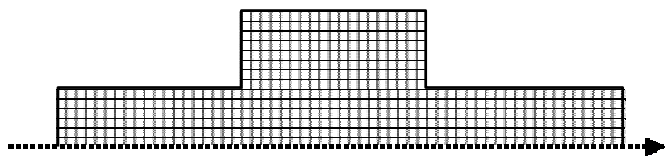
解析的（近似形状）

パイプの影響
を無視し解析

格子状に離散化
して数値解析

FDTD/FIT 法（差分法）

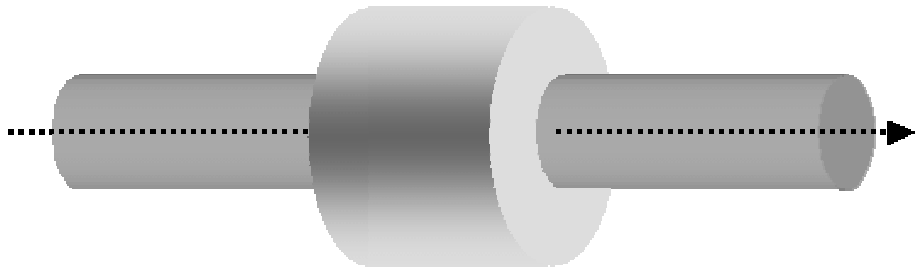




計算速度　速
対称性のある形状（円柱、球）のみ

計算速度　速
任意形状
直線軌道のみ
航跡場＋自己場
グリッド分散

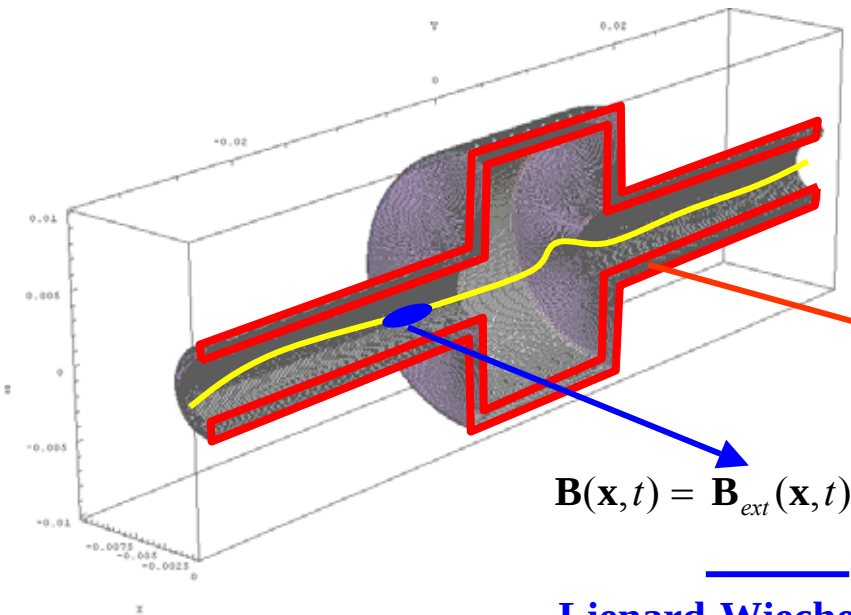
１．３　時間領域境界要素法の利点



時間領域境界要素法

境界のみを離散化し
積分方程式解析

TDBEM { 3D / 2D軸対称 / 2.5次元



計算速度　遅
安定性
曲線軌道も可
航跡場＋自己場の分離が可能
任意形状
グリッド分散なし

$$\mathbf{B}(\mathbf{x},t)=\mathbf{B}_{ext}(\mathbf{x},t)-\frac{1}{4\pi}\int_S\left[\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}+\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t}\right]\times\mathbf{B}_t\left(\mathbf{x}',t-\frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)dS'$$

Lienard-Wiechert fields　　**Wade fields**

２．時間領域境界要素法（ＴＤＢＥＭ）

２．１　電磁場のタイプと電磁界数値解析

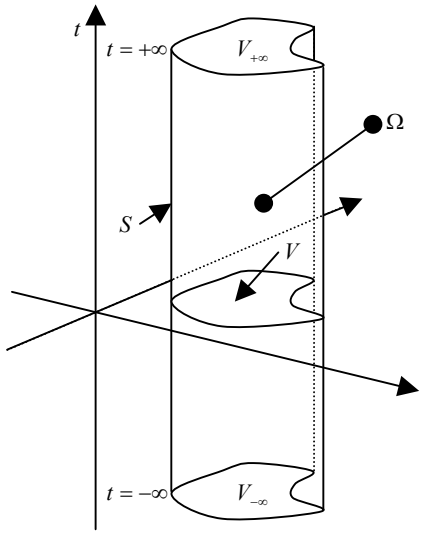
em fields \ method		FDM	FEM	BEM
Static				
Quasi-Static	Frequency Domain			
	Time Domain			
High-Freq.	Frequency Domain			
	Time Domain	FDTD /FIT		

2D	2D Systems
	Axis-symm. Sys.
3D	Full 3D Systems

For open boundary problems
For coupled problems with charged particles

Unstable in long time range calculation
Heavy calculation cost
Large required memory

2．2　時間領域境界積分方程式



Green's theorem in time domain

$$\int_{\Omega}\left(\frac{\partial^2\Phi}{\partial x_\lambda \partial x^\lambda}\Psi-\Phi\frac{\partial^2\Psi}{\partial x_\lambda \partial x^\lambda}\right)d\Omega=\int_{S+V_\infty+V_{-\infty}}\left(\frac{\partial\Phi}{\partial x^\nu}\Psi-\Phi\frac{\partial\Psi}{\partial x^\nu}\right)dV^\nu$$

scalar & vector potentials

$$\Phi \leftarrow \left(\nabla^2-\frac{\partial^2}{c^2\partial t^2}\right)\phi=-\frac{\rho}{\varepsilon}\quad\left(\nabla^2-\frac{\partial^2}{c^2\partial t^2}\right)\mathbf{A}=-\mu\mathbf{J}$$

fundamental solutions

$$\Psi \leftarrow G(ct,\mathbf{x};ct',\mathbf{x}')=\frac{1}{4\pi}\frac{\delta\left(t'-t+\frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)}{|\mathbf{x}-\mathbf{x}'|}$$

Boundary conditions

$$\phi=0\quad \mathbf{A}=0$$

$$\mathbf{E}\times\mathbf{n}=0\quad \mathbf{B}\cdot\mathbf{n}=0$$

Boundary integral equations

$$\phi(ct,\mathbf{x})=\frac{1}{4\pi\varepsilon}\int_{\Omega}\frac{\rho\left(t-\frac{|\mathbf{x}-\mathbf{x}'|}{c},\mathbf{x}'\right)}{|\mathbf{x}-\mathbf{x}'|}dV'+\frac{1}{4\pi}\int_{S}\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\frac{\partial\phi\left(t-\frac{|\mathbf{x}-\mathbf{x}'|}{c},\mathbf{x}'\right)}{\partial n}\right)dS'+\frac{\partial G}{\partial t}$$

$$\mathbf{A}(ct,\mathbf{x})=\frac{\mu}{4\pi}\int_{\Omega}\frac{\mathbf{J}\left(t-\frac{|\mathbf{x}-\mathbf{x}'|}{c},\mathbf{x}'\right)}{|\mathbf{x}-\mathbf{x}'|}dV'+\frac{1}{4\pi}\int_{S}\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\frac{\partial\mathbf{A}_t\left(t-\frac{|\mathbf{x}-\mathbf{x}'|}{c},\mathbf{x}'\right)}{\partial n}\right)dS'-\nabla G$$

Elimination of gauge term

２．２　時間領域境界積分方程式

Magnetic Field Integral Equation (MFIE)

$$\mathbf{B}(\mathbf{x},t) = \mathbf{B}_{ext}(\mathbf{x},t) - \frac{1}{4\pi}\int_S \left[\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right] \times \mathbf{B}_t\left(\mathbf{x}', t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right) dS'$$

Electric Field Integral Equation (EFIE)

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}_{ext}(\mathbf{x},t) - \frac{1}{4\pi}\int_S \frac{1}{|\mathbf{x}-\mathbf{x}'|}\mathbf{B}_t\left(\mathbf{x}', t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right) dS'$$

$$- \frac{1}{4\pi}\int_S \left[\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right] E_n\left(\mathbf{x}', t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right) dS'$$

$$\mathbf{E}\cdot\mathbf{n} = E_n = -\frac{\partial\phi}{\partial n} = -\frac{\sigma}{\varepsilon}$$

$$\mathbf{B}\times\mathbf{n} = \mathbf{B}_t = \frac{\partial \mathbf{A}_t}{\partial n} = -\mu\mathbf{K}$$

Conservation law

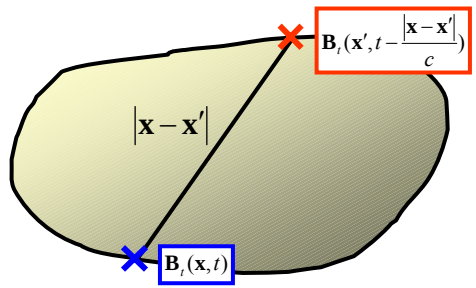
$$\frac{\partial\sigma}{\partial t} + div\mathbf{K} = 0$$

３．時間領域境界要素法による航跡場解析

３．１　粒子加速器と航跡場解析

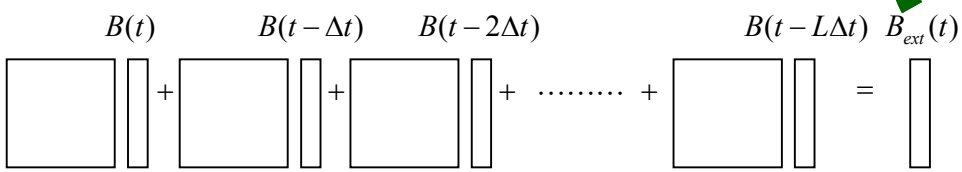
Time Domain MFIE

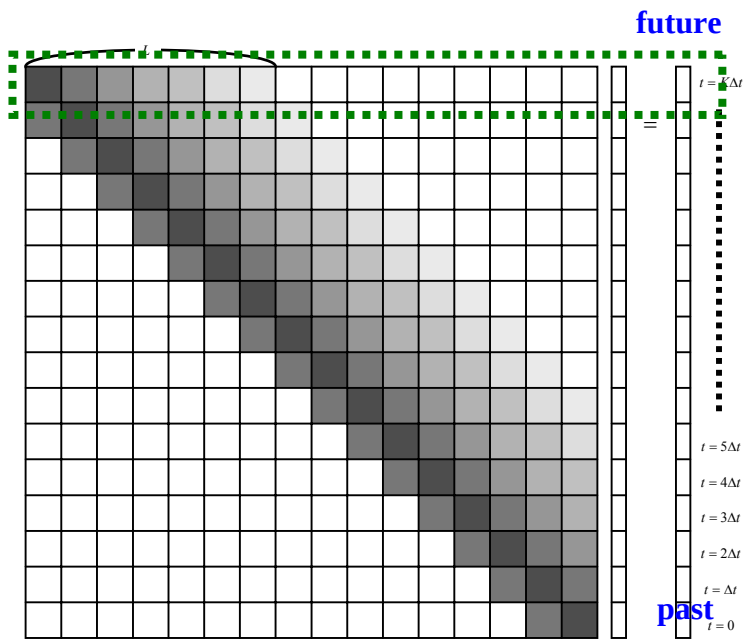
$$\mathbf{B}(\mathbf{x},t) = \mathbf{B}_{ext}(\mathbf{x},t) - \frac{1}{4\pi}\int_S \left[\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t} \right] \times \mathbf{B}_t\left(\mathbf{x}', t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right) dS'$$



$$G_0 B_t + \sum_{k=1}^{L} G_k B_{t-k\Delta t} = B_{ext}$$

Matrix equation of TDBEM



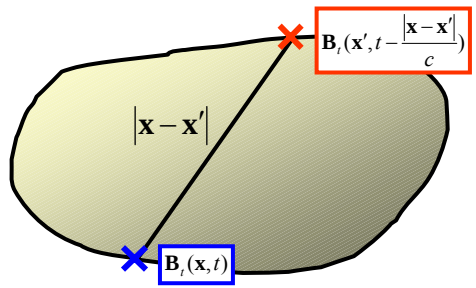


３．時間領域境界要素法による航跡場解析

３．１　粒子加速器と航跡場解析

Time Domain MFIE

$$\mathbf{B}(\mathbf{x},t) = \mathbf{B}_{ext}(\mathbf{x},t) - \frac{1}{4\pi}\int_S \left[\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t}\right] \times \mathbf{B}_t\left(\mathbf{x}', t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}\right) dS'$$



$$G_0 B_t + \sum_{k=1}^{L} G_k B_{t-k\Delta t} = B_{ext}$$

Matrix equation of TDBEM

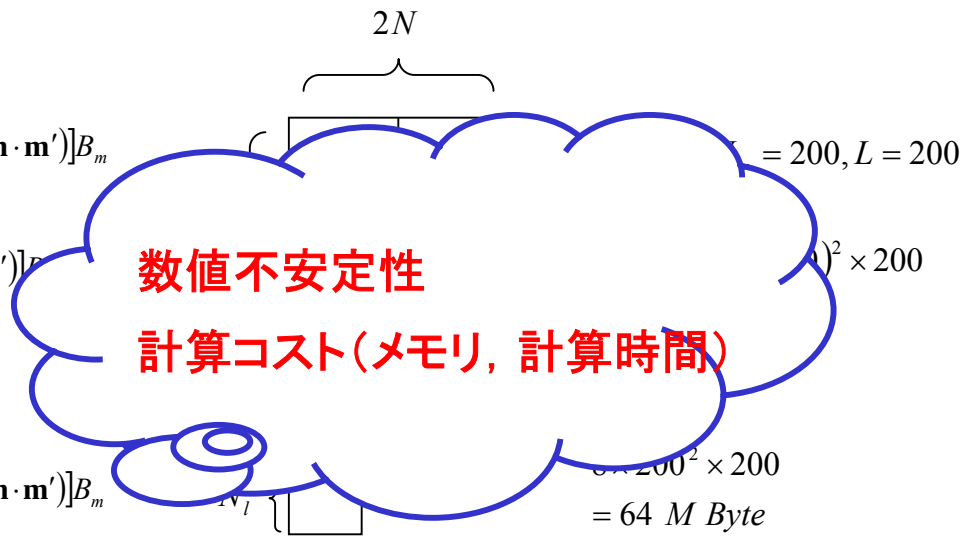
3D　$2N = 2(N_l + N_m)$

$$\mathbf{m}\cdot\mathbf{B} = \mathbf{m}\cdot\mathbf{B}_{ext} - \frac{1}{4\pi}\int_{S'} dS'\{[-(\mathbf{m}'\cdot\mathbf{R})(\mathbf{m}\cdot\mathbf{n}') + (\mathbf{n}'\cdot\mathbf{R})(\mathbf{m}\cdot\mathbf{m}')]B_m + [-(\mathbf{l}'\cdot\mathbf{R})(\mathbf{m}\cdot\mathbf{n}') + (\mathbf{n}'\cdot\mathbf{R})(\mathbf{m}\cdot\mathbf{l}')]B_l\}$$

$$\mathbf{l}\cdot\mathbf{B} = \mathbf{l}\cdot\mathbf{B}_{ext} - \frac{1}{4\pi}\int_{S'} dS'\{[-(\mathbf{m}'\cdot\mathbf{R})(\mathbf{l}\cdot\mathbf{n}') + (\mathbf{n}'\cdot\mathbf{R})(\mathbf{l}\cdot\mathbf{m}')]B_m + [-(\mathbf{l}'\cdot\mathbf{R})(\mathbf{l}\cdot\mathbf{n}') + (\mathbf{n}'\cdot\mathbf{R})(\mathbf{l}\cdot\mathbf{l}')]B_l\}$$

2D軸対称　N_l

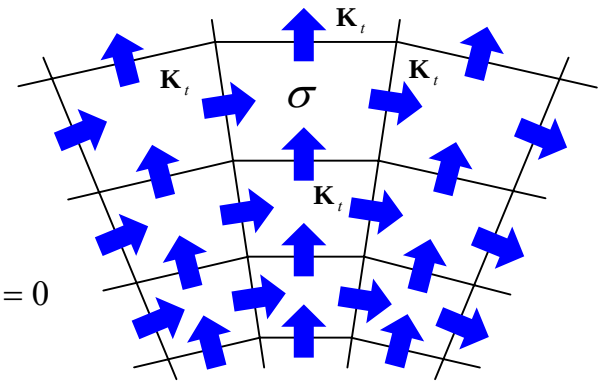
$$\mathbf{m}\cdot\mathbf{B} = \mathbf{m}\cdot\mathbf{B}_{ext} - \frac{1}{4\pi}\int_{S'} dS'[-(\mathbf{m}'\cdot\mathbf{R})(\mathbf{m}\cdot\mathbf{n}') + (\mathbf{n}'\cdot\mathbf{R})(\mathbf{m}\cdot\mathbf{m}')]B_m$$



４．数値解析コード

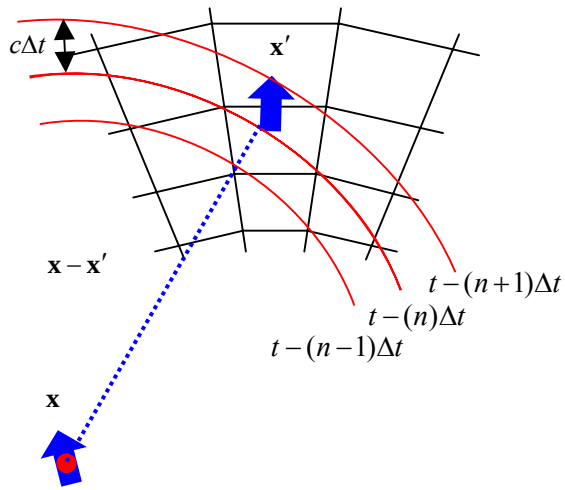
４．１　数値不安定性

未知変数の配置

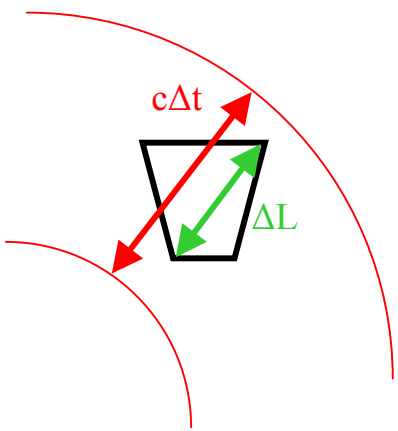


$$div\mathbf{K}_t + \frac{\partial \sigma}{\partial t} = 0$$

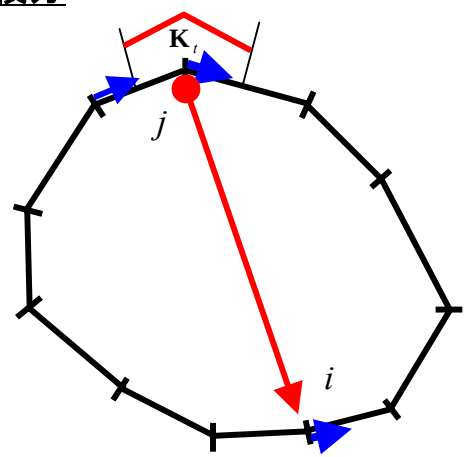
因果律の評価



陰的スキーム

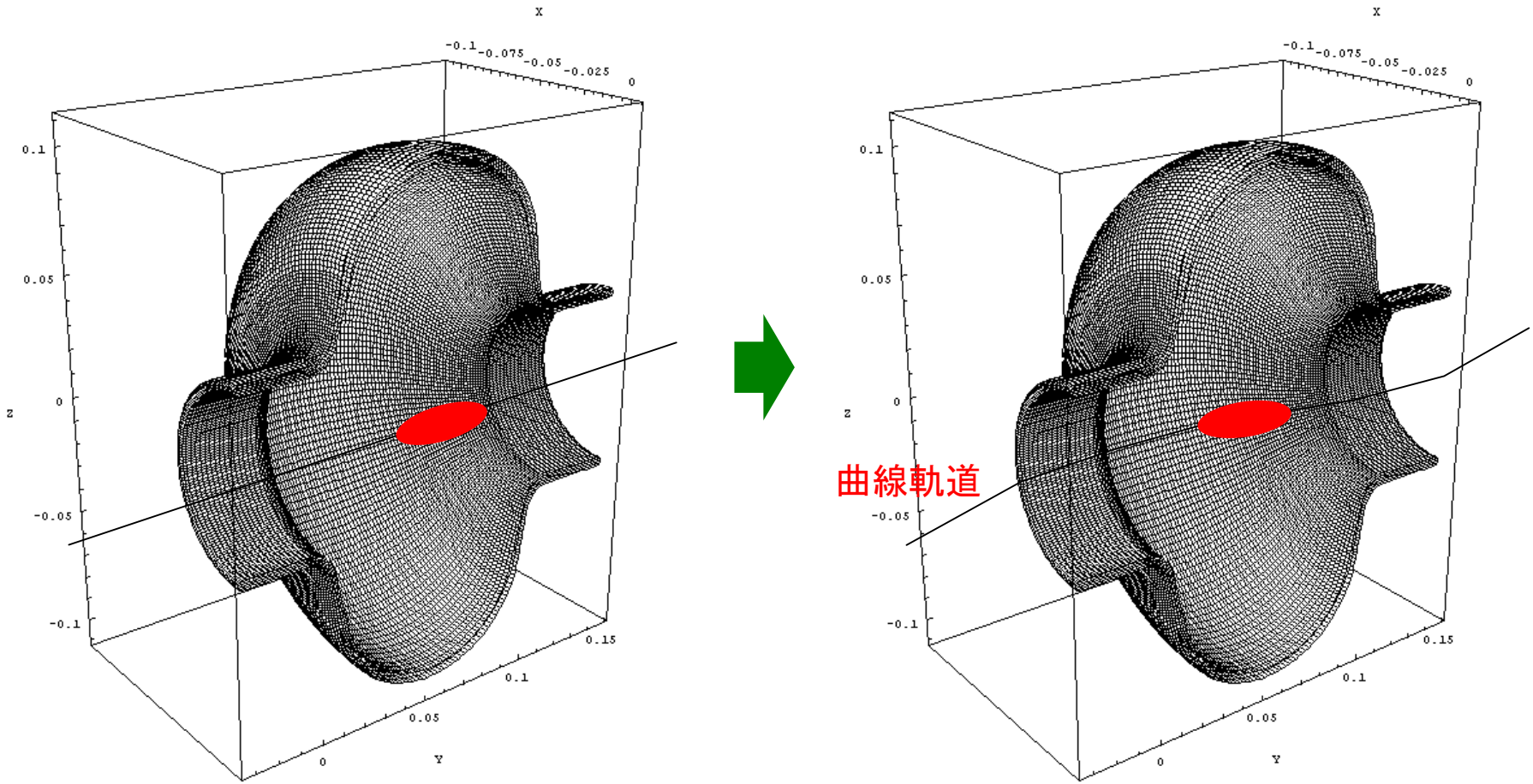


２メッシュ補間積分



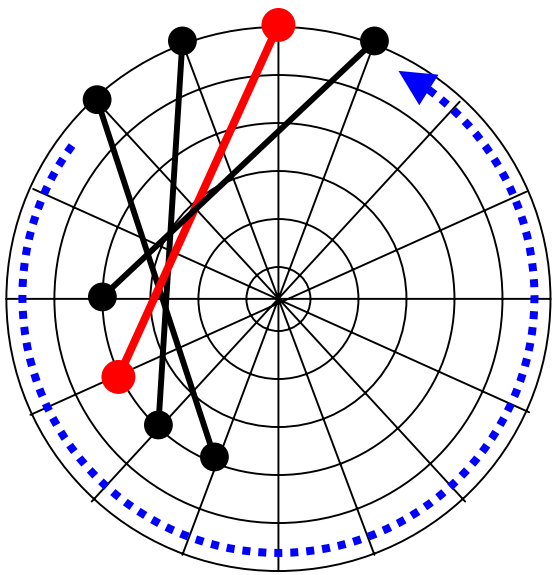
４．２　計算コスト

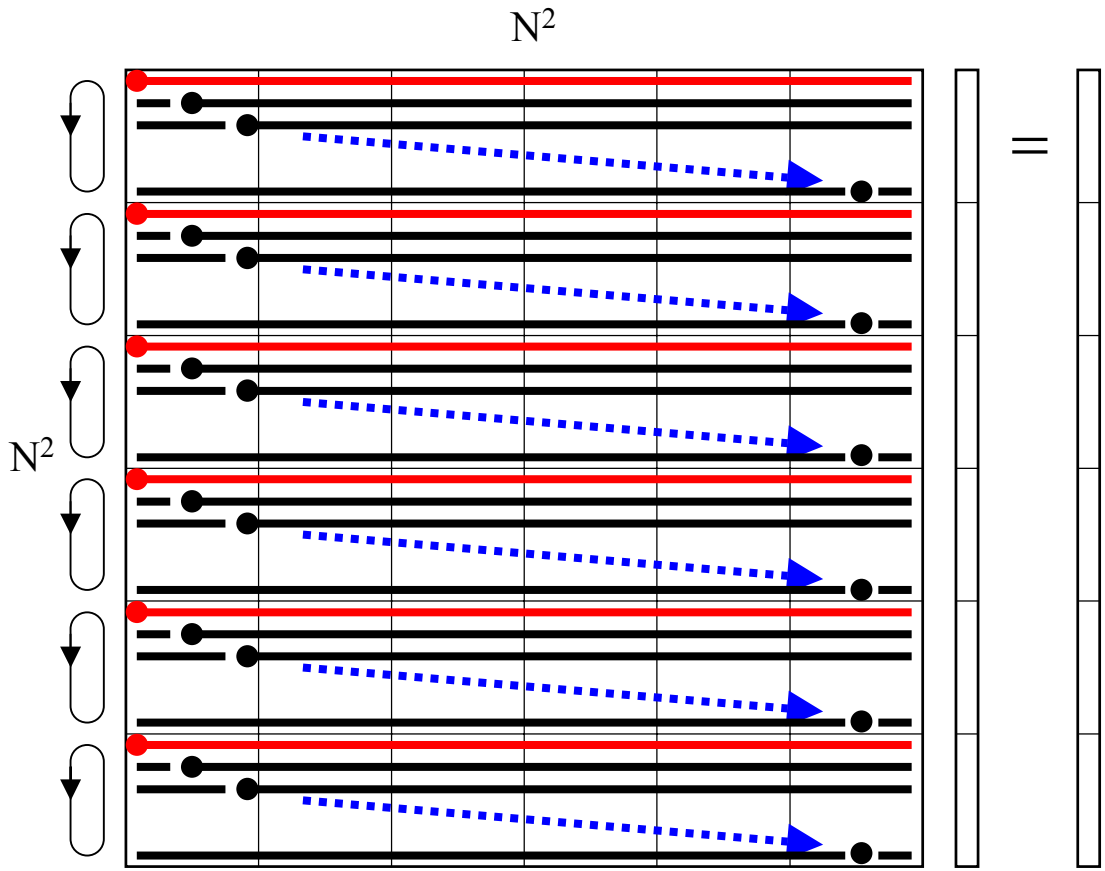
メモリ削減　　**数値モデルの軸対称性**



4．2　計算コスト

メモリ削減　　**数値モデルの軸対称性**



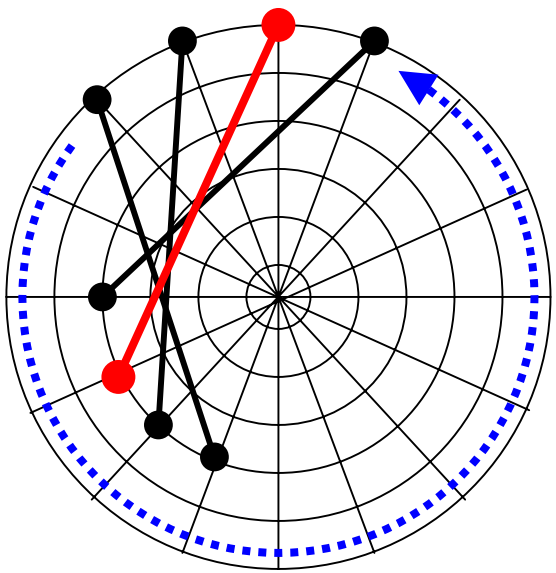


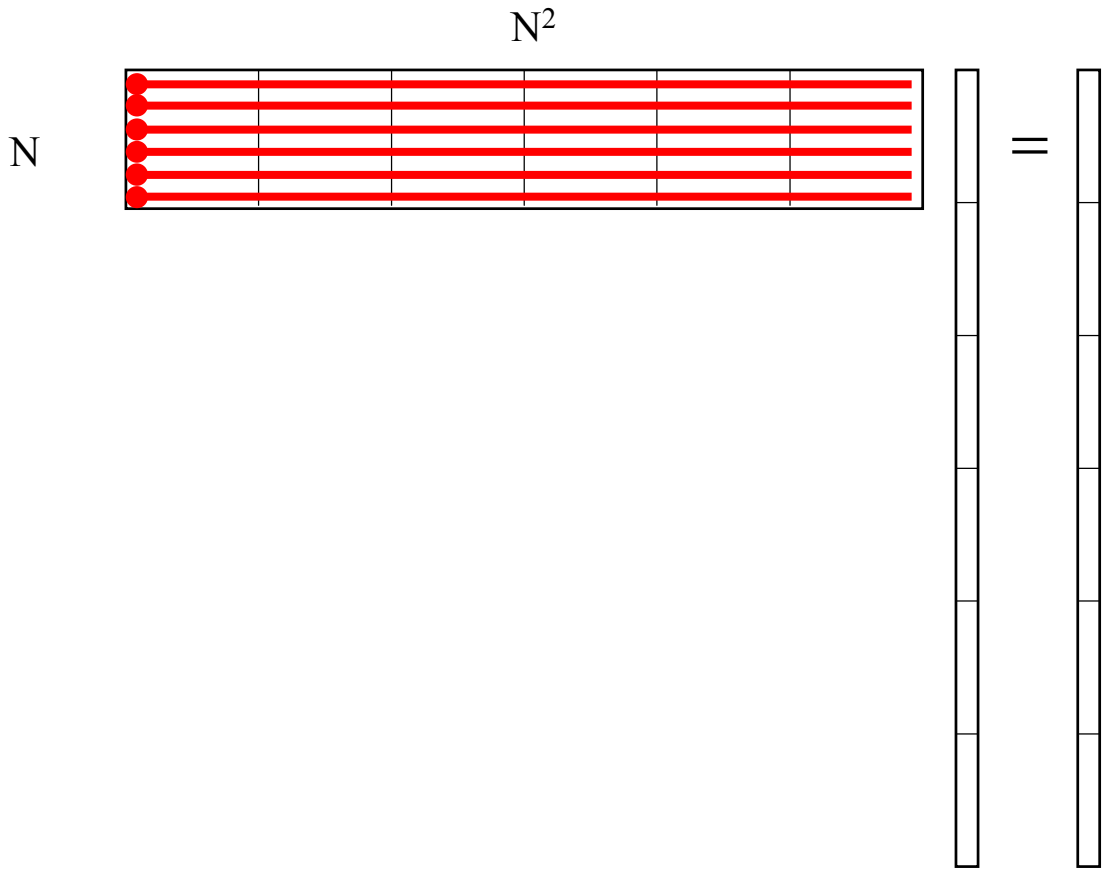
$$\mathbf{B}(\mathbf{x},t)=\mathbf{B}_{LW}(\mathbf{x},t)$$
$$-\frac{1}{4\pi}\int_{S'}\{[-(\mathbf{m}'\cdot\mathbf{R})\mathbf{n}'+(\mathbf{n}'\cdot\mathbf{R})\mathbf{m}']B_m(\mathbf{x}',t')$$
$$+[-(\mathbf{l}'\cdot\mathbf{R})\mathbf{n}'+(\mathbf{n}'\cdot\mathbf{R})\mathbf{l}']B_l(\mathbf{x}',t')\}dS'$$

$$\mathbf{R}=\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}+\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t}$$

４．２　計算コスト

メモリ削減　　**数値モデルの軸対称性**





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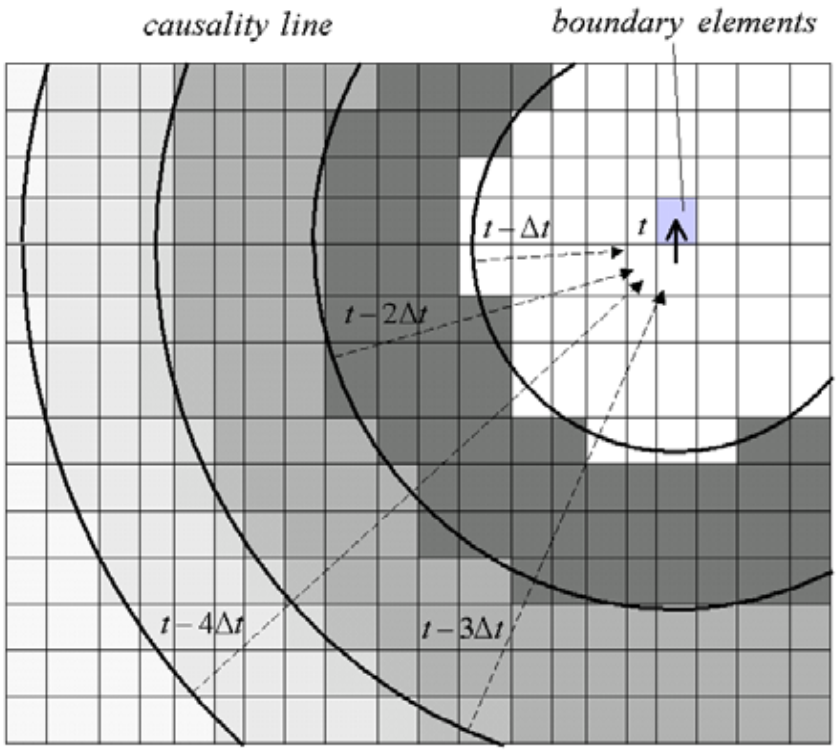
$$\mathbf{R}=\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}+\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t}$$

4．2　計算コスト

メモリ削減　　**行列のスパース性**



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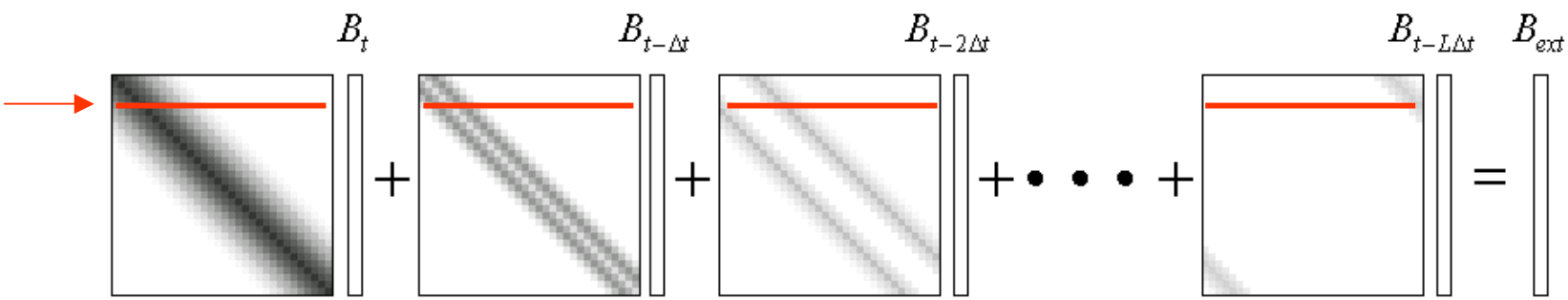
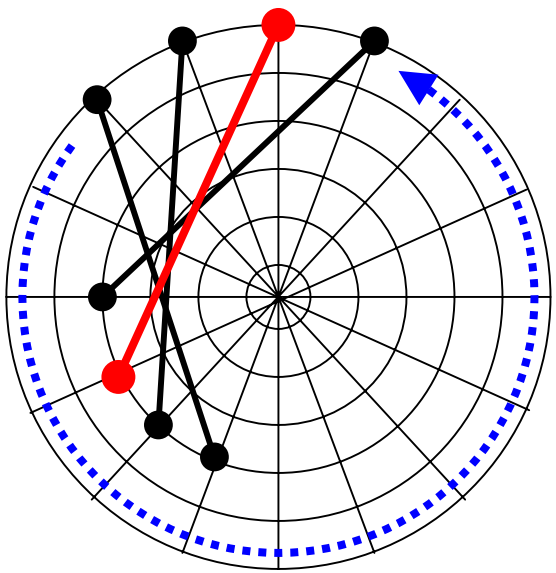
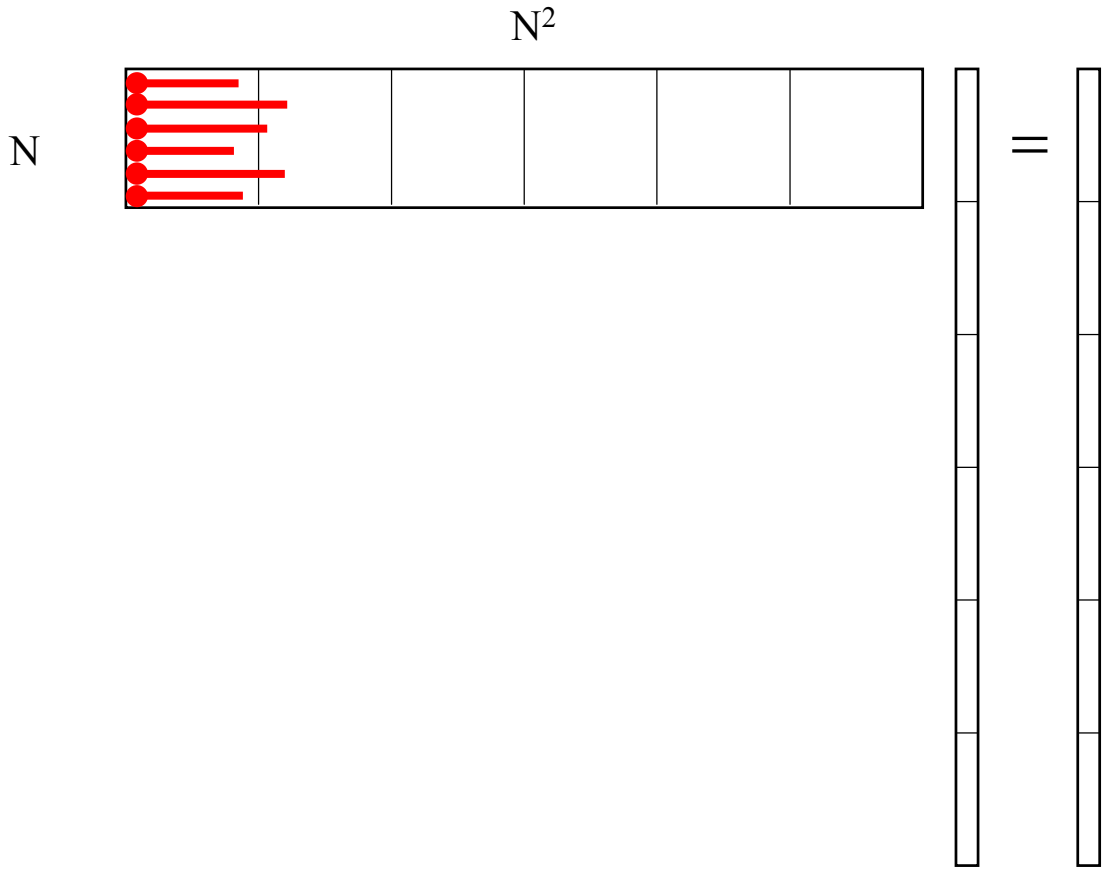


4．2　計算コスト

メモリ削減　　**数値モデルの軸対称性＋行列のスパース性**





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$$-\frac{1}{4\pi}\int_{S'}\{[-(\mathbf{m}'\cdot\mathbf{R})\mathbf{n}'+(\mathbf{n}'\cdot\mathbf{R})\mathbf{m}']B_m(\mathbf{x}',t')$$
$$+[-(\mathbf{l}'\cdot\mathbf{R})\mathbf{n}'+(\mathbf{n}'\cdot\mathbf{R})\mathbf{l}']B_l(\mathbf{x}',t')\}dS'$$
$$\mathbf{R}=\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}+\frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t}$$

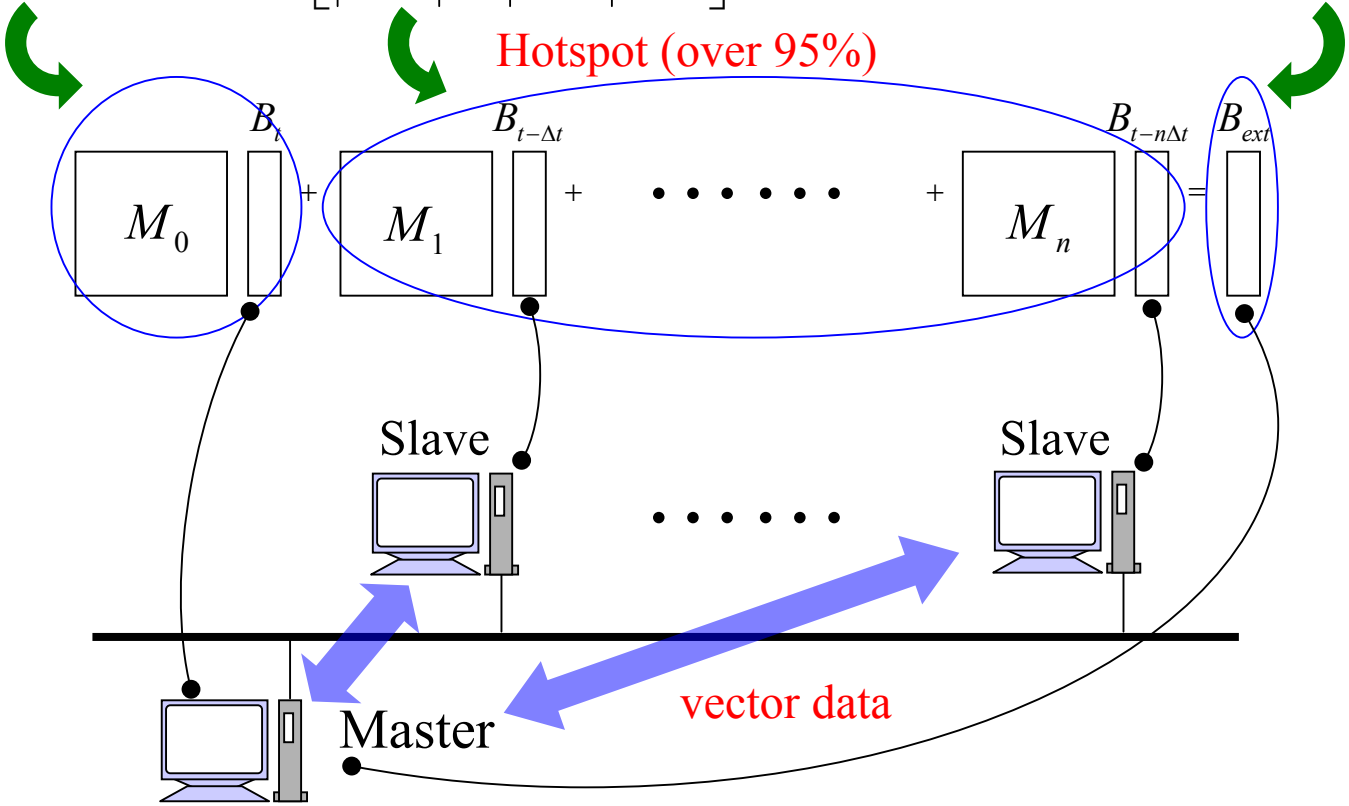
10 T Byte ➡ 51.2 G Byte ➡ 64 M Byte

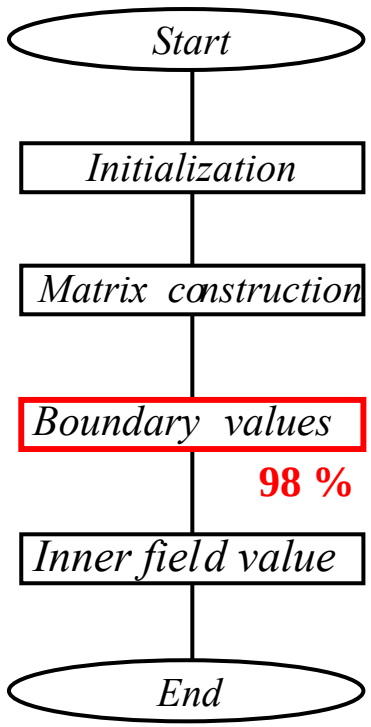
3D　　2.5D　　軸対称2D

4．2　計算コスト

計算時間　　**並列計算化**

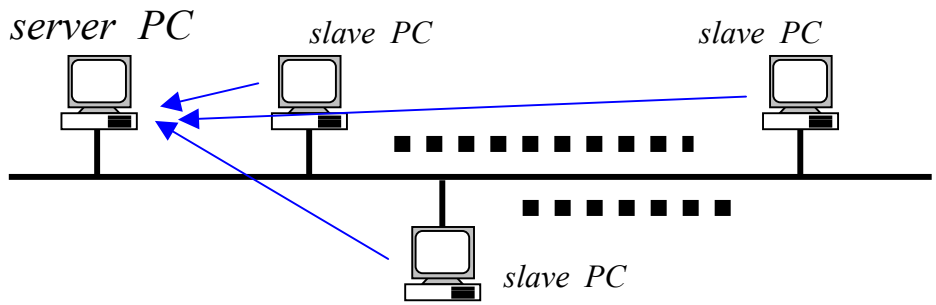
$$\mathbf{B}(\mathbf{x},t)+\frac{1}{4\pi}\int_S\left[\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}+\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^2}\frac{\partial}{c\partial t}\right]\times\mathbf{B}_t\left(\mathbf{x}',t-\frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)dS'=\mathbf{B}_{ext}(\mathbf{x},t)$$

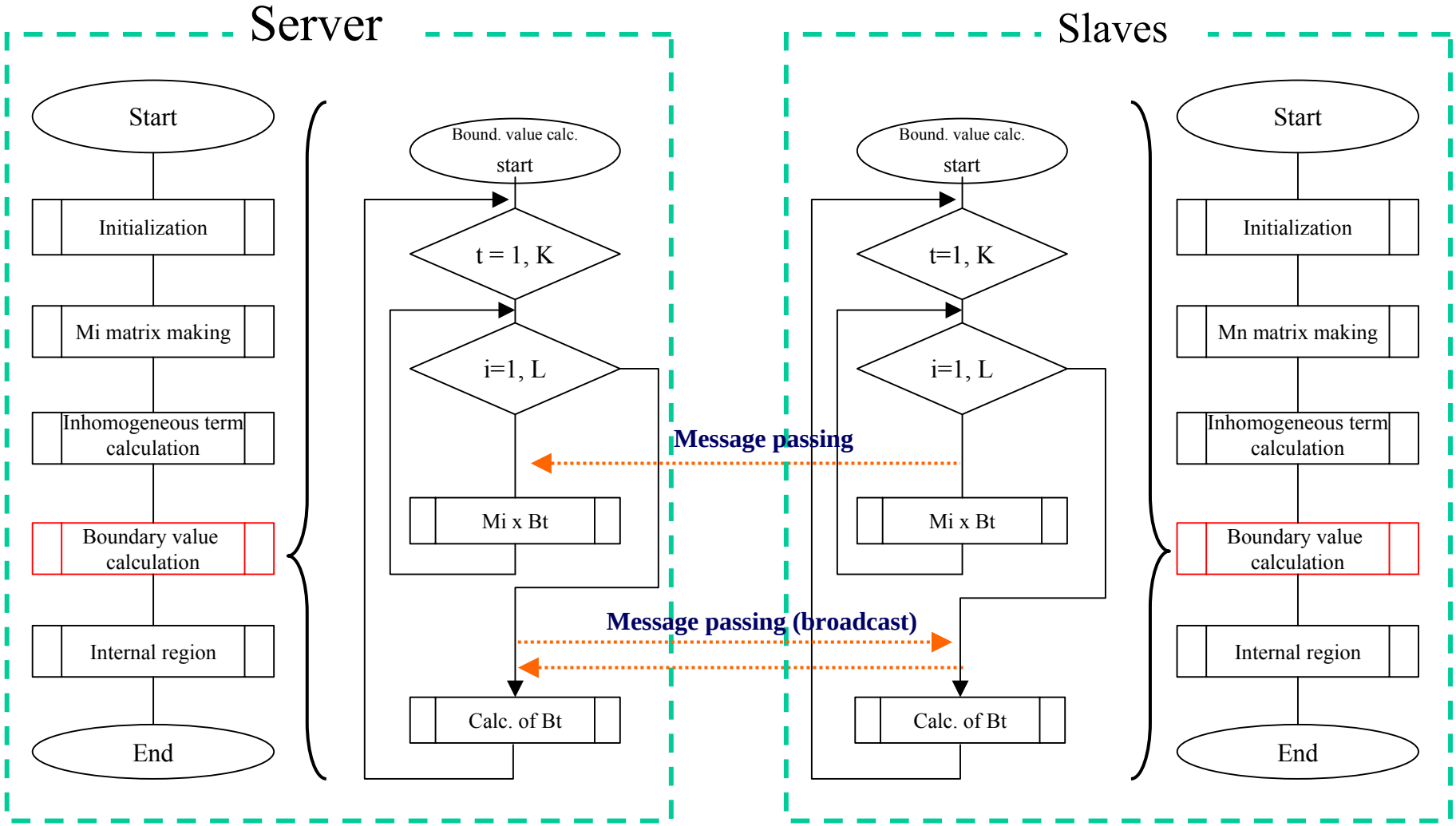




4．2　計算コスト

計算時間　　**並列計算化**



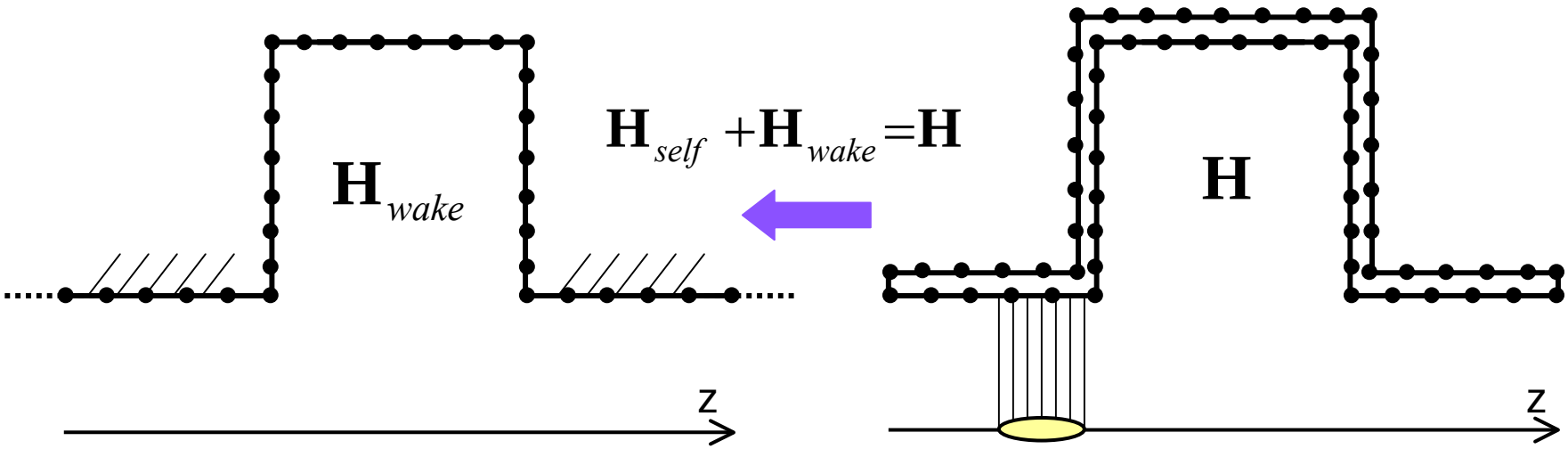


4．3　散乱場表示定式化

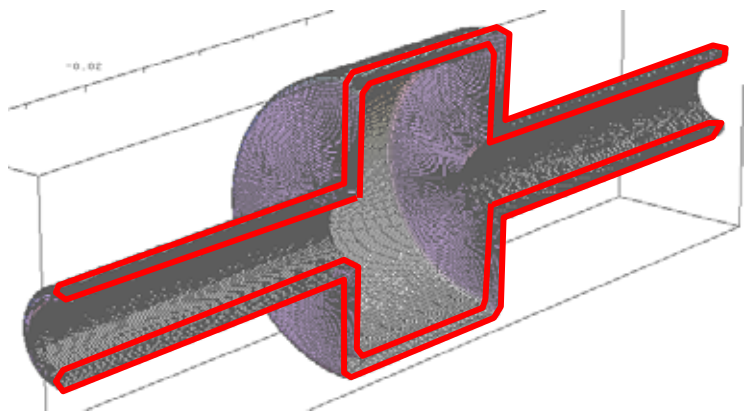
その他のTDBEMの問題

バンチ入射（体積積分＋特異性，非物理的擾乱），無駄なメッシュ，内部共振解＋コホモロジー解

Scattered field TDBEM(S-TDBEM)　　Conventional TDBEM

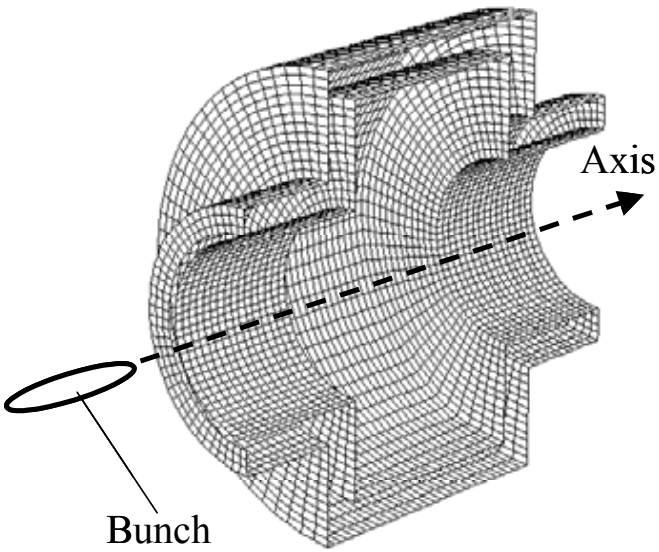


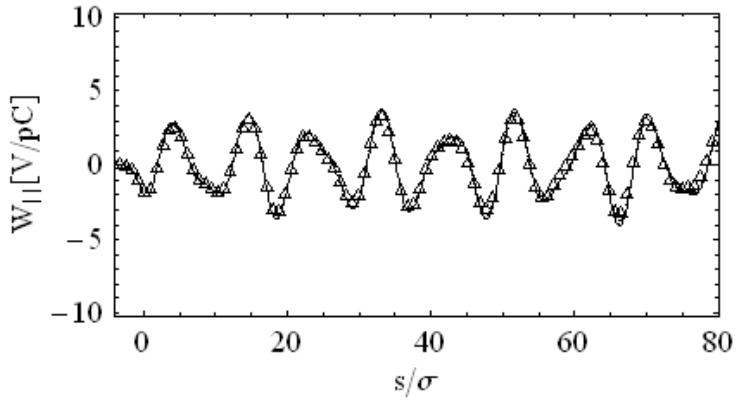
- スムースなバンチ入射
- メッシュ数削減
- 内部共振解の除去（安定化）
- 陽的スキームが可能
- 計算時間増



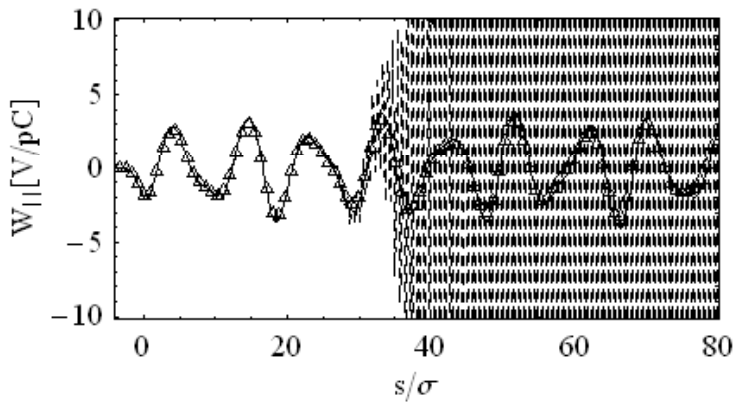
４．３　散乱場表示定式化

内部共振による不安定性（？）

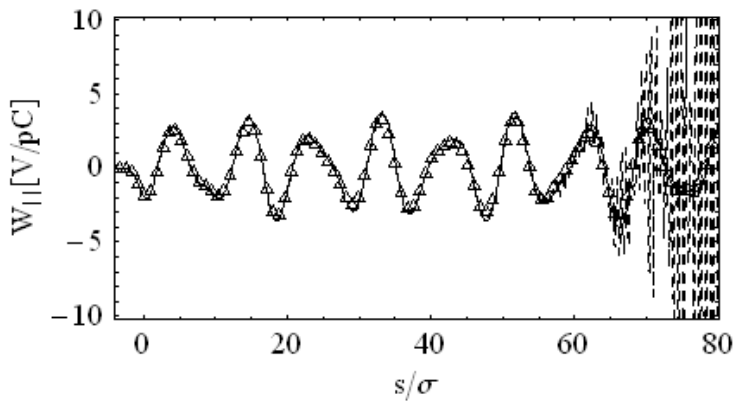




(a) d=2 mm ($c\Delta t = 1.08h$)

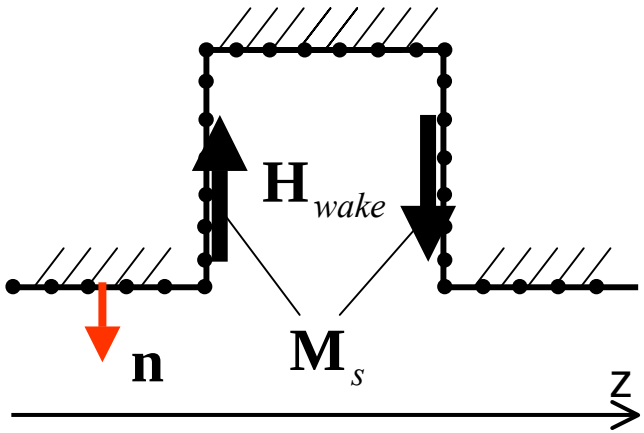


(b) d=2 mm ($c\Delta t = 0.38h$)



(c) d=4 mm ($c\Delta t = 0.38\ h$)

4．3　散乱場表示定式化



Integral representation in axis-symmetric 2D

$$\mathbf{H}_{wake}(\mathbf{r},t) = -\frac{1}{4\pi}\oint_S \left\{ \left(\mathbf{n}'\times\mathbf{H}_{wake}(\mathbf{r}',t')\right)\times grad'\frac{1}{R} - \frac{1}{R}\left(\mathbf{n}'\times\frac{\partial\mathbf{H}_{wake}}{c\partial t}(\mathbf{r}',t')\right)\times grad'R + \frac{1}{R}\left(\mathbf{n}'\times\varepsilon\frac{\partial\mathbf{E}_{wake}}{\partial t}(\mathbf{r}',t')\right)\right\} dS'$$

$$H_{wake}(t) = G_0 \quad = \quad H_{wake}(t-\Delta t)\ H_{wake}(t-2\Delta t) \quad G_1 \quad + \quad G_2 \quad + \cdots + \quad H_{wake}(t-L\Delta t) \quad G_L$$

$$+ \quad E_{wake}(t-\Delta t)\ \ E_{wake}(t-2\Delta t) \quad M_1 \quad M_2 \quad + \cdots + \quad E_{wake}(t-L\Delta t) \quad M_L$$

Required memeory: same

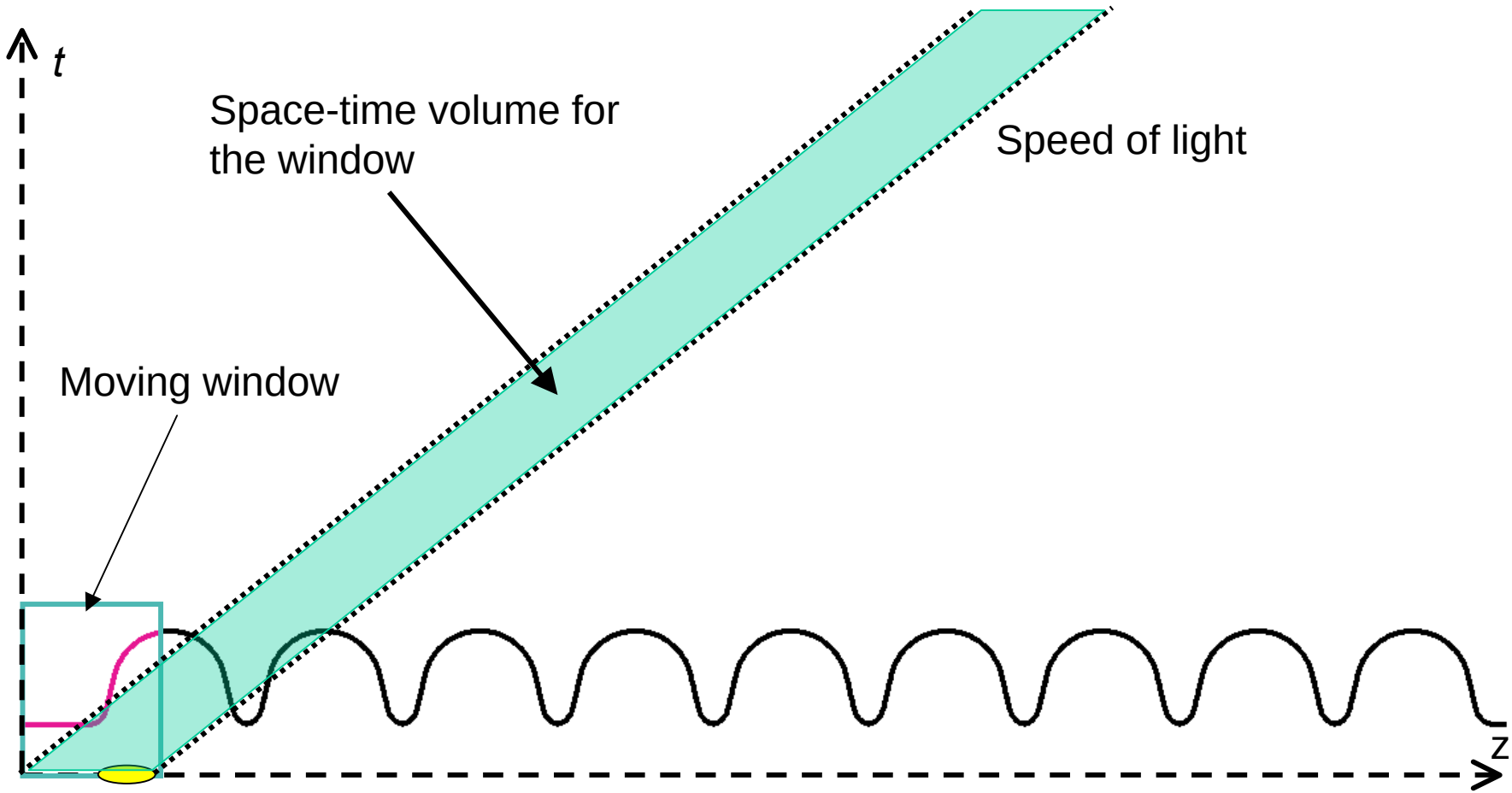
Calculation time: double

$$H_{wake}(t) = G_0 \quad = \quad H_{wake}(t-\Delta t)\ H_{wake}(t-2\Delta t) \quad G_1 \quad + \quad G_2 \quad + \cdots + \quad H_{wake}(t-L\Delta t) \quad G_L \quad + \quad H_{ext}(t)$$

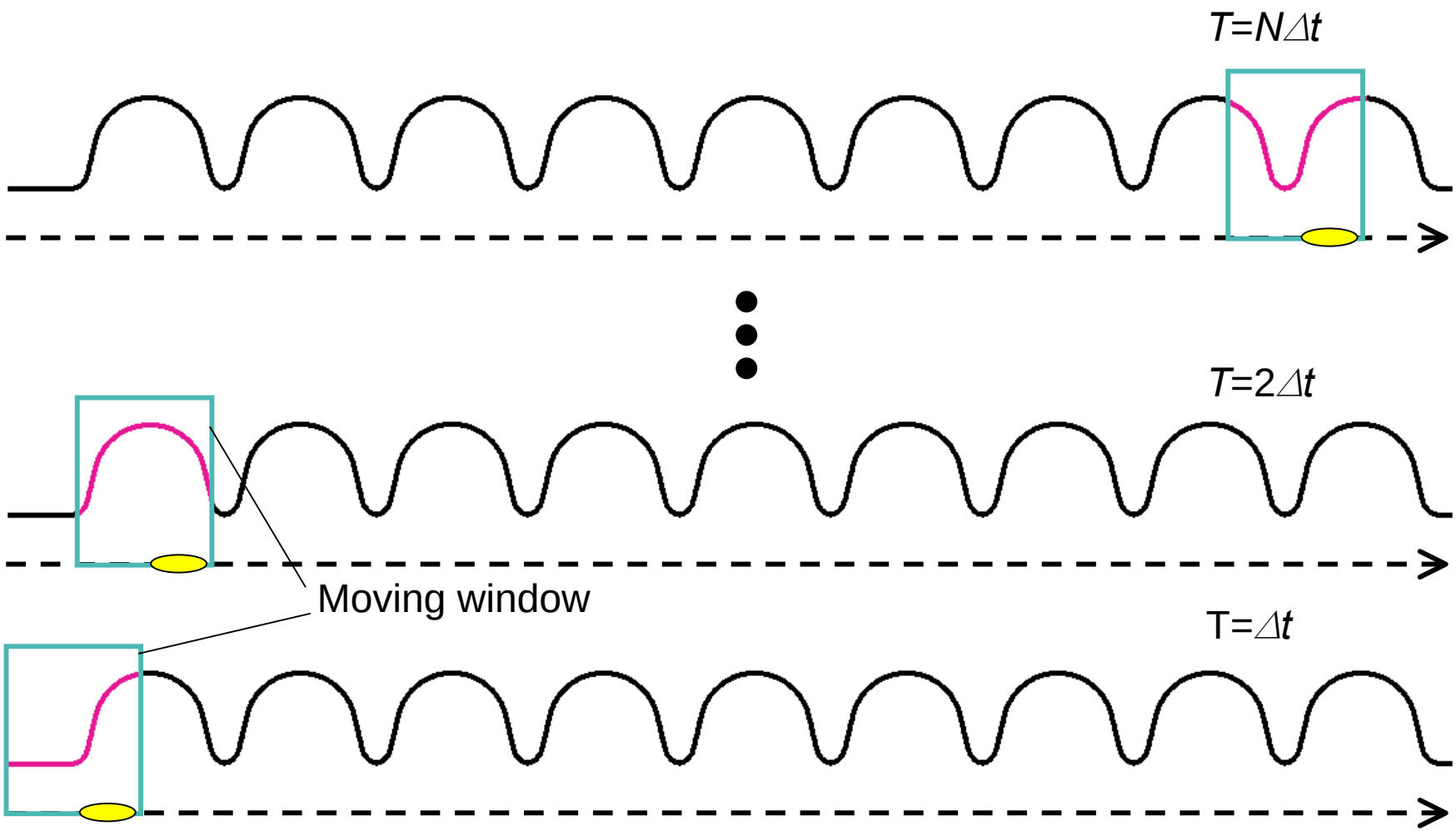
４．４　ウィンドウオプション

K.Bane and T.Weiland,
“Wake force computation in the time domain for long structures”,
Proc. 12th Int. Conf. High Energy Accelerators, pp.314-316, 1983

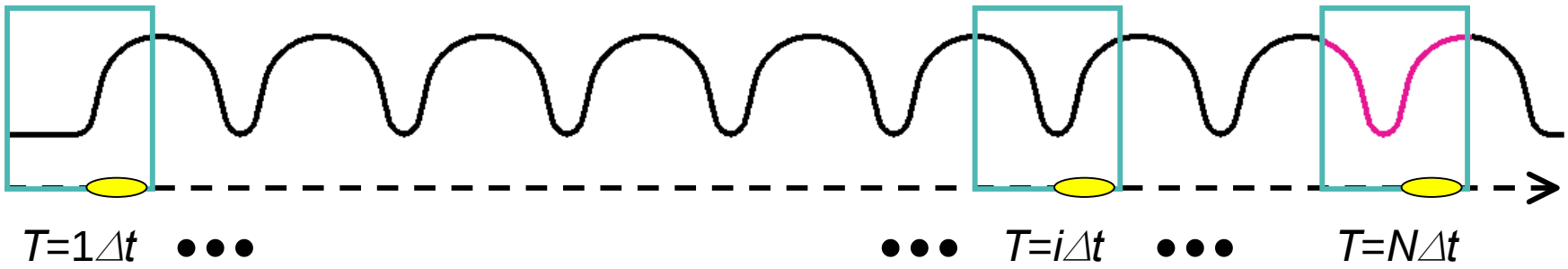
ビームダイナミクスのみに興味がある場合

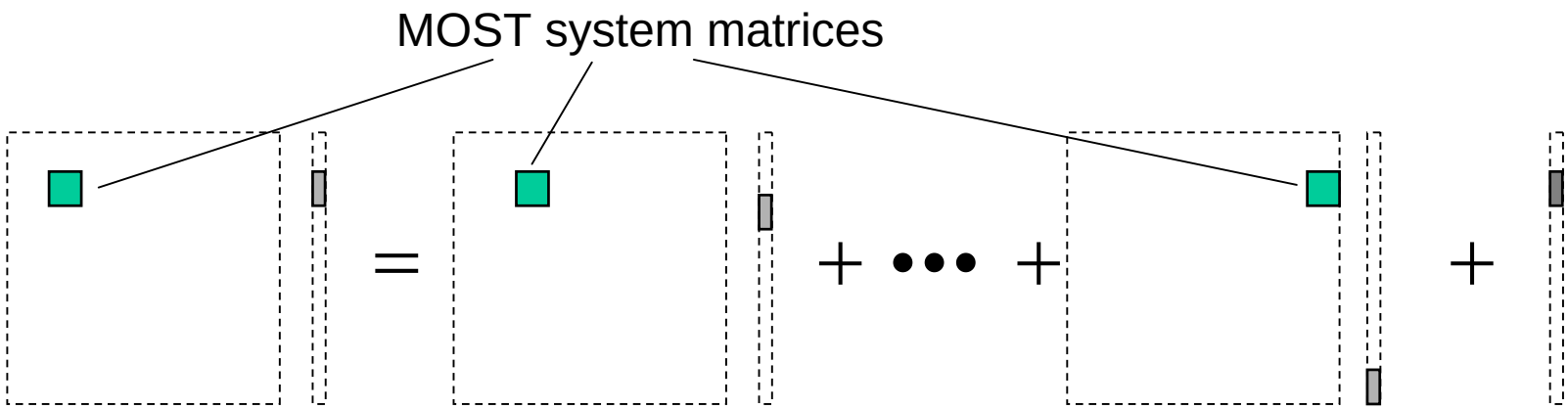


４．４　ウィンドウオプション



4．4　ウィンドウオプション



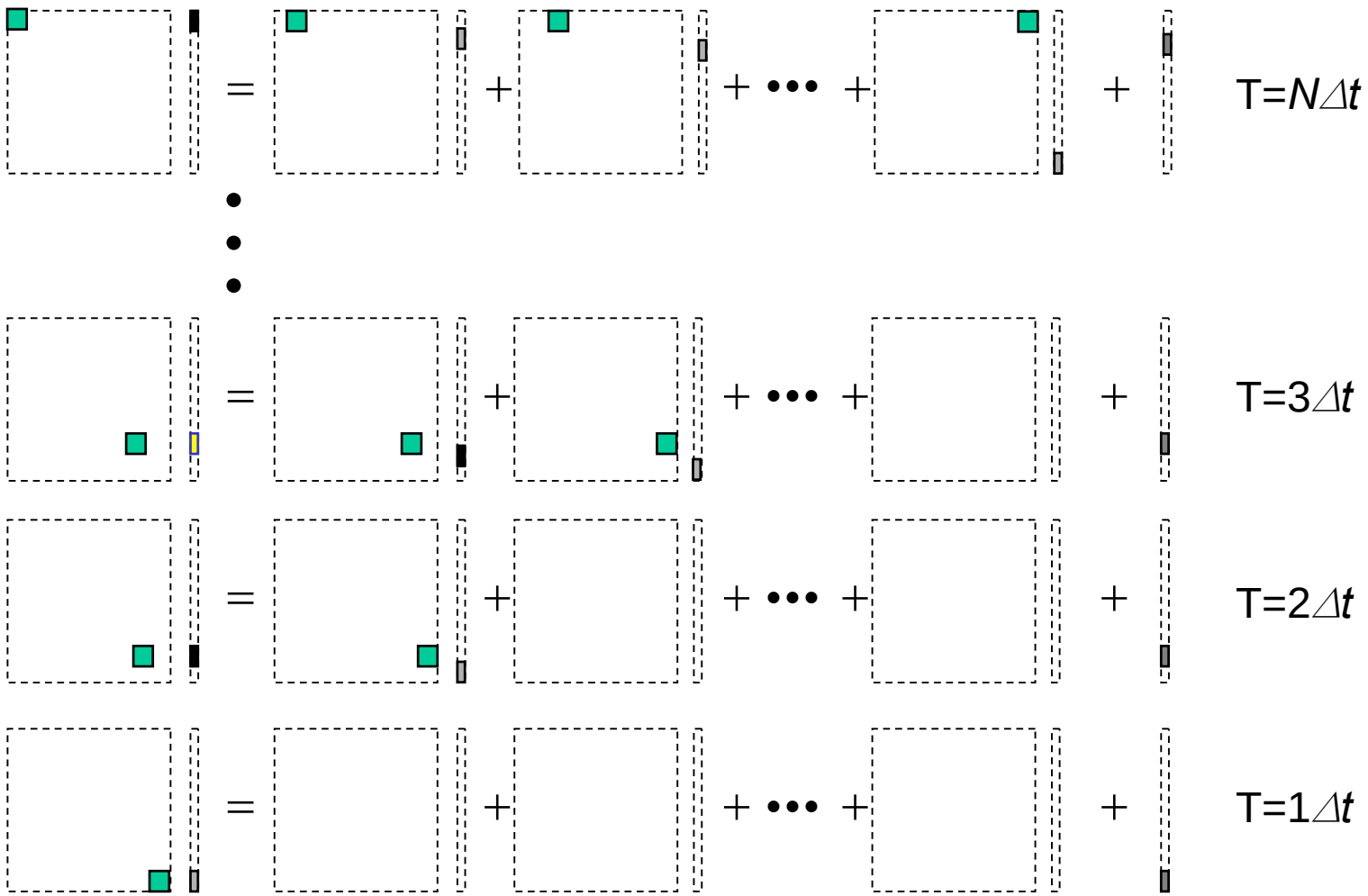


For N = 10000, L_{matrix}=15000,

4 TB(full matrix) → 5 GB(moving window) with N_{window}=200

4．4　ウィンドウオプション

システム行列

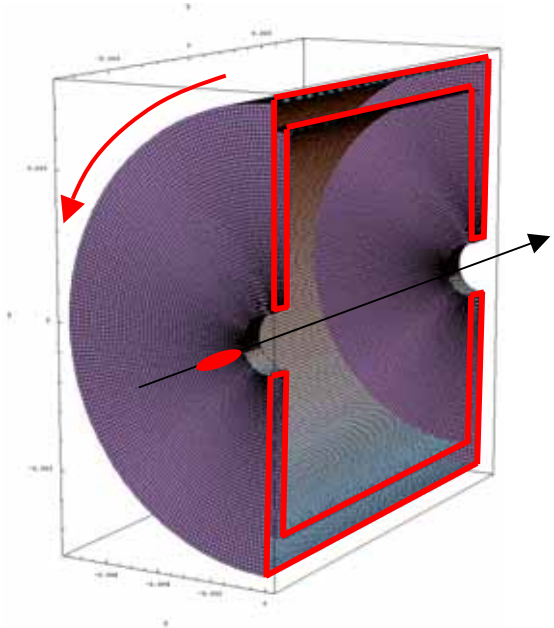


陽的スキームが必要！

５．数値解析例

５．１　ピルボックスキャビティ

数値モデル



回転方向　400分割

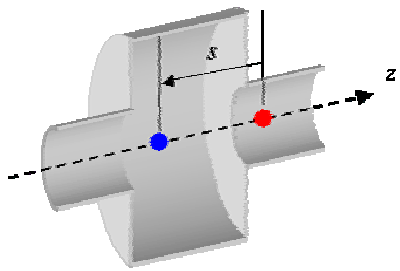
断面方向　660分割

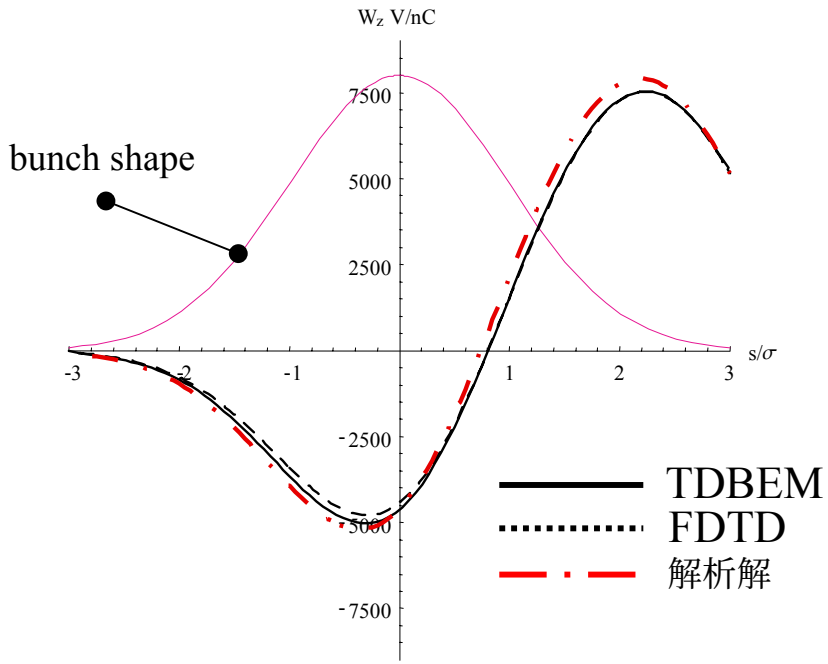
全メッシュ数　240000

所要メモリ　320MB

Wake potential (σ=3mm)

$$W_z(\mathbf{r},s) = -\frac{1}{q}\int_{-\infty}^{\infty} E_z(\mathbf{r},z,t=\frac{z+s}{v})dz$$

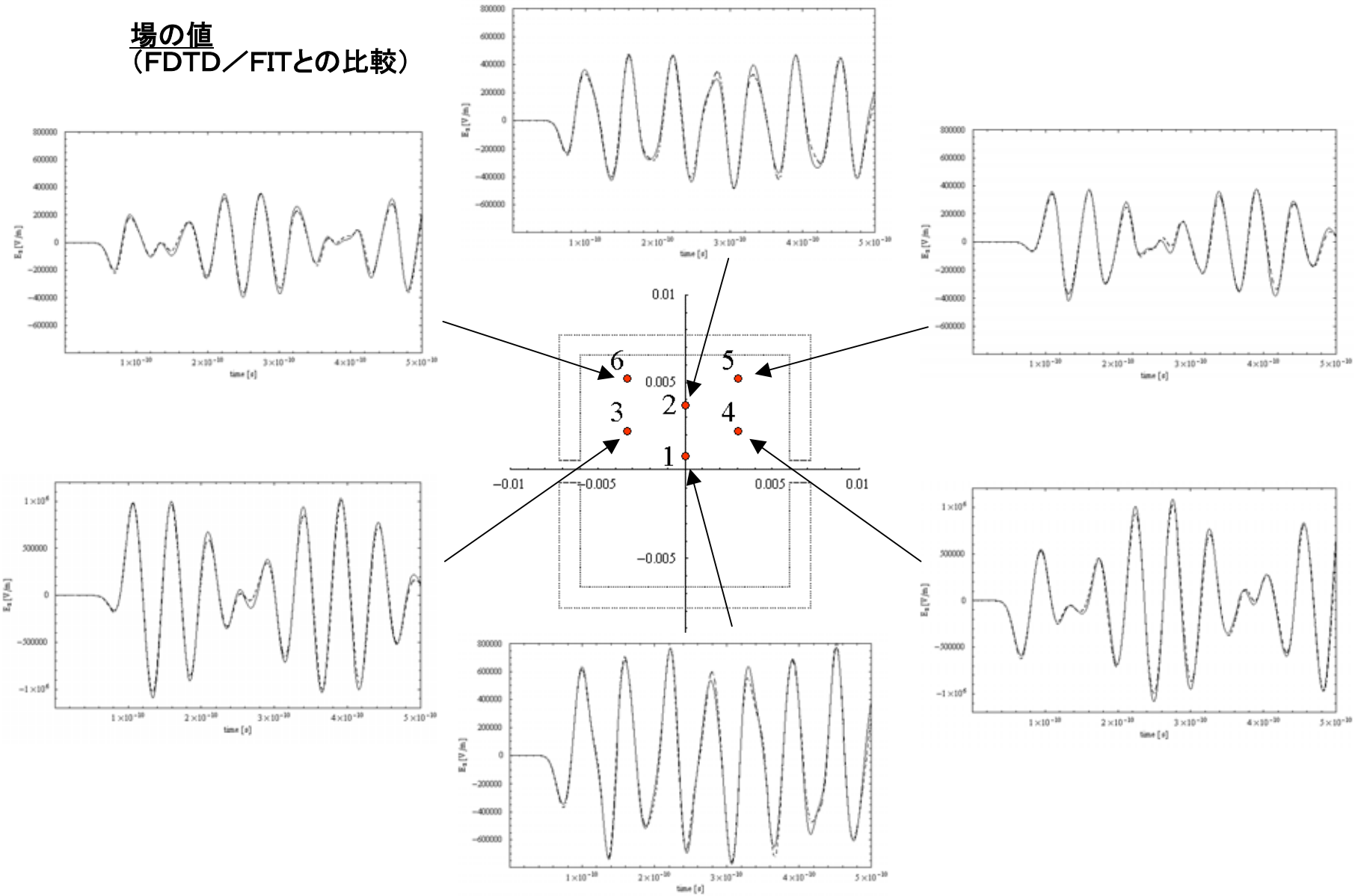




K. Fujita, H. Kawaguchi: Computational engineering I, JASCOME, 2004, pp.101-108

５．１　ピルボックスキャビティ

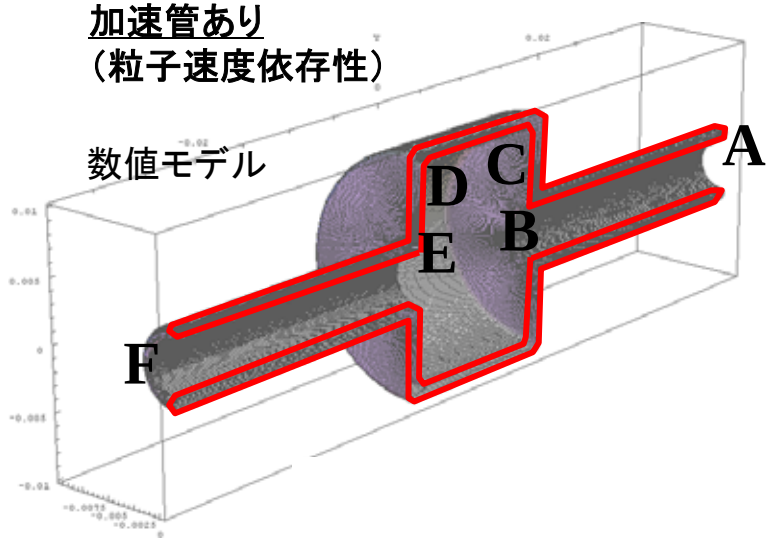
場の値
（FDTD／FITとの比較）

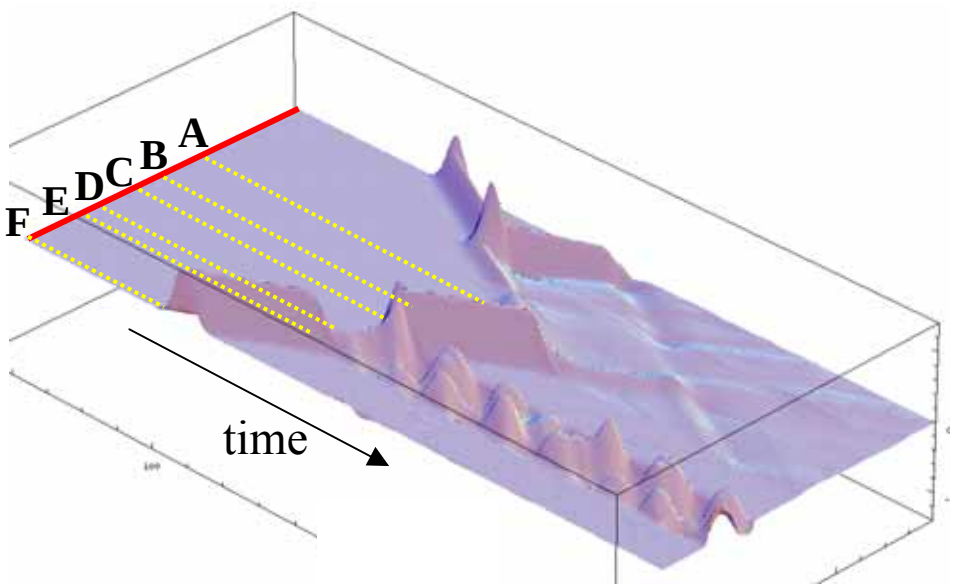


５．１　ピルボックスキャビティ

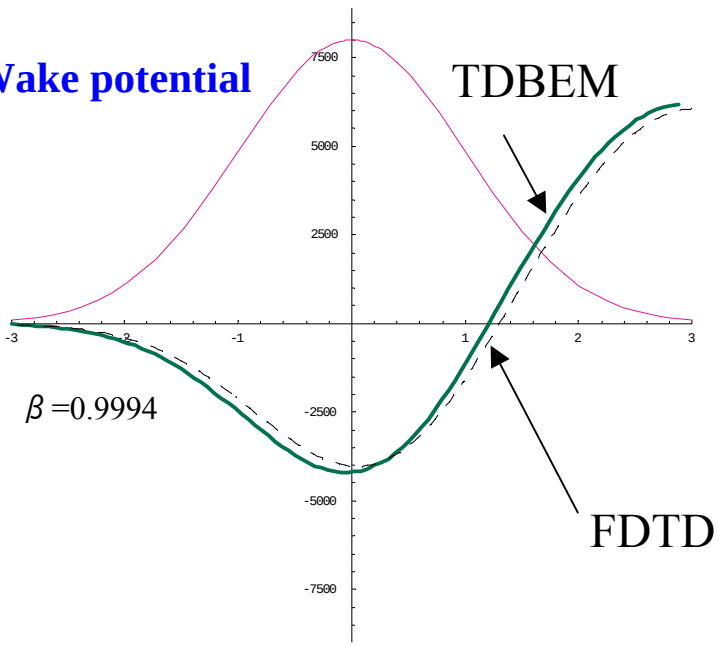
加速管あり
（粒子速度依存性）

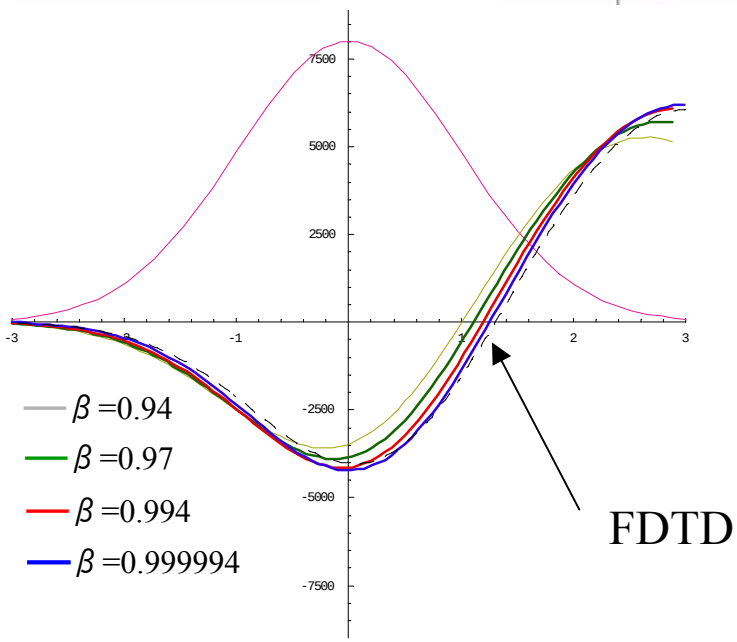
数値モデル





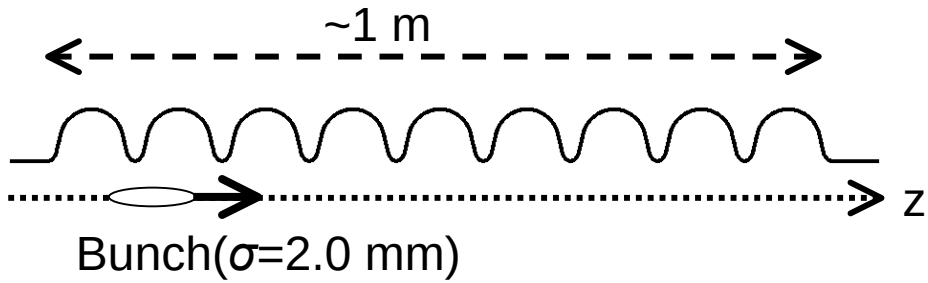
Wake potential

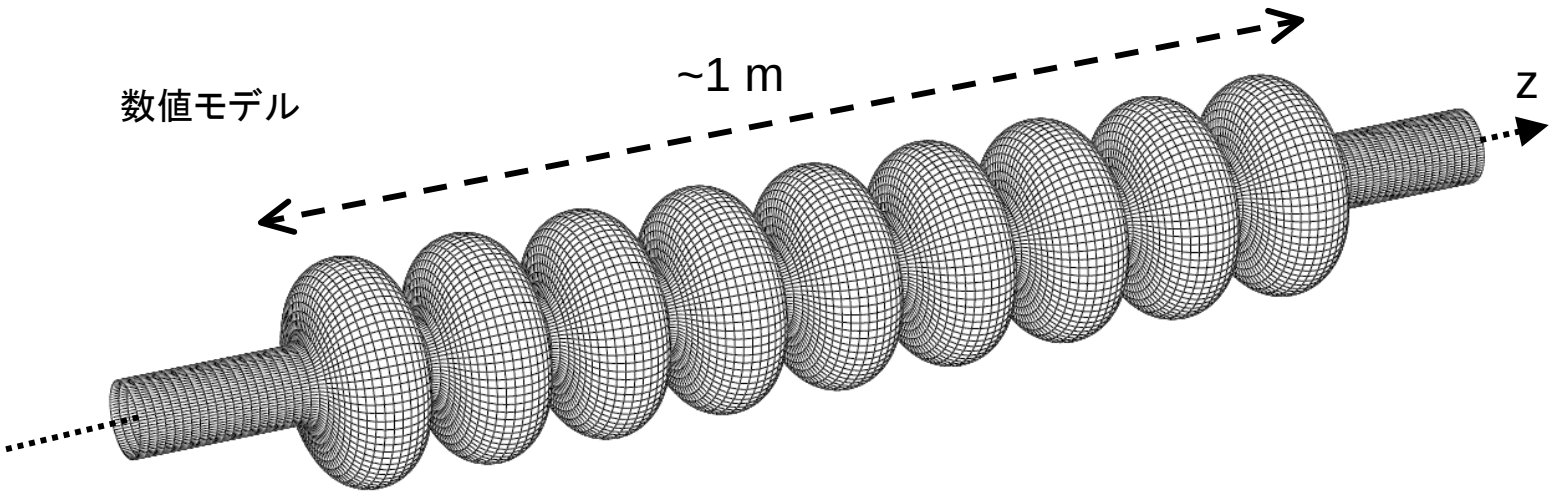




５．２ TESLA ９セルキャビティ

軸対象2次元＋ウィンドウオプション





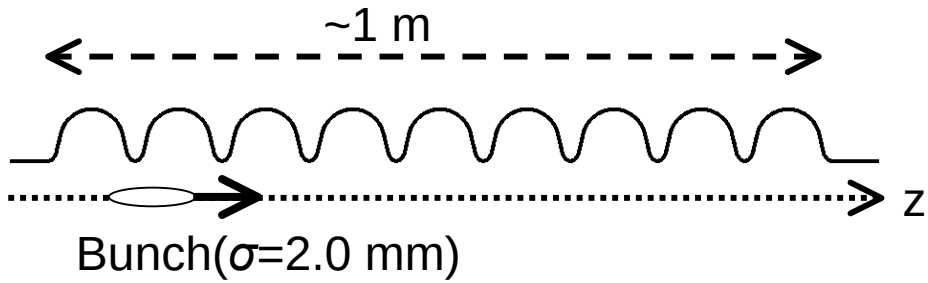
For the number of unknows N=5300, time step L = 8000, N_{window}=200,

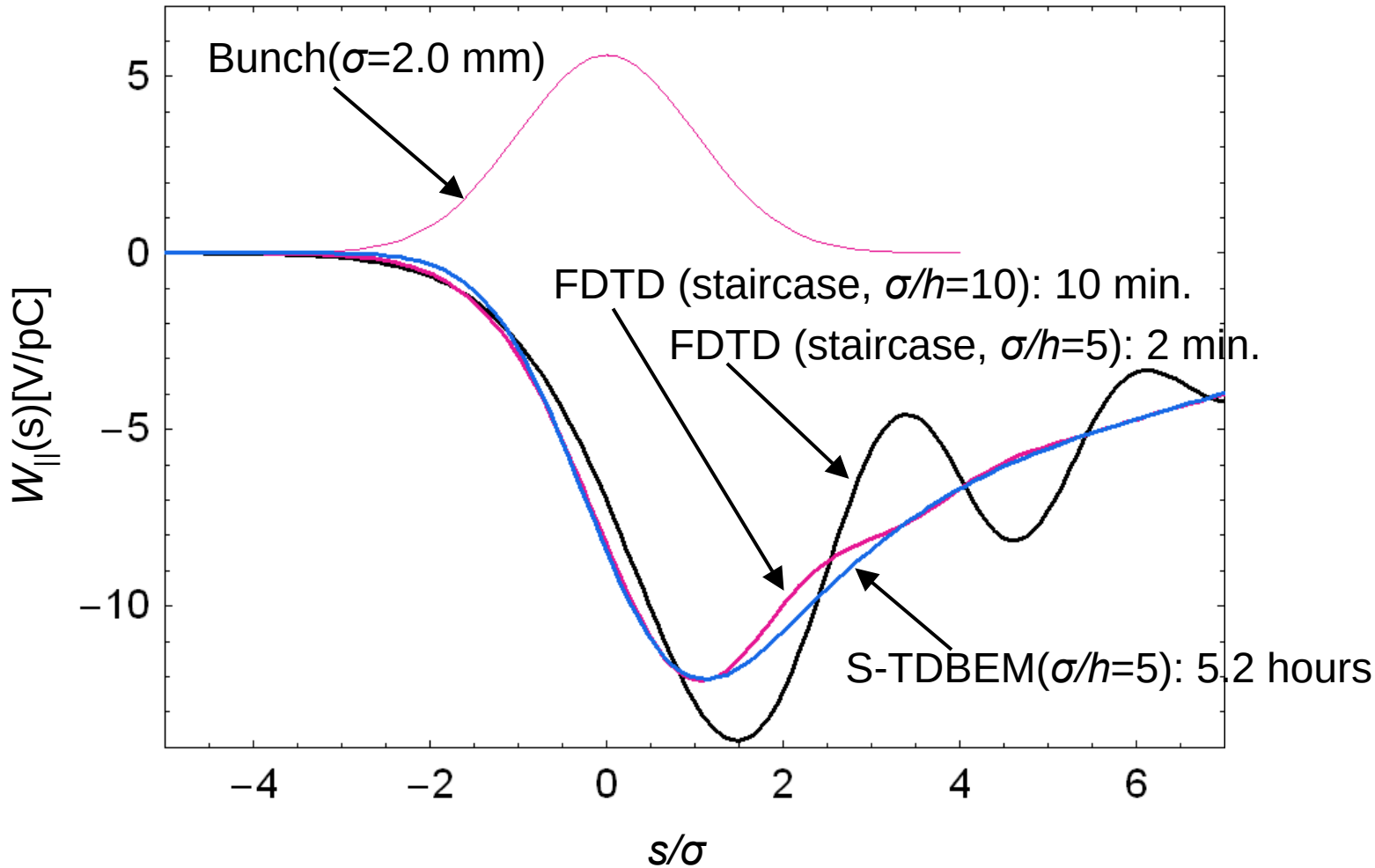
-Full matrix: 1.8 TB

-Moving window (with N_t=20) : 2.5 GB, 5.2 hours with HITACHI SR-11000/K1

-Moving window (no matrix): less than 500 MB, about 4 days

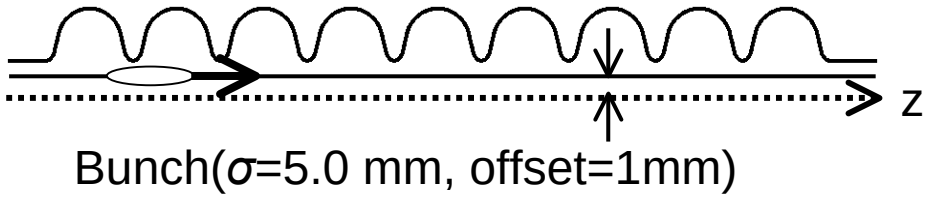
５．２　TESLA ９セルキャビティ

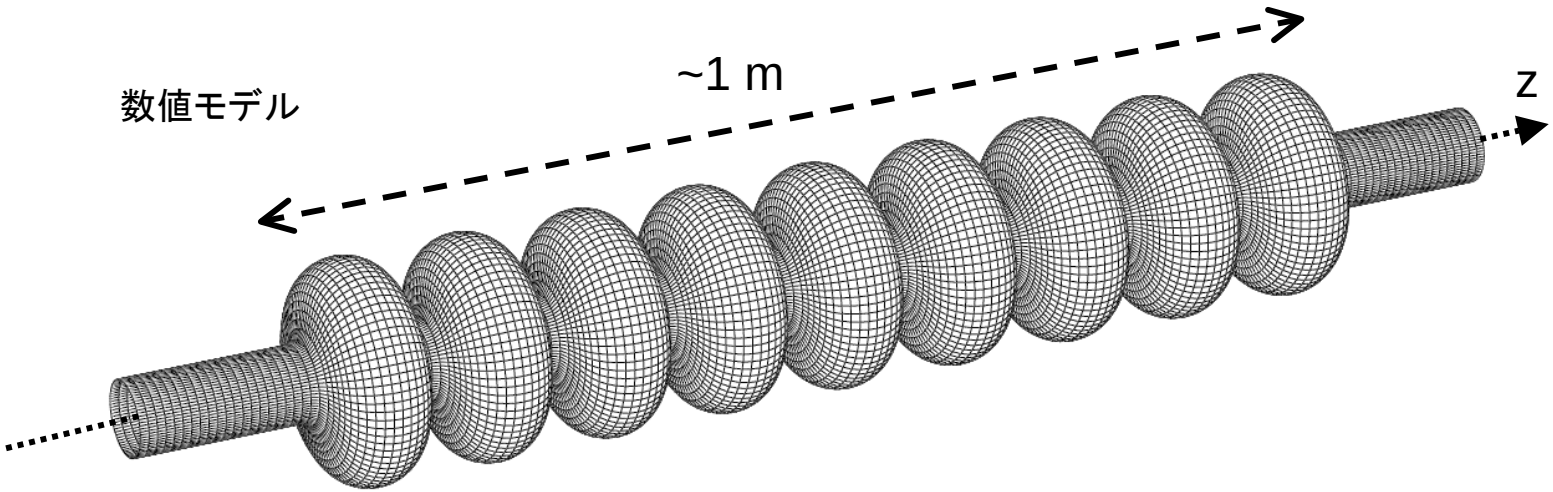




５．２　TESLA ９セルキャビティ

軸対象２．５次元＋ウィンドウオプション
（ダイポールウェーク）





For the number of unknows N=3720, time step L = 7600, N_{window}=290,

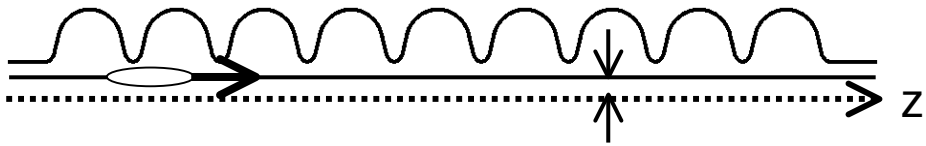
-Full matrix: 3.4 TB

-Moving window (with N_t=40) : 20.4 GB, 15.3 hours with HITACHI SR-11000/K1

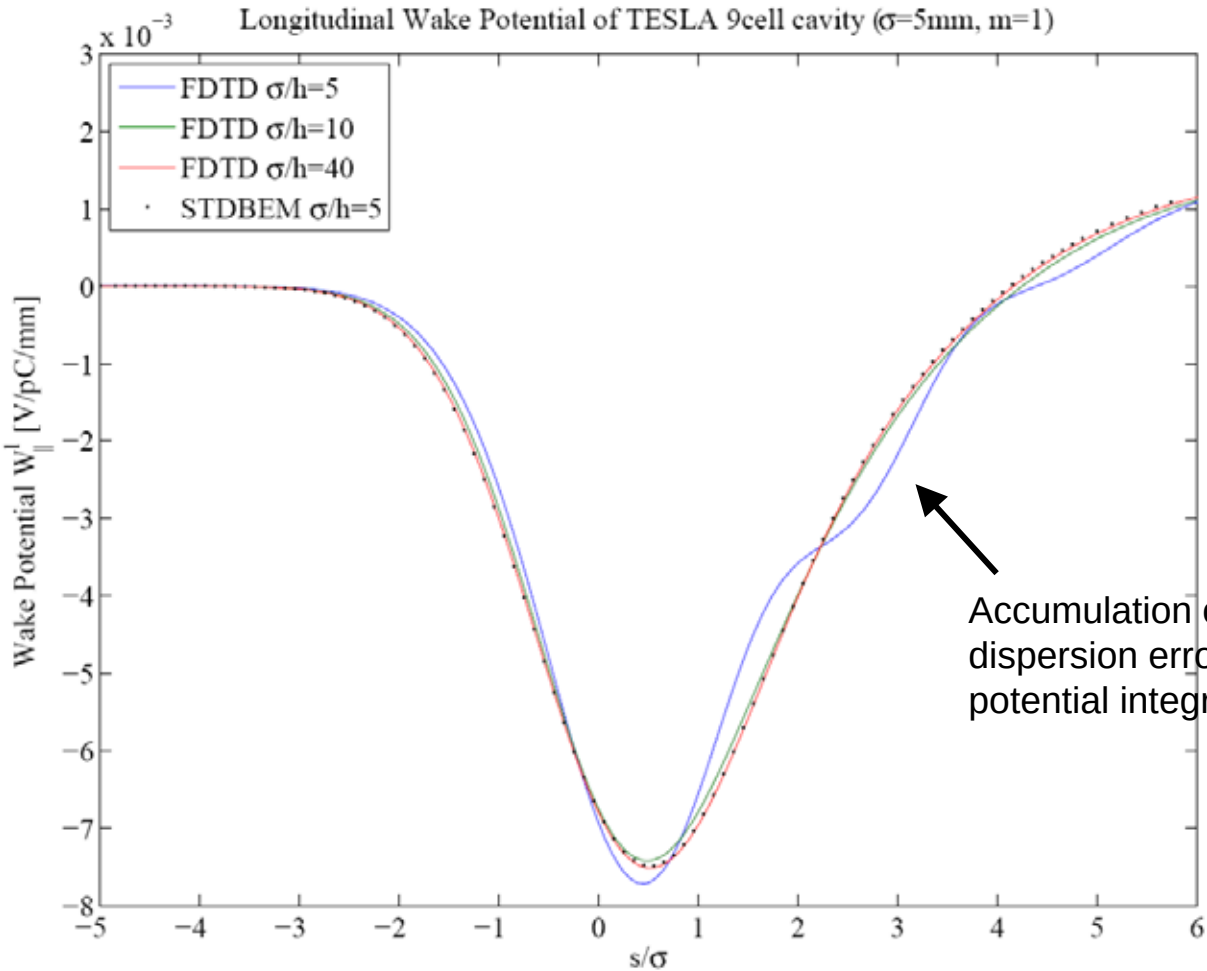
-Moving window (no matrix): less than 500 MB, about 25 days

５．２　TESLA ９セルキャビティ

軸対象２．５次元＋ウィンドウオプション
（ダイポールウェーク）



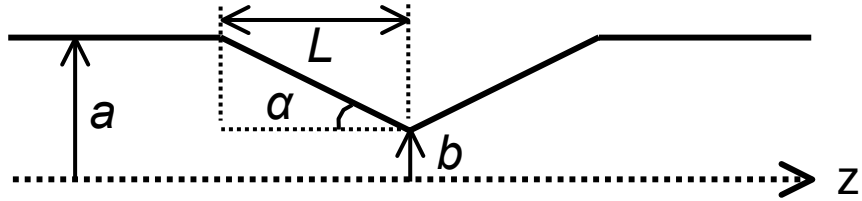
Bunch(σ=5.0 mm, offset=1mm)



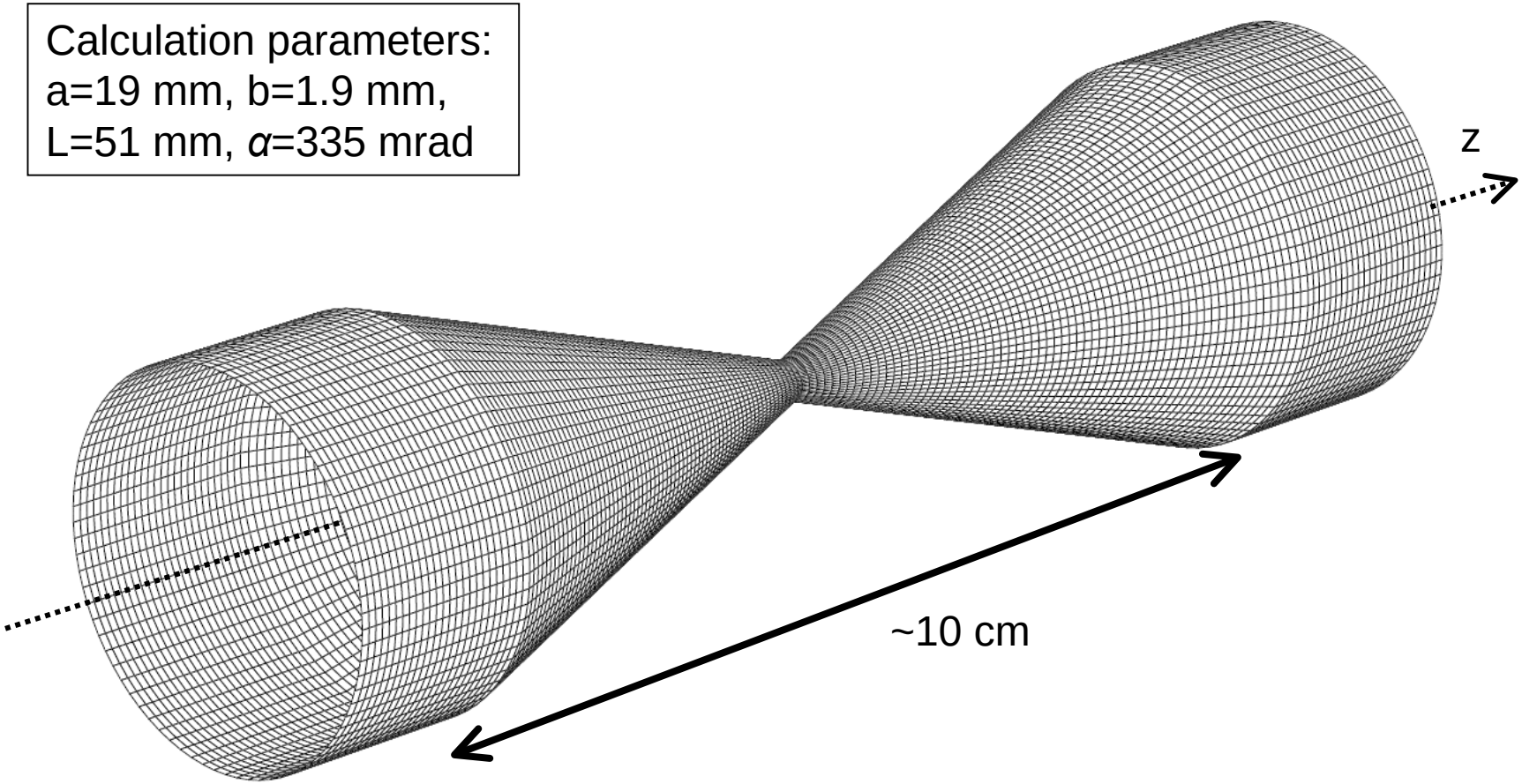
Accumulation of grid dispersion error in wake potential integraion

5．3 Tapered Collimeter

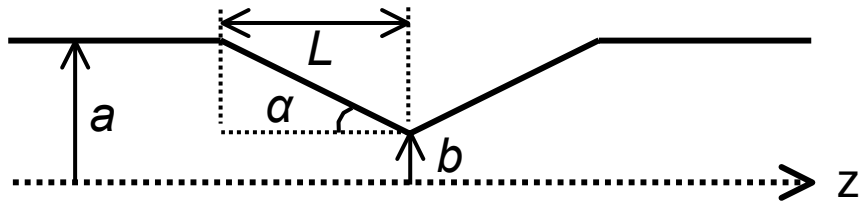
$\rho = \tan(\alpha) b/\sigma = 0.1653$



Calculation parameters:
a=19 mm, b=1.9 mm,
L=51 mm, α=335 mrad



5．3 Tapered Collimeter



$\rho=\tan(\alpha)b/\sigma$ =0.1653

Calculation parameters:
a=19 mm, b=1.9 mm,
L=51 mm, α=335 mrad

Bunch(σ=4.0 mm)

ECHO(conformal, σ/h=10)

S-TDBEM(σ/h=10)

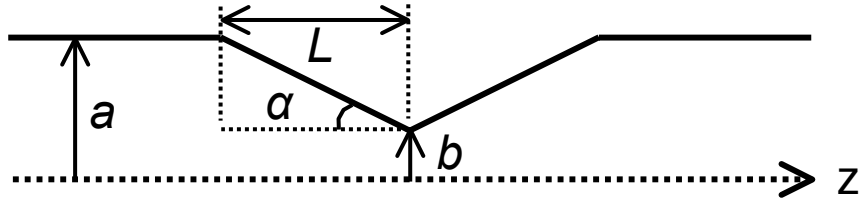
ECHO(staircase)
σ/h=20
σ/h=10
σ/h=5

Yokoya's formula for
ρ<<1(estimation)

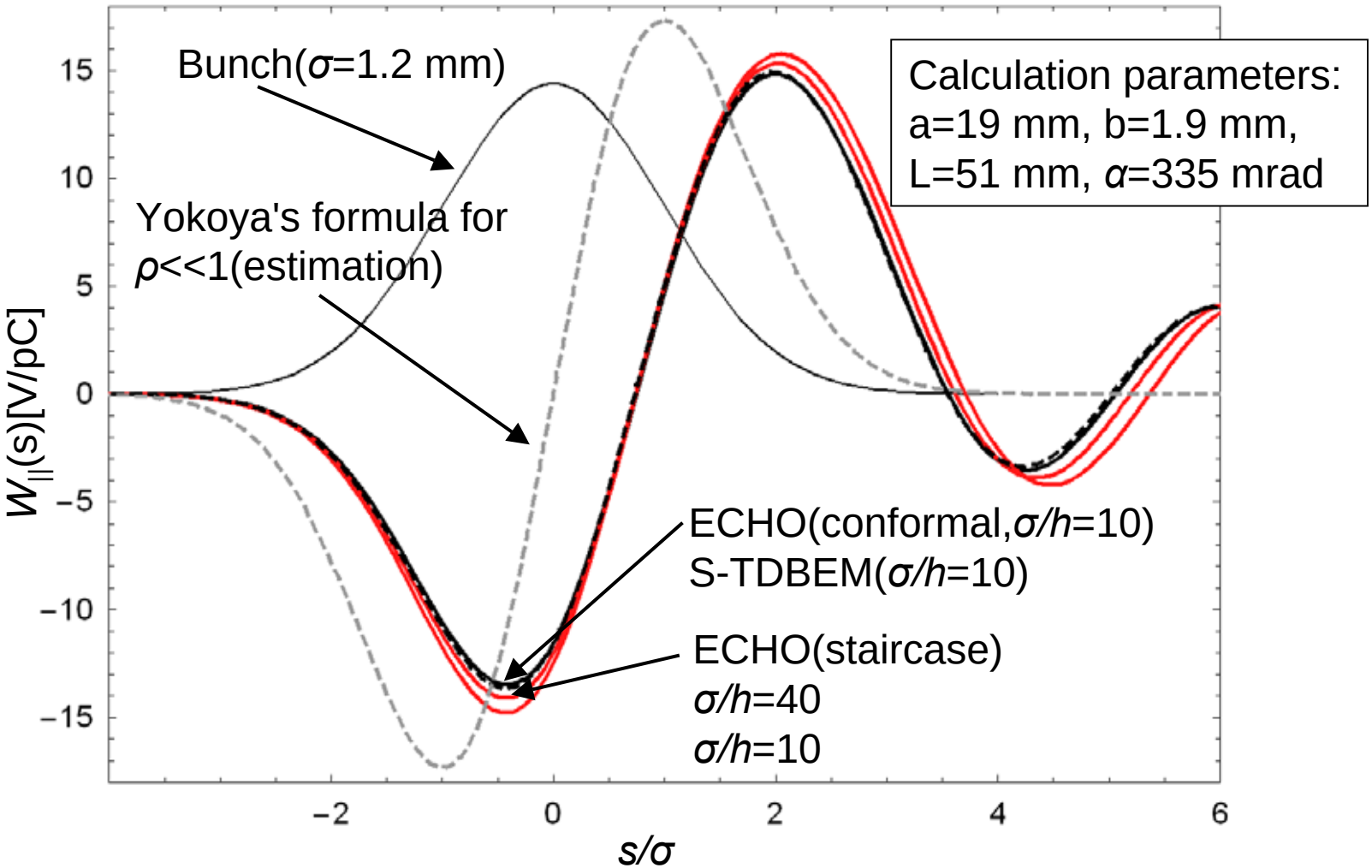
$W_{\parallel}(s)$[V/pC]

s/σ

5．3 Tapered Collimeter



$\rho=\tan(\alpha)b/\sigma$ =0.5511



６．今後の課題

フル3D化

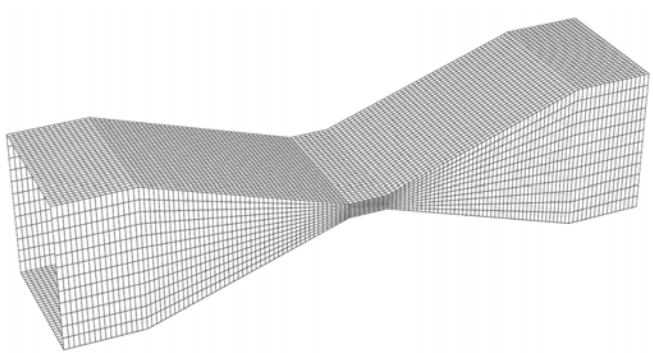
ウィンドウオプション化のコード作成済，実際の問題への適用

自己無撞着解析

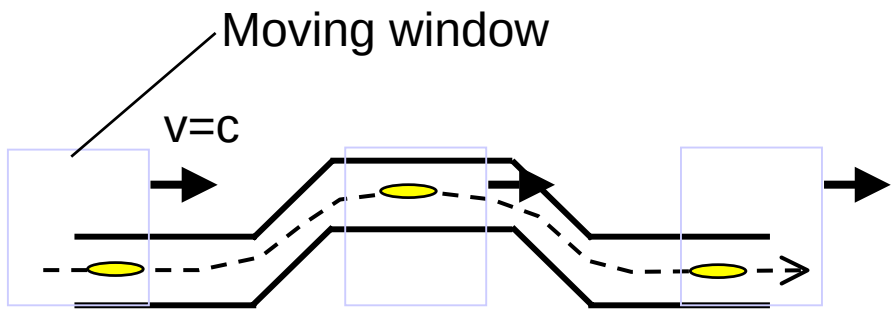
M. Dohlus のイメージ電荷法との比較

加速器科学への応用

３次元構造コリメーター



バンチコンプレッサー



ハイパフォーマンスコンピューティング

専用計算機の開発

A. 1 時間領域境界要素法専用計算機

システム構成