細かく刻めば、ぼくでもできる

高輝度ビーム生成のための
３次元差分法シミュレーション

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東北大学大学院理学研究科原子核理学研究施設

東北大核理研の電子銃開発

１　熱陰極RF電子銃

コヒーレントTHz放射光源のための高輝度超短パルスビーム生成

２　熱陰極DC電子銃

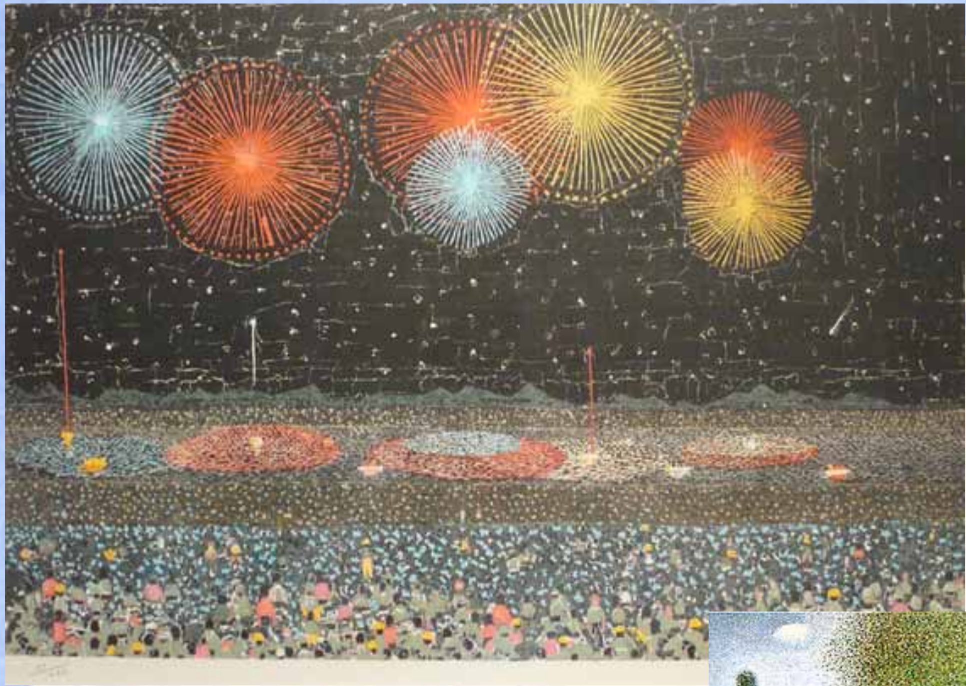
Smith-Purcell BWO-FELのための超高輝度DC電子ビーム生成

シミュレーションコード

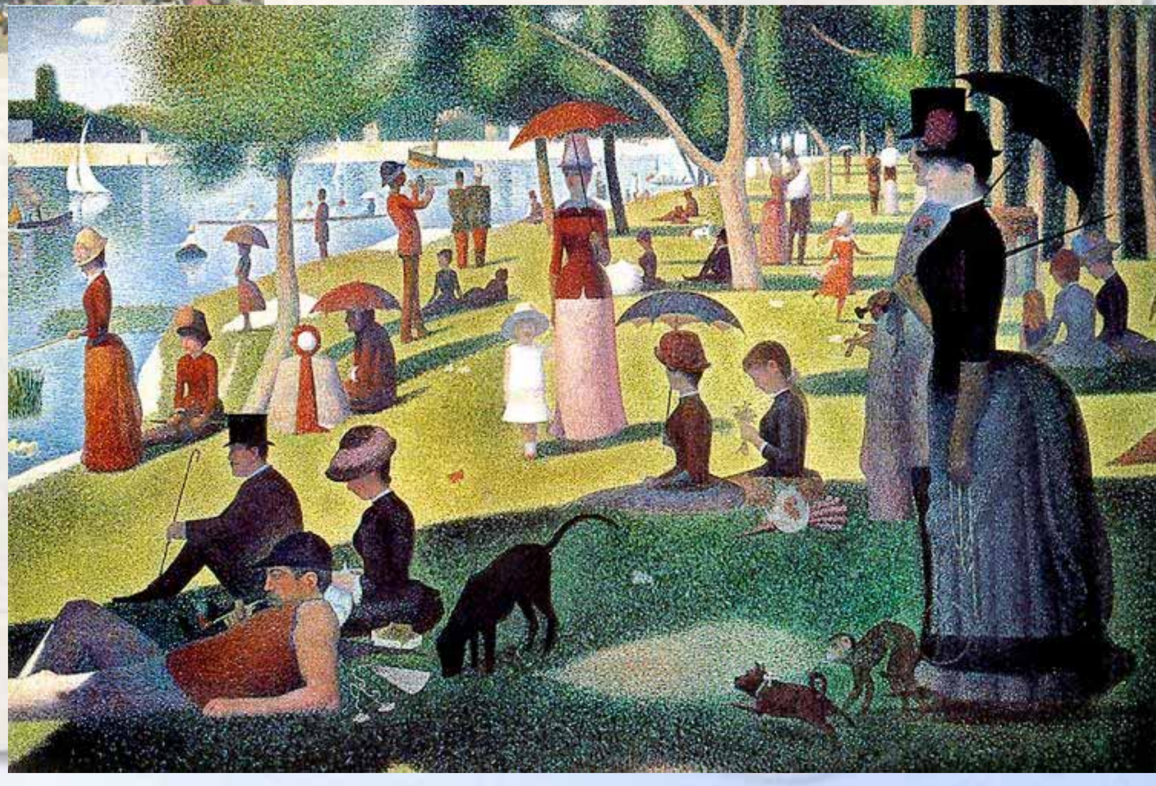
１　３次元時間発展電磁場計算（FDTD法）　コードX

２　３次元静電磁場計算（差分法）コードY

３　もらったり、買ったりしたもの
　EGUN, MWSTUDIO, SUPEFISH, POISSON, PARMERA



「長岡の花火」 山下清

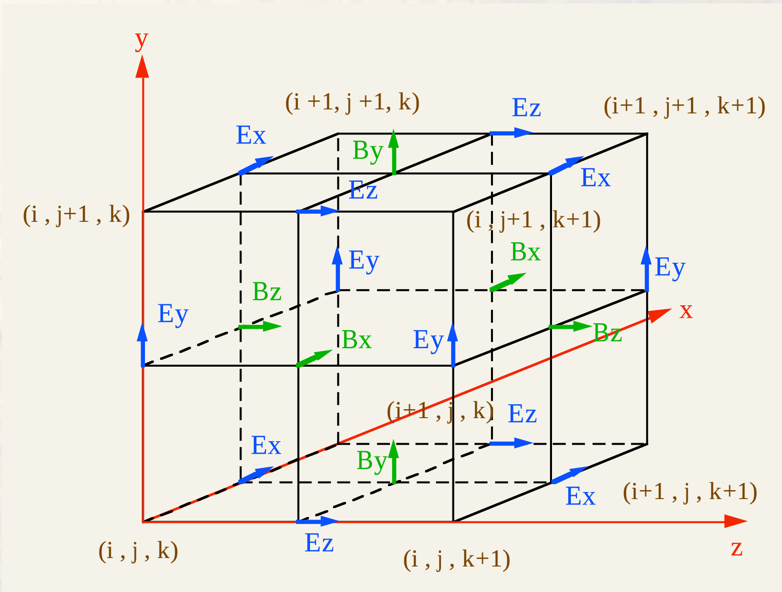


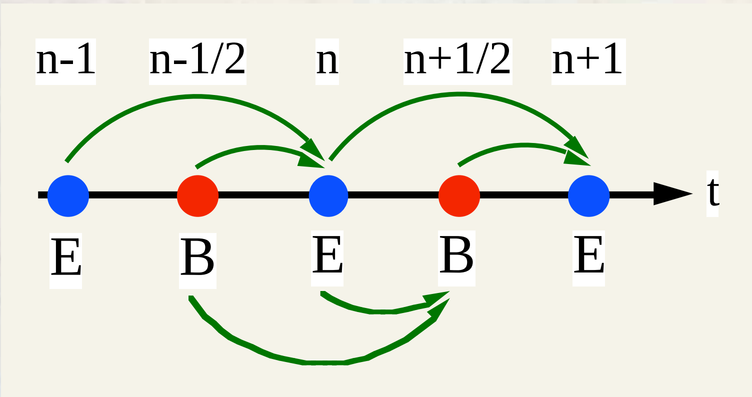
“Sunday Afternoon on the Island of La Grande Jatte” Georges Seurat

3-Dimensional FDTD

Finite Difference Time Domain

Yee cell (grid)





Leap-Frog algorism

Finite Difference Equation

Spatial Domain

$$\frac{\partial F^n(i,j,k)}{\partial x} = \frac{F^n\left(i+\frac{1}{2},j,k\right) - F^n\left(i-\frac{1}{2},j,k\right)}{\Delta x} + O\left(\Delta x^2\right)$$

Time Domain

$$\frac{\partial F^n(i,j,k)}{\partial t} = \frac{F^{n+\frac{1}{2}}(i,j,k) - F^{n-\frac{1}{2}}(i,j,k)}{\Delta t} + O\left(\Delta t^2\right)$$

First order of Taylor expansion of differential equation
Error comes from 2nd order expansion
Symplectic ! within 1 st order

Maxwell's Equations

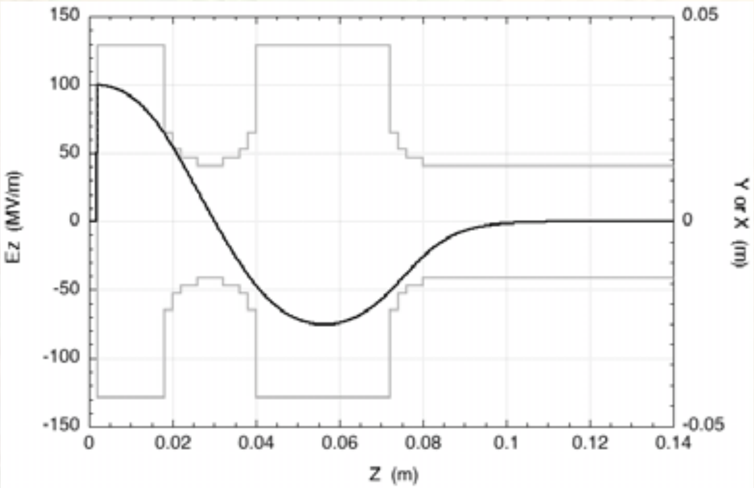
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$$

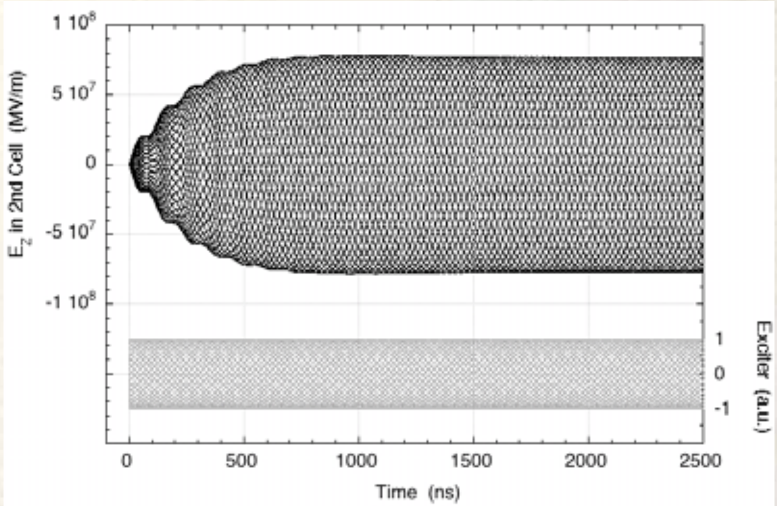
Example solution (E_x) of finite difference equation,

$$E_x^{n+1}\left(i+\frac{1}{2},j,k\right) = c^2\Delta t\left[\frac{B_z^{n+\frac{1}{2}}\left(i+\frac{1}{2},j+\frac{1}{2},k\right)-B_z^{n+\frac{1}{2}}\left(i+\frac{1}{2},j-\frac{1}{2},k\right)}{\Delta y}-\frac{B_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k+\frac{1}{2}\right)-B_y^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k-\frac{1}{2}\right)}{\Delta z}\right]$$
$$-\frac{\Delta t}{\varepsilon_0}J_x^{n+\frac{1}{2}}\left(i+\frac{1}{2},j,k\right)+E_x^{n}\left(i+\frac{1}{2},j,k\right)$$

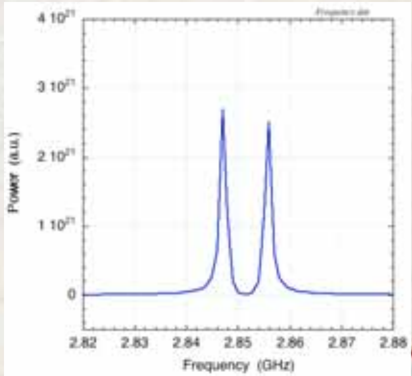
Yee cell restriction $\quad (c\Delta t)^{-1} \geq \sqrt{\Delta x^{-2}+\Delta y^{-2}+\Delta z^{-2}}$

BNL 1.5-Cell-RF Gun

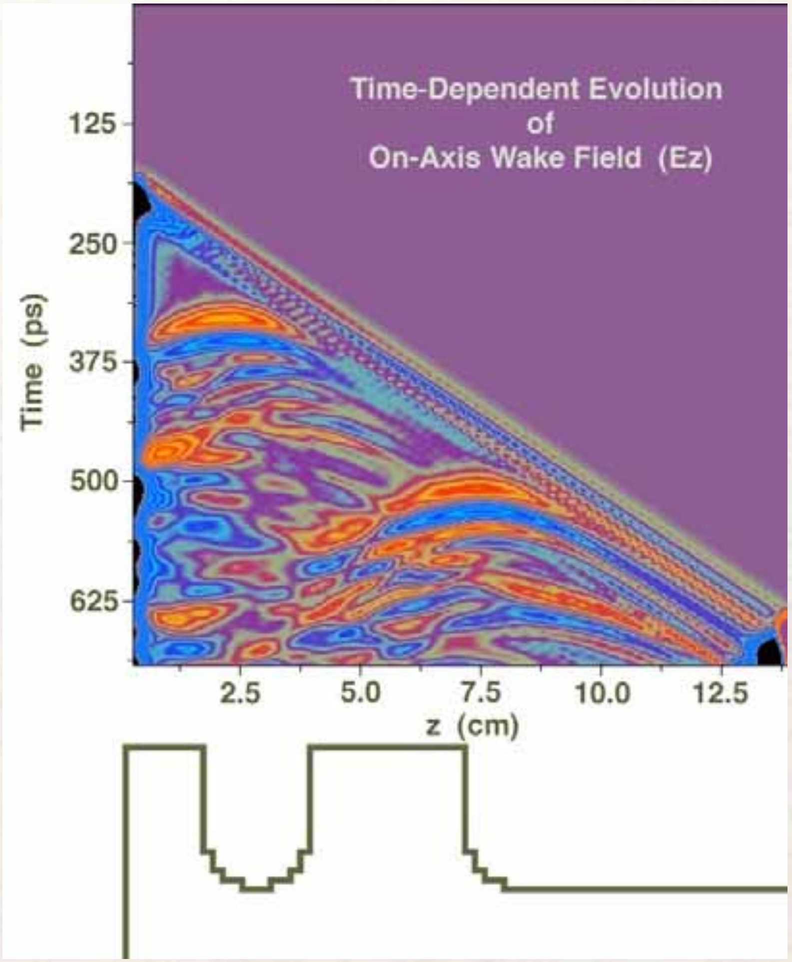




Two modes are excited !

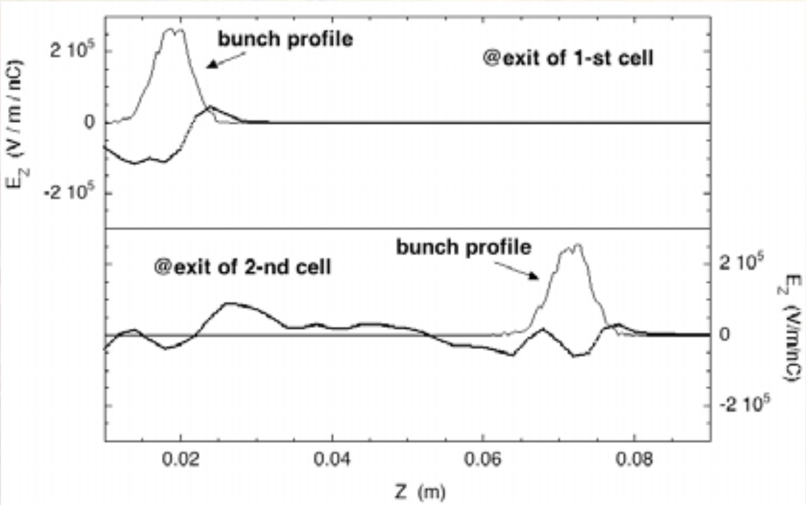


Time Evolution of Wakefield

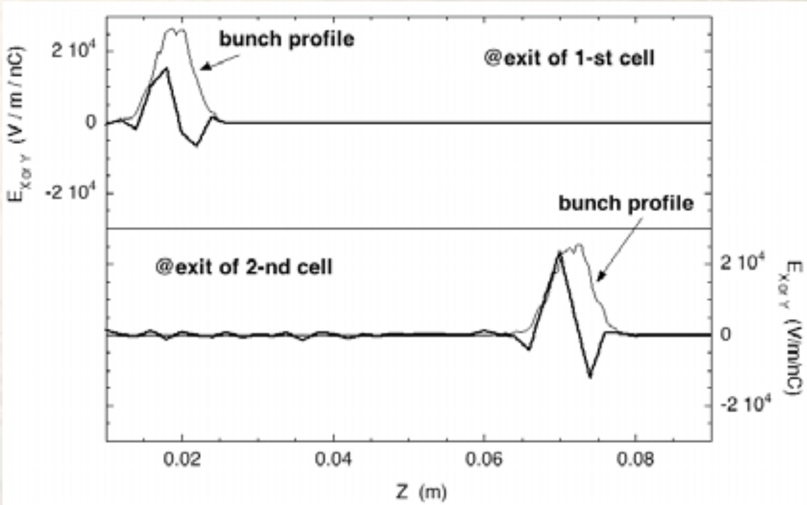


Wakefields

Longitudinal

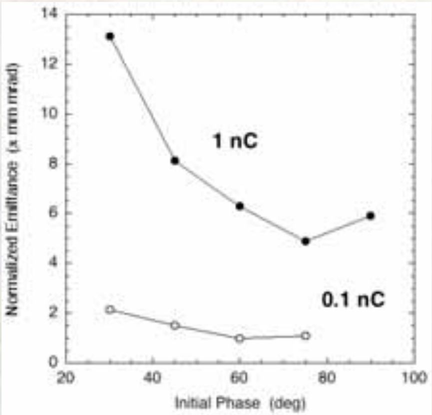


Transverse

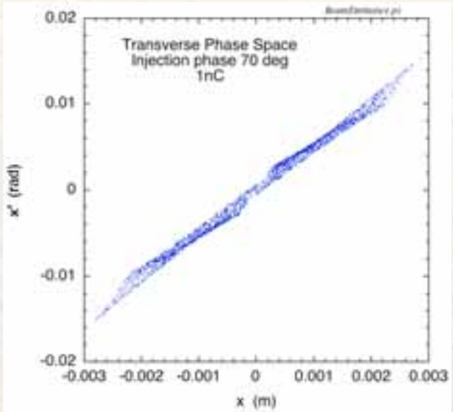


Emittance

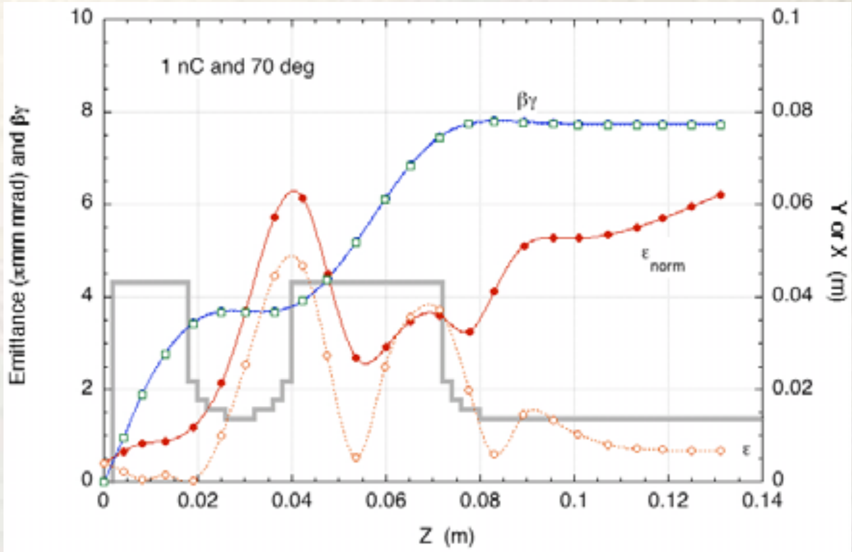
Charge and Phase dependence



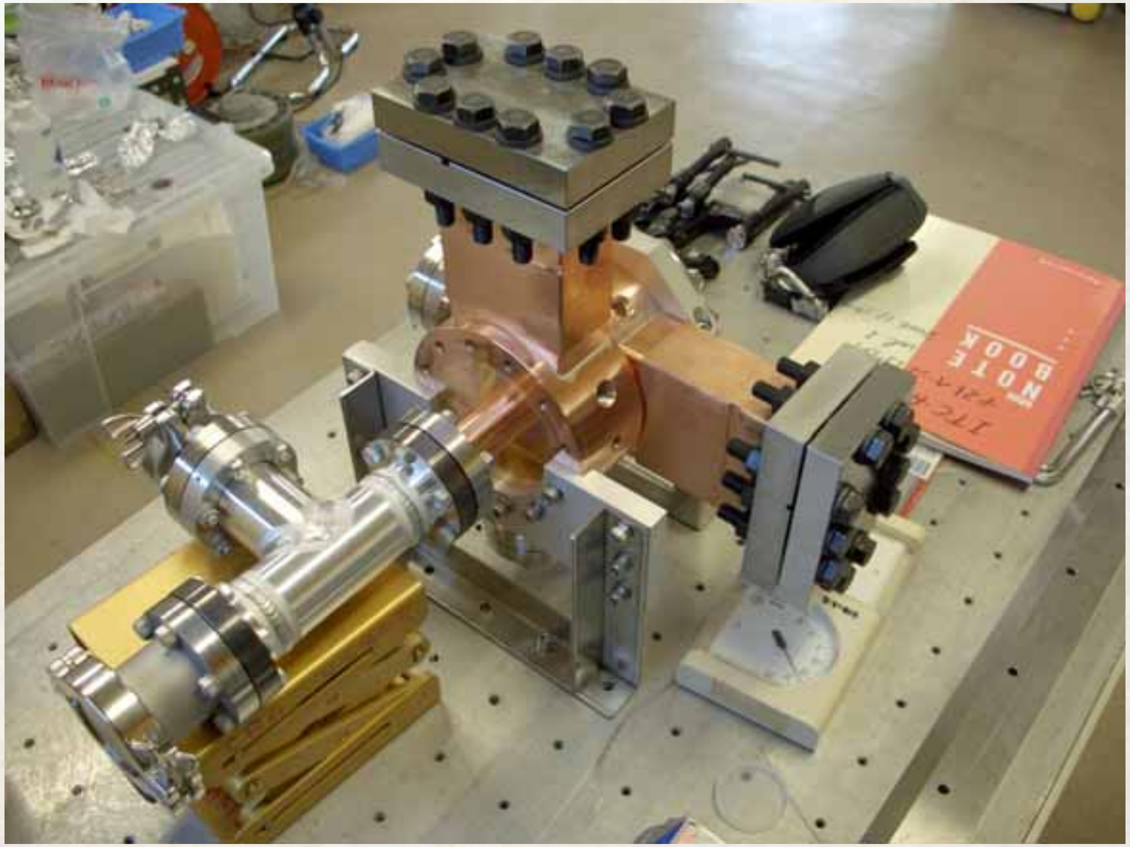
Phase space



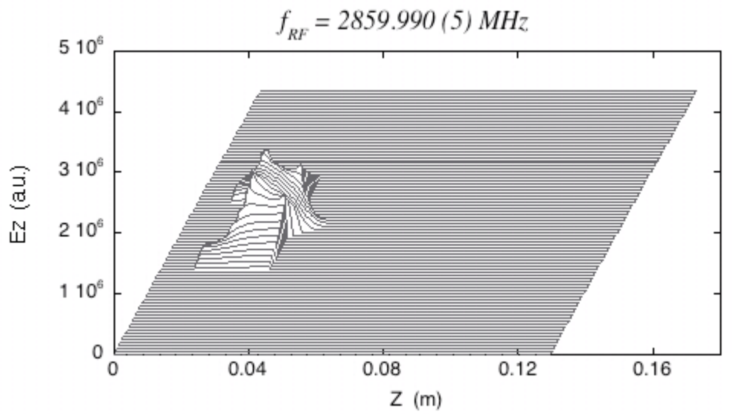
Emittance evolution

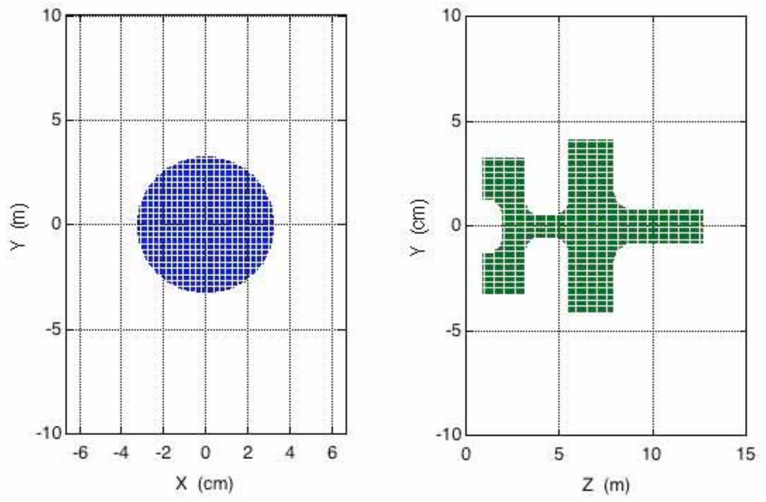


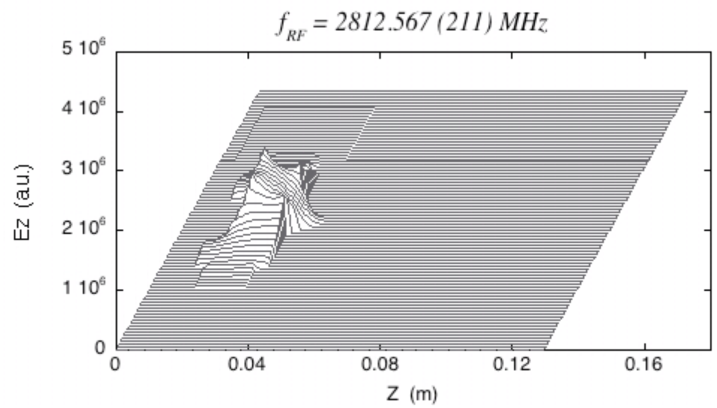
3次元解析　ITC-RF GUN



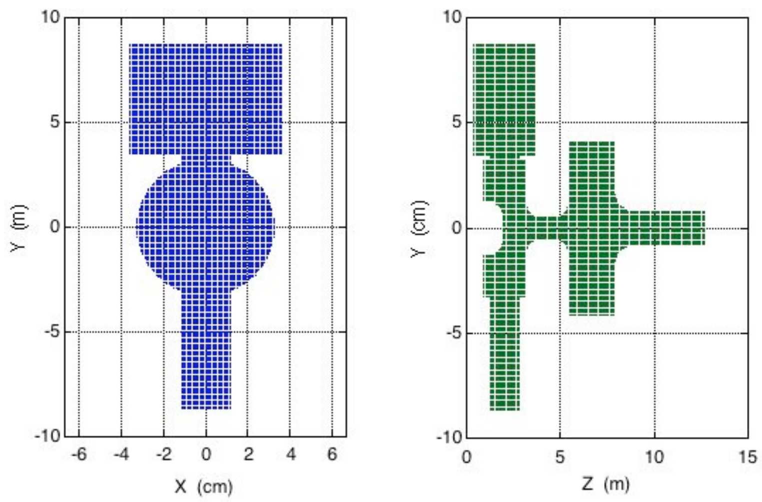
軸対称　　　　　　　　　　　　軸非対称



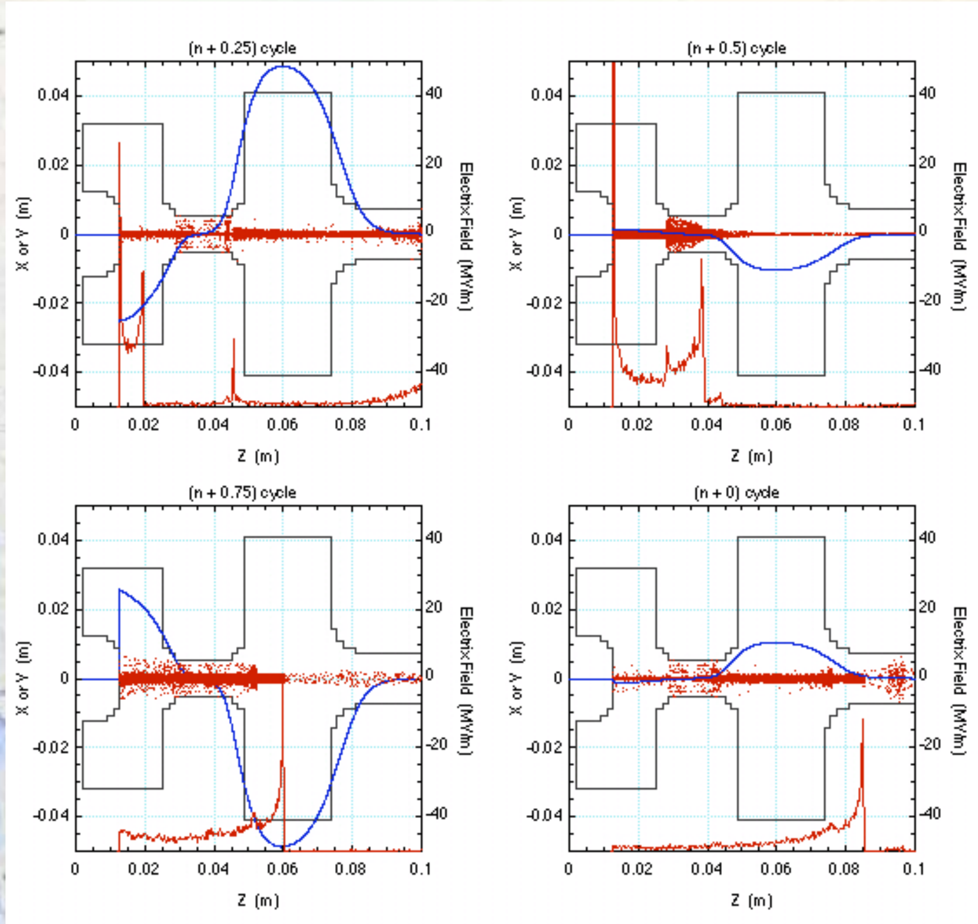


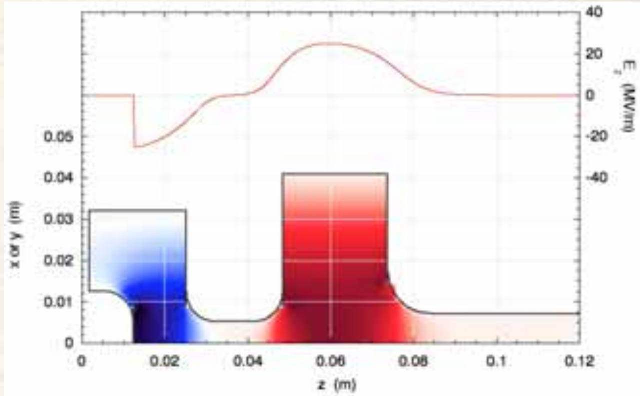


$f_{RF}^{measured} = 2810.8\ MHz$



Beam Simulation





Equation of motion

$$\frac{d\vec{\beta}}{dt} = -\frac{\sqrt{1-\beta^2}\,e}{m_0 c}\left[E + c\vec{\beta}\times\vec{B} - \left(E\cdot\vec{\beta}\right)\vec{\beta}\right]$$

3-D Elec.Mag. Static State

静電場

$$\nabla^2 \cdot \phi = -\frac{\rho}{\varepsilon_0}$$

２次補間差分化

$$\frac{\partial^2 \phi}{\partial x^2} \Rightarrow \Delta x^2 \phi_{i-1,j,k} = \frac{\phi_{i+1,j,k} - 2\phi_{i,j,k} + \phi_{i-1,j,k}}{h^2}$$

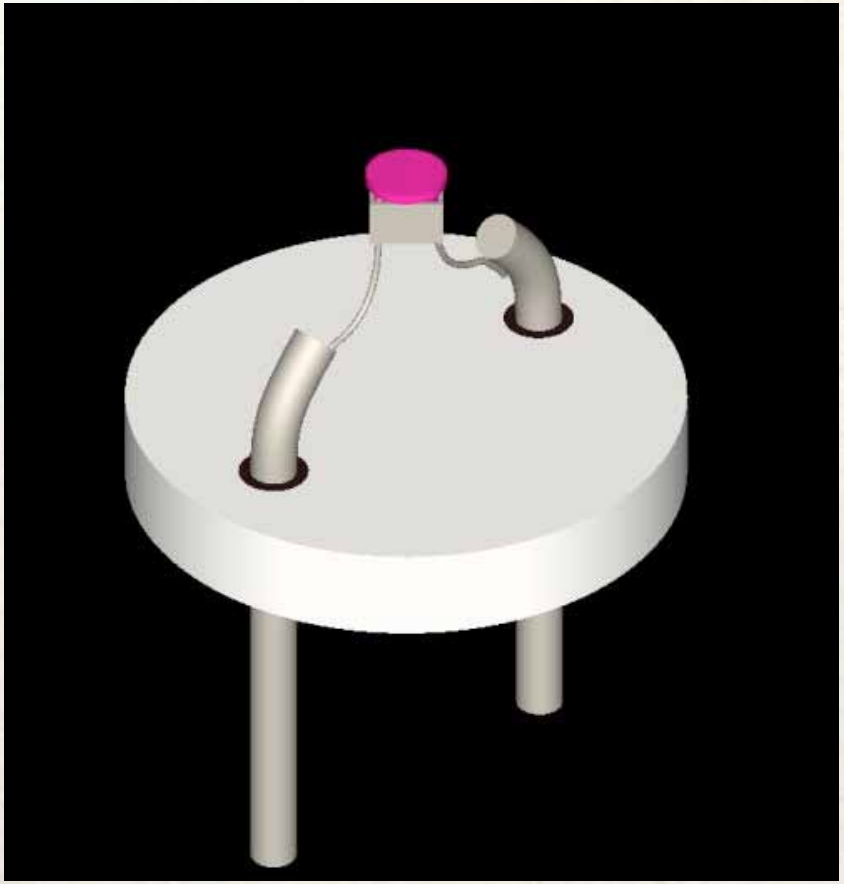
等グリッドサイズ

$$\phi_{i,j,k} = \frac{\phi_{i+1,j,k} + \phi_{i-1,j,k} + \phi_{i,j+1,k} + \phi_{i,j-1,k} + \phi_{i,j,k+1} + \phi_{i,j,k-1}}{6}$$

静磁場

$$\nabla \times \vec{B} = 0 + \mu_0 \vec{J}$$

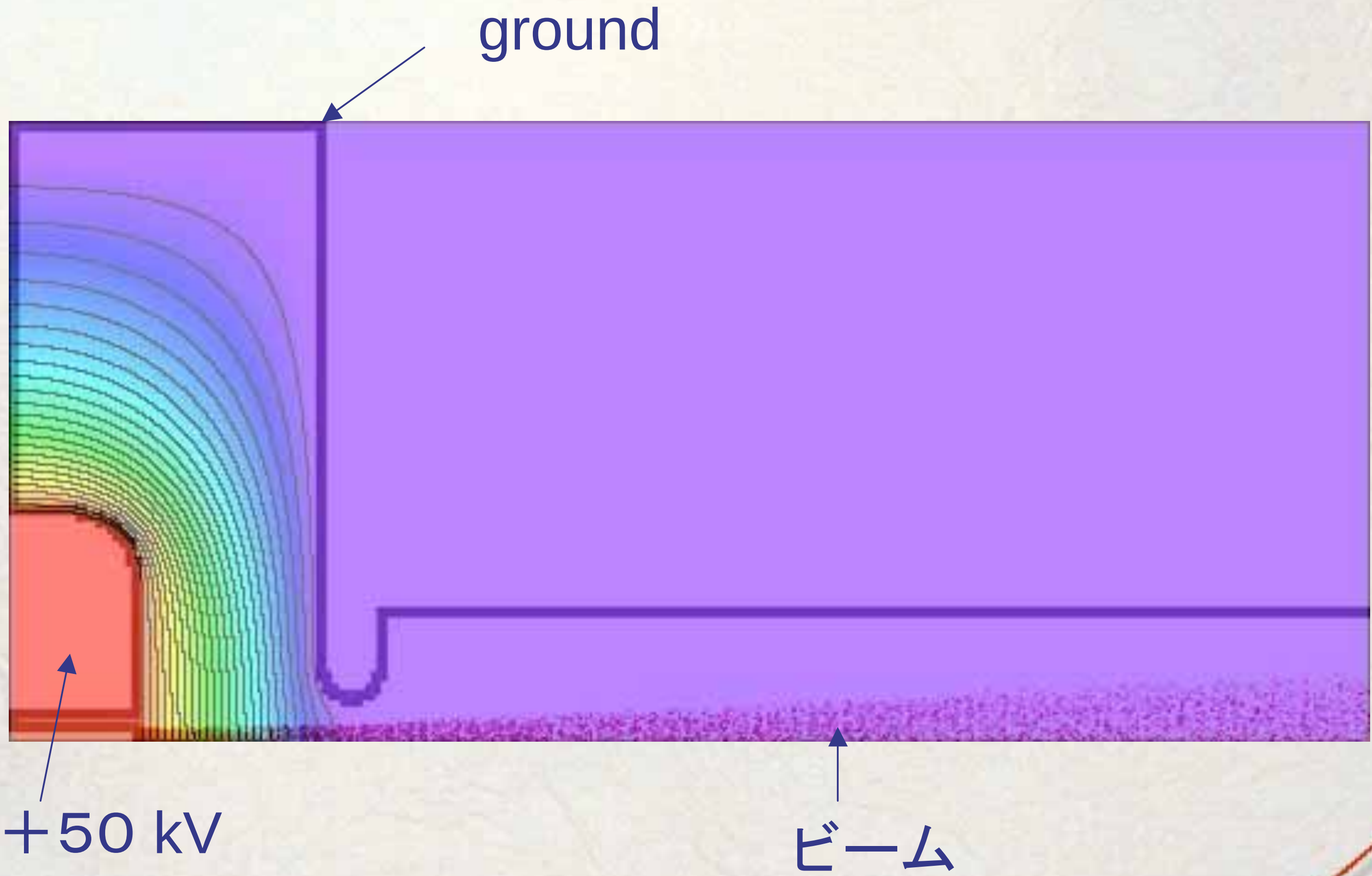
小径カソードを用いた高輝度DC電子銃



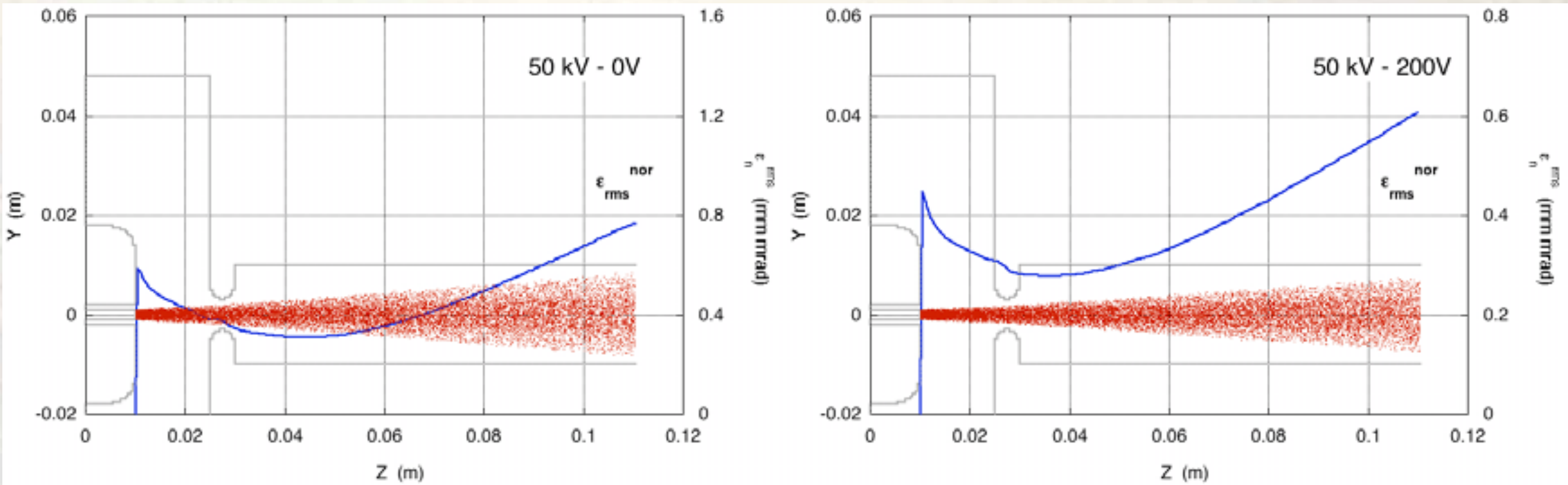
LaB_6カソード

$\Phi = 1.8$ mm

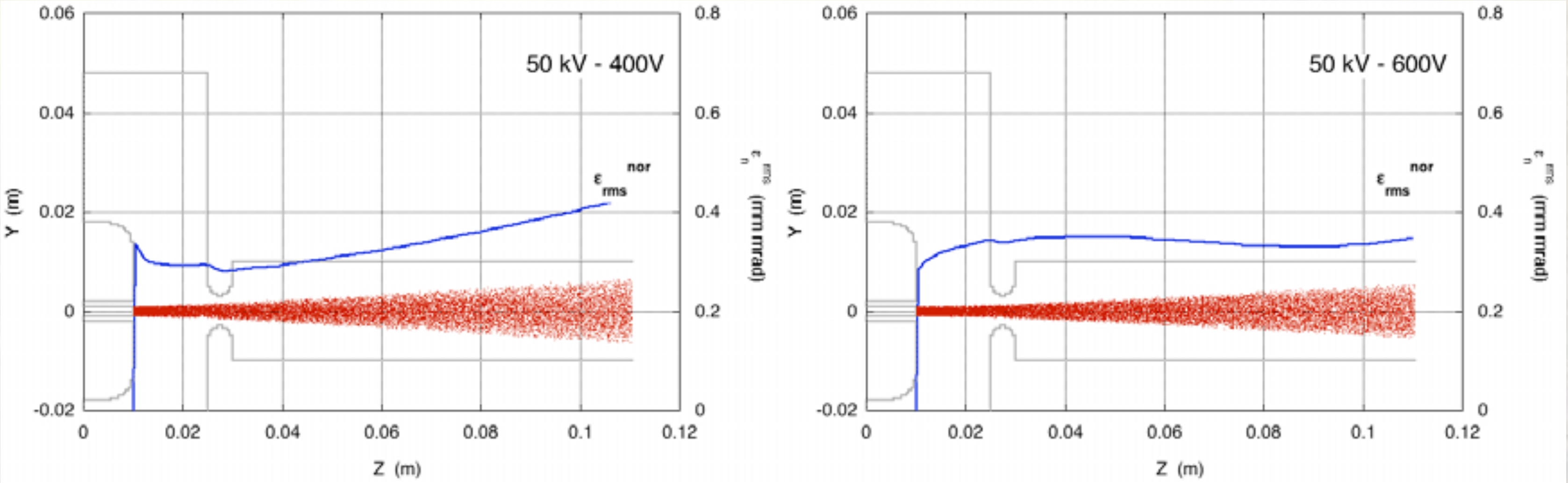
Equi-potential with a Beam



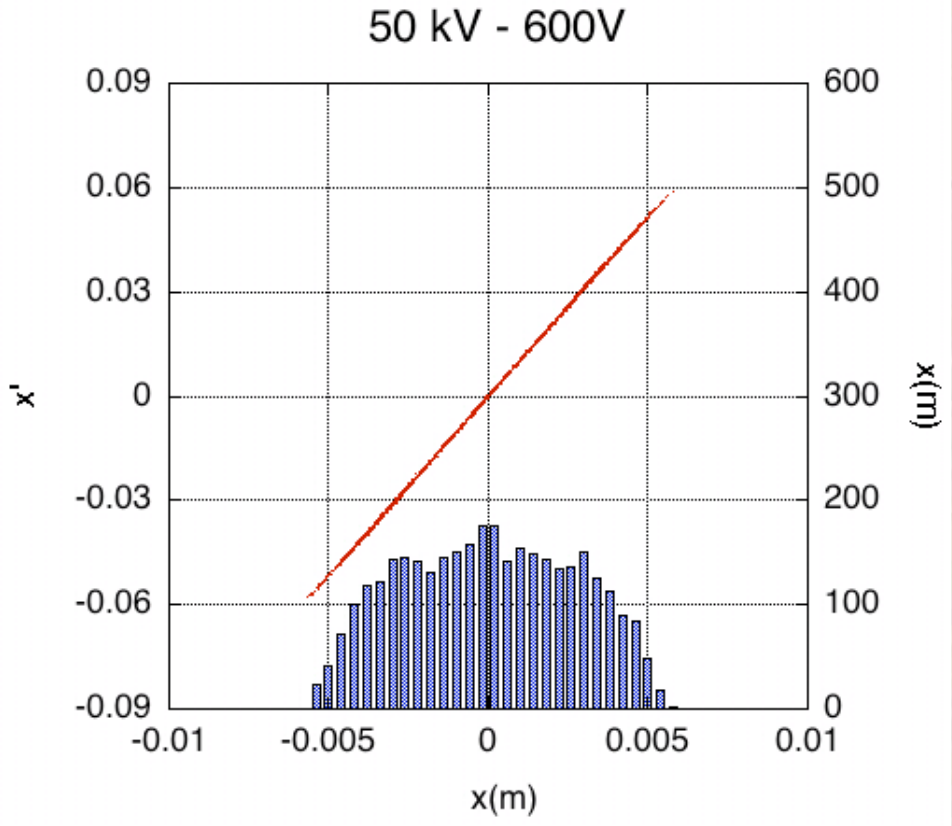
Manipulation of the Potential Surface



Emittance trend can be changed by a bit manipulation of potential surface



50kV - 600 V



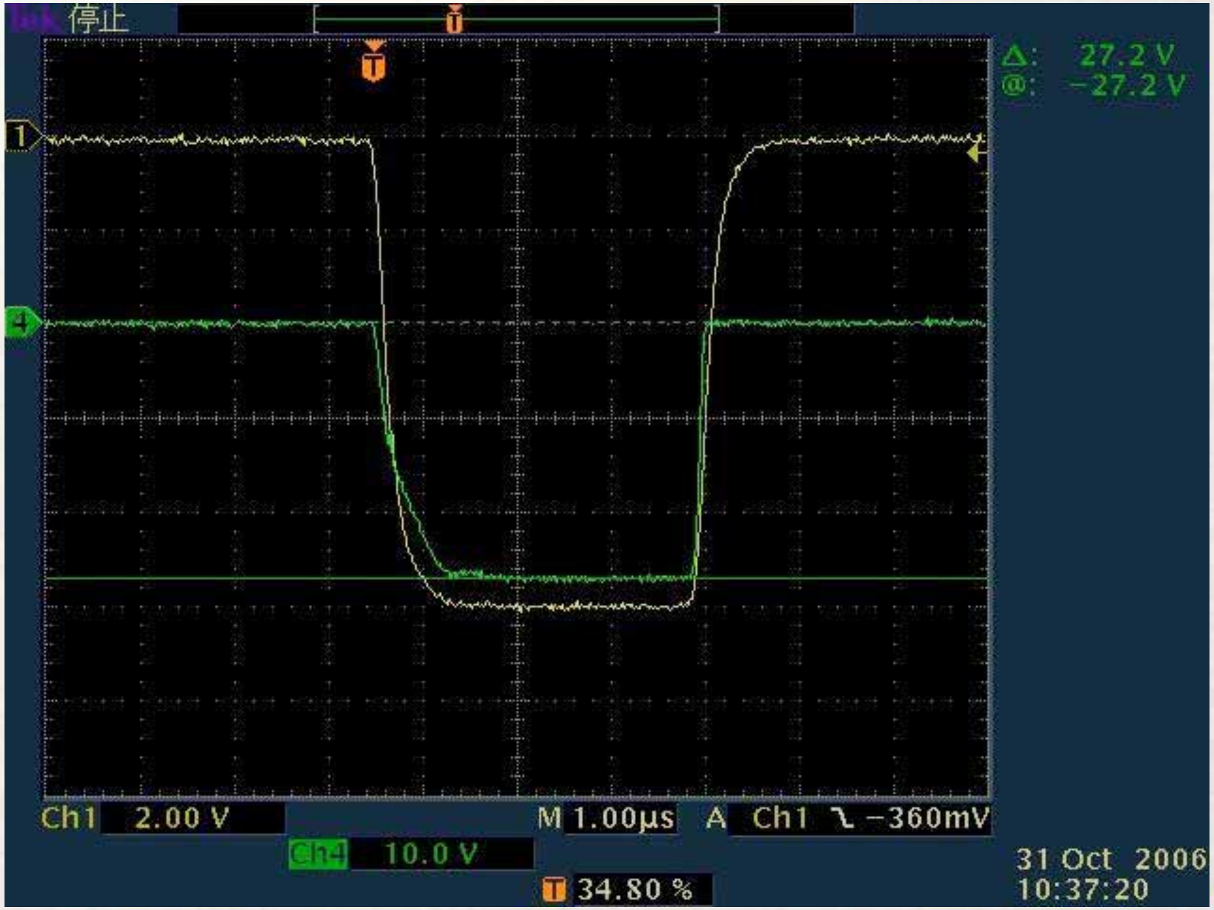
50 kV - 600V
0.09
0.06
0.03
0
-0.03
-0.06
-0.09
600
500
400
300
200
100
0
-0.01
-0.005
0
0.005
0.01
x(m)
x'
x(m)

LaB$_6$ DC電子銃

高圧

ビーム電流

550mA



Conclusions

What is the role of simulation ?

To design the hard wares ?

To know physics aspects?

Hobby ?

It is a Physics itself !