

Lists of Cyclotrons

Ultra-Precise Beams of RCNP Cyclotron  
 - Practice and New Non-Linear Orbit Theory -  
 16 October 2001, Spring-8  
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RCNPサイクロトロンの高品質ビーム  
 - その実践と新しい非線形軌道理論 -  
 平成13年10月16日 Spring-8  
 大阪大学核物理研究センター  
 佐藤健次

Successful Achievement of Ultra-Precise Beams  
 But  
 Happenings of Deterioration of Ultra-Precise Beams  
 Reveals Necessity of  
 New Non-Linear Orbit Theory

A thing left in cyclotrons  
 without being noticed by anyone so far

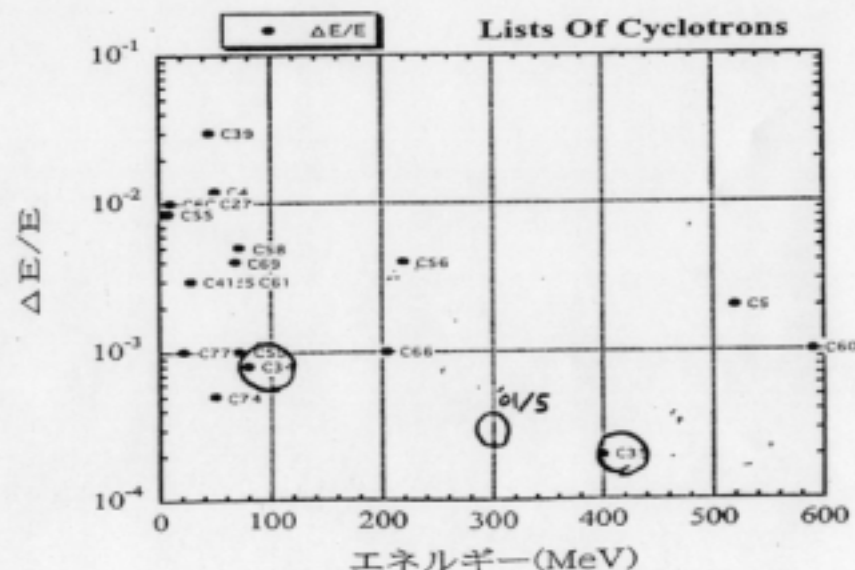
超高品質ビームの実現に成功  
 しかし  
 超高品質ビームが悪化する現象が発生し、その説明には  
 新しい非線形軌道理論  
 が必要

誰も気付かなかった、サイクロトロンのお忘れ物

Further Achievement of Hyper-Precise Beams  
 Beyond Ultra-Precise Beams  
 due to Development and Invention of a New Device  
 for Curing a Non-Linear Behavior

超高品質ビームを超える  
 極超高品質ビームの実現を目指して  
 非線形ビーム挙動を矯正する新装置の開発と発明

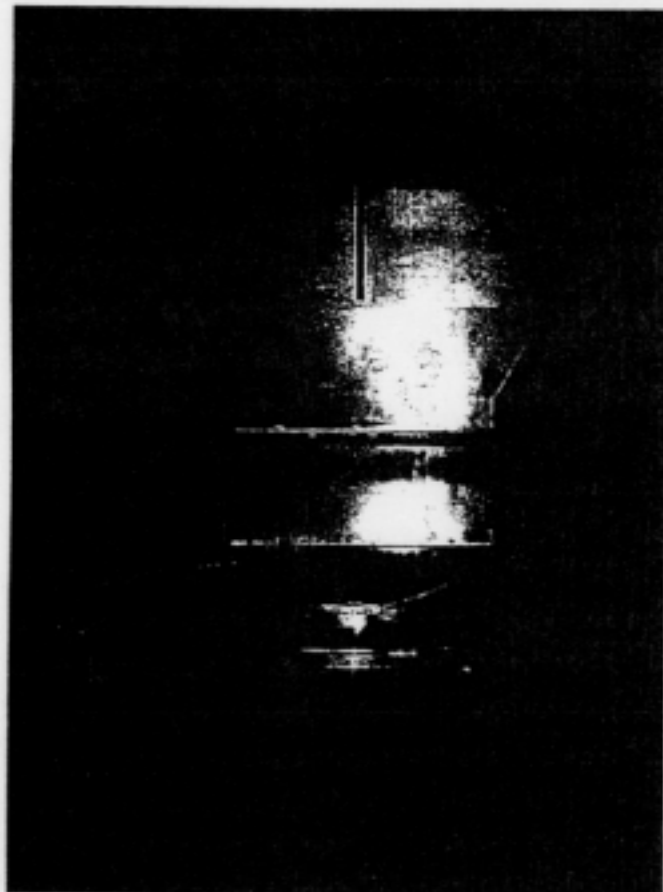
識別番号	研究所の名称		k 値	$\Delta E/E$
C1	Louvain-la-Neuve(BELGIUM)	85MeV, p, h.l.	K=110	0.003
C4	Winnipeg(U.of Man)(CANADA)	50MeV, p, ...		0.012
C5	Vancouver(TRIUMF)(CANADA)	520MeV, H-		0.002
C6	Santiago(U.de Chile)(CHILE)	10MeV, p, ...		0.01
C10	Shanghai(INR)(CHINA)	30MeV, p, ...		0.01
C23	Berlin(HMI)(GERMANY)	72MeV, p, h.l.	k=130	0.001
C25	Julich(KFK)(GERMANY)	45MeV, H-, ...	k=180	0.003
C27	Karlsruhe(KFK)(GERMANY)	42MeV, H-		0.01
C28	Munich(Tech. U.)(GERMANY)	22MeV, p, ...		0.001
C34	Osaka(RCNP)	80MeV, p, ...	k=140	0.0009
C35	Osaka(RCNP)	400MeV, p, ...	k=400	0.0002
C39	Tokyo(INS)	45MeV, p, h.l.	k=68	0.03
C40	Alma Ata(INP)(KAZAKHSTAN)	30MeV, p, ...		
C41	Amsterdam(Vrije U.)(NETHERLANDS)	28MeV, p, ...		0.003
C43	Eindhoven(U. of Tech)(NETHERLANDS)	29.5MeV, p, ...		
C54	Faure(NAC)(SOUTH AFRICA)	8MeV, p, ...		0.0085
C55	Faure(NAC)(SOUTH AFRICA)	8MeV, p, h.l.	k=11	0.0085
C56	Faure(NAC)(SOUTH AFRICA)	220MeV, p, h.l.	k=220	0.004
C58	Villigen(PSI)(SWITZERLAND)	72MeV, p, ...		0.005
C59	Villigen(PSI)(SWITZERLAND)	72MeV, p		0.001
C60	Villigen(PSI)(SWITZERLAND)	590MeV, p		0.001
C61	Kiev(INR-UAS)(UKRAINE)	78MeV, p, ...		0.003
C66	Bloomington(IUCF)(UNITED STATES)	205MeV, p, ...		0.001
C66	Bloomington(IUCF)(UNITED STATES)	205MeV, p, ...		0.001
C69	Davis(U. of Cal.)(UNITED STATES)	68MeV, p, ...		0.004
C74	Princeton(U.)(UNITED STATES)	50MeV, p, ...		0.0005
C77	Tashkent(NP)(UZBEKISTAN)	22MeV, p, ...		0.001



Proposal of Upgrading Project: Part 1  
Flat-topping of rf accelerating voltage

The 5th higher harmonics for flat topping:  
contaminated by a parasitic half-wave resonance  
Control of a resonant frequency of a parasite:  
by installing a center slit for Dee electrode

Investigations by Model Dee Electrode with a Slit

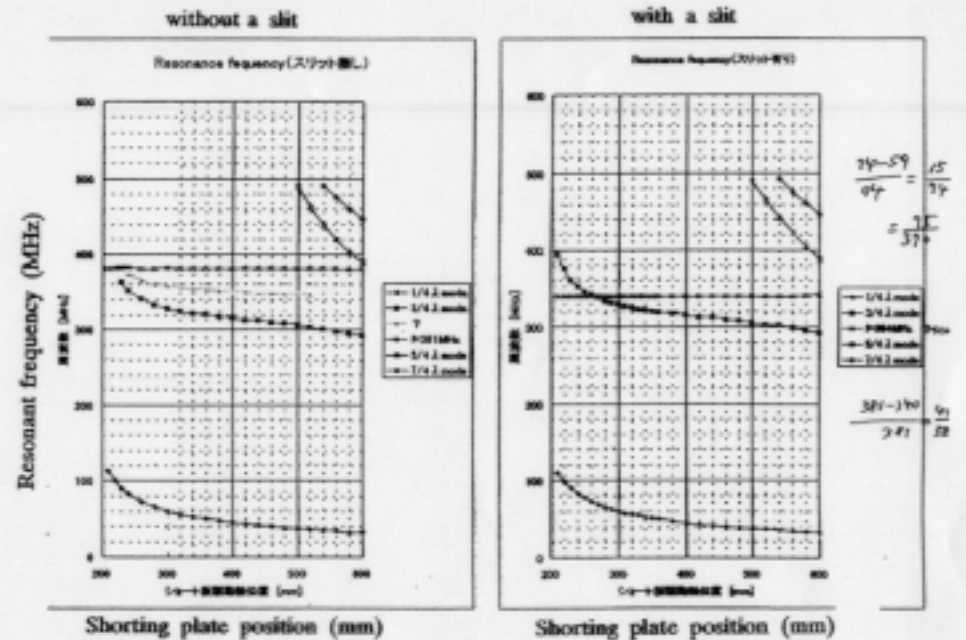


Measurement of Resonant Frequencies in a Model

Dependence on a shorting plate position

Without a center slit

With a center slit



Frequency shift of a half-wave resonance with a center slit:

10 % down as expected and  
enough for avoidance of overlapping of resonances

VII. 当面, 何をすれば良いのか (阪大 RCNP の場合)

- ① 電磁石の鉄芯温度の制御による磁場の安定化
- ② AVF: 取り出し半径での 2 種類のビームスクレイプ
- ③ AVF: 加速電圧のフラット・トップ化
- ④ 加速電圧の振幅, 周波数, 位相の変調
- ⑤ これらに加えて, フラズ・アルファの何かが必要!

イオン源

↓ トランスポートでの整形: 縦方向マッピング

AVF 入射

↓

AVF 取り出し

↓

トランスポートでの整形: 縦方向マッピング

リング入射

↓

リング取り出し

軌道理論として今後解きたいこと

横方向運動と縦方向運動, 結合運動

||  
調和振動 = 重なる非線形振動  
cosφ, r<sup>2</sup>

Isochronous cyclotrons

$$\dot{x}_n = \dot{x}_c$$

General equations of motion

$$\ddot{x} + \omega_n^2 \dot{x}^2 x = \frac{c^2 \dot{\theta}_n}{\omega_n r_n} r - 2 \left( \frac{\partial}{\partial r} - \frac{1}{\omega_n^2} \right) \frac{c^2}{\omega_n^2 r_n^3} r^2 + \frac{\partial \omega_n \dot{\theta}_n}{m_0 \omega_n} b$$

$$\dot{\phi} - 2 \left( \frac{\partial}{\partial r} - \frac{5}{4} \frac{1}{\omega_n^2} + \frac{1}{\omega_n^4} \right) \frac{A c^4}{\omega_n^4 r_n^4 \dot{\theta}_n} r^2$$

$$= - \frac{A}{\omega_n^2 r_n \dot{\theta}_n} \ddot{x} + \frac{A \delta}{m_0 \omega_n^3} b + \omega_{cf} - \omega_{rf, n} + \dot{\phi}_{cf}$$

$$\dot{r} = \frac{\partial V_n}{2\pi m_0 c^2} (\cos\phi - \cos\phi_n) + \frac{\partial V}{2\pi m_0 c^2} \cos\phi_n$$

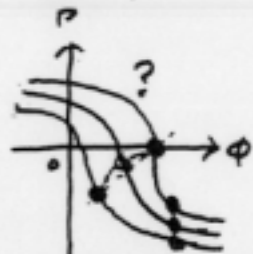
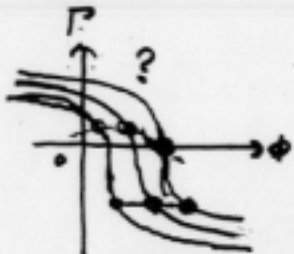
## Problems

Problem 1. What is an initial beam condition to realize a desired final beam performance?

Narrow energy spread

Narrow phase spread

Narrow position spread



See the following relation between  $x$  and  $\Gamma$  and  $\dot{\phi}$ .

$$\dot{\phi} + \frac{kc^2}{\gamma_n^2 \beta_n^2} \Gamma - \left(\frac{3}{2} - \frac{1}{\gamma_n^2}\right) \frac{kc^2}{\gamma_n^2 \beta_n^2 \dot{\theta}_n} \Gamma^2 - \frac{kc^2}{\gamma_n} x = u_{\dot{\phi}} - u_{\dot{\theta}_n} + \dot{\phi}_i$$

Note: It can be considered that the tuning of the deviation of the magnetic field controls a shape of the final beam for the given initial beam condition.

Problem 2. How an initial beam condition can be realized at injection to cyclotrons?

Invent a non-linear device for an injection beam transport line in order to control a relation among  $\Gamma$ ,  $\phi$  and  $x$ .

Problem Make clear beam dynamics at flat-top of rf accelerating voltage.

Ideal flat-top

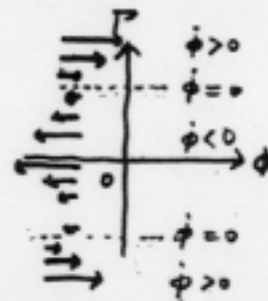
$$\begin{cases} \dot{\phi} = 2 \left( \frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4} \right) \frac{kc^2}{\gamma_n^2 \beta_n^2 \dot{\theta}_n} \Gamma^2 + \frac{kc^2}{\gamma_n^2 \beta_n^2} b \\ \dot{\Gamma} = 0 \end{cases}$$

$\Gamma = \text{const.}$

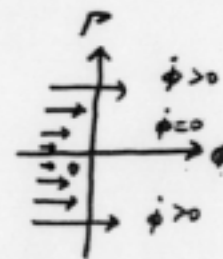
$$\dot{\phi} = 2 \left( \frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4} \right) \frac{kc^2}{\gamma_n^2 \beta_n^2 \dot{\theta}_n} \left\{ \Gamma^2 + \frac{\frac{kc^2}{\gamma_n^2 \beta_n^2} b}{2 \left( \frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4} \right) \frac{kc^2}{\gamma_n^2 \beta_n^2 \dot{\theta}_n}} \right\} x + \dot{\phi}_i$$

$b < 0$

Increase of magnetic field



$b = 0$



$b > 0$

Decrease of magnetic field



Increase of magnetic field :  $b < 0$

$$\Gamma_u - \Gamma_d = 2\Gamma_u = 2 \frac{\gamma_n}{\beta_n} \left( \frac{r_n \theta}{c} \right)^2 \sqrt{-\frac{1}{2} \frac{1}{\frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4}} \cdot \frac{b}{|B_{en}|}}$$

$$\text{where } \left( \frac{r_n \theta}{c} \right)^2 = \frac{\gamma_n^2 - 1}{\gamma_n^2} = \frac{(\gamma_n + 1)(\gamma_n - 1)}{\gamma_n^2}$$

$$(\phi_u - \phi_d) \cos \phi_n - (\sin \phi_u - \sin \phi_d)$$

$$= -\frac{2\pi \frac{\gamma_n + 1}{\gamma_n^2}}{\frac{3}{2} \frac{1}{2\pi} \frac{8V_n}{m_0 c^2 (\gamma_n - 1)}} \cdot \frac{b}{|B_{en}|} \sqrt{-\frac{1}{2} \frac{1}{\frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4}} \cdot \frac{b}{|B_{en}|}}$$

Relative energy difference to kinetic energy

$$\frac{\Delta E}{E} = \frac{\gamma_u - \gamma_d}{\gamma_u - 1} = \frac{\dot{\theta}_n (\Gamma_u - \Gamma_d)}{\gamma_n - 1} = 2 \frac{\gamma_n + 1}{\gamma_n} \sqrt{-\frac{1}{2} \frac{1}{\frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4}} \cdot \frac{b}{|B_{en}|}}$$

Polynomial approximation of phase difference

$$(\phi_u - \phi_d) (\cos \phi_n - \cos \phi_d) + \frac{1}{6} (\phi_u - \phi_d)^2 \sin \phi_d + \frac{1}{6} (\phi_u - \phi_d)^3 \cos \phi_d$$

$$= -\frac{2\pi \frac{\gamma_n + 1}{\gamma_n^2}}{\frac{3}{2} \frac{1}{2\pi} \frac{8V_n}{m_0 c^2 (\gamma_n - 1)}} \cdot \frac{b}{|B_{en}|} \sqrt{-\frac{1}{2} \frac{1}{\frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4}} \cdot \frac{b}{|B_{en}|}}$$

In case of  $\phi_d = \phi_n = 0$ ,

$$\frac{1}{6} (\phi_u - \phi_d)^3 = -\frac{2\pi \frac{\gamma_n + 1}{\gamma_n^2}}{\frac{3}{2} \frac{1}{2\pi} \frac{8V_n}{m_0 c^2 (\gamma_n - 1)}} \cdot \frac{b}{|B_{en}|} \sqrt{-\frac{1}{2} \frac{1}{\frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4}} \cdot \frac{b}{|B_{en}|}}$$

$$\phi_u - \phi_d = \left( + \frac{2\pi \frac{\gamma_n + 1}{\gamma_n^2}}{\frac{3}{2} \frac{1}{2\pi} \frac{8V_n}{m_0 c^2 (\gamma_n - 1)}} \sqrt{-\frac{1}{2} \frac{1}{\frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4}} \cdot \frac{b}{|B_{en}|}} \right)^{\frac{1}{3}} \sqrt{\frac{b}{|B_{en}|}}$$

Numerical example :  $b < 0$

50 MeV proton;  $m_0 c^2 (\gamma_n - 1) = 50 \text{ MeV}$ ,  $\gamma_n \approx 1.053$

100 kV rf accelerating voltage;  $8V_n = 100 \text{ kV}$

harmonic number;  $h = 1$

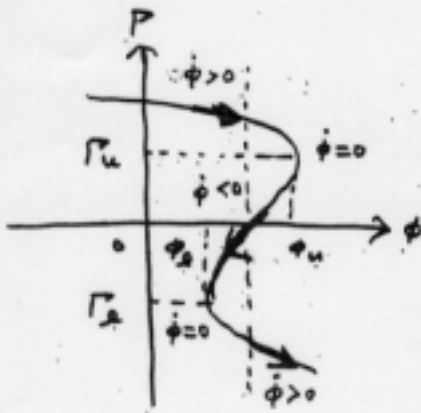
$$\frac{\Delta E}{E} \approx 4.18 \sqrt{-\frac{b}{|B_{en}|}}$$

$$\phi_u - \phi_d \approx 36.2 \sqrt{-\frac{b}{|B_{en}|}}$$

$-\frac{b}{ B_{en} }$	$\frac{\Delta E}{E}$	$\Delta E$	$\phi_u - \phi_d$ (rad)	$\phi_u - \phi_d$ (deg)
1 ppm = $10^{-6}$	$4.18 \times 10^{-3}$	209 keV	$3.62 \times 10^{-2}$	$2.09^\circ$
10 ppm = $10^{-5}$	$1.32 \times 10^{-2}$	661 keV	$1.14 \times 10^{-1}$	$6.56^\circ$

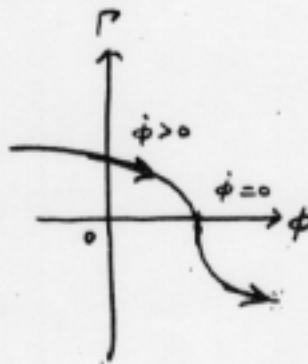


$b < 0$   
Increase of  
magnetic field



$\Gamma_s = -\Gamma_u$

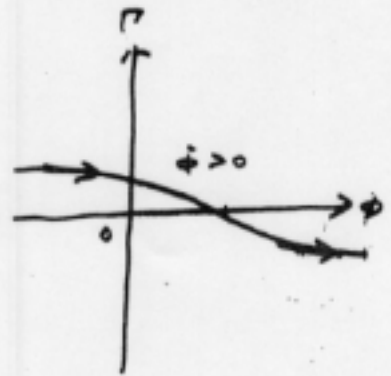
$b = 0$



no isochronism!

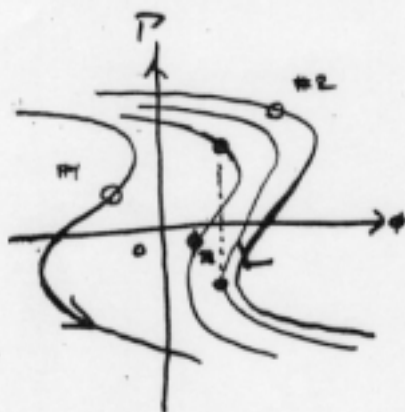
$b > 0$

Decrease of  
magnetic field

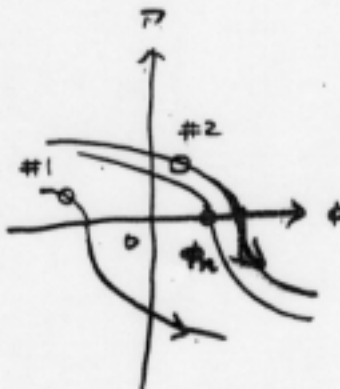


Trajectory of different particles

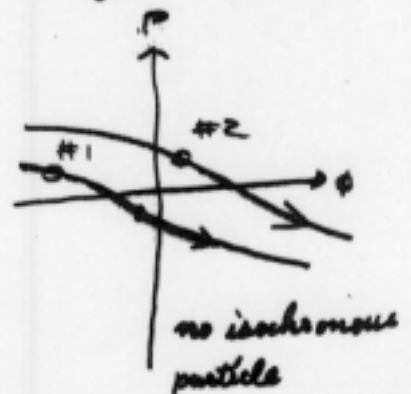
$b < 0$   
Increase of  
magnetic field



$b = 0$



$b > 0$   
Decrease of  
magnetic field



- o : initial condition of each particle at  $t=0$
- : unstable fixed points = isochronous particle

Ordinary theory of longitudinal motion

$$\begin{cases} \dot{\phi} = \frac{4\mathcal{E}}{m_0\gamma_n^3} b & \textcircled{1} \\ \dot{\Gamma} = \frac{2V_n}{2\pi m_0 c^2} (\cos\phi - \cos\phi_n) & \textcircled{2} \end{cases}$$

Useful relation

$$\dot{\phi}\dot{\Gamma} = \frac{2V_n}{2\pi m_0 c^2} (\dot{\phi}\cos\phi - \dot{\phi}\cos\phi_n) = \frac{d}{dt} \left\{ \frac{2V_n}{2\pi m_0 c^2} (\sin\phi - \phi\cos\phi_n) \right\}$$

$\otimes \times \dot{\Gamma}$

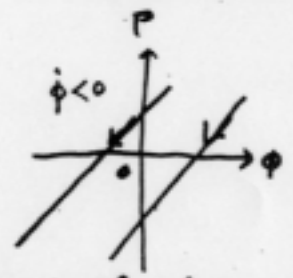
$$\dot{\phi}\dot{\Gamma} = \frac{4\mathcal{E}}{m_0\gamma_n^3} b \dot{\Gamma}$$

Integration by time

$$-\frac{4\mathcal{E}}{m_0\gamma_n^3} b \Gamma = \frac{2V_n}{2\pi m_0 c^2} (\phi\cos\phi_n - \sin\phi - K)$$

$b < 0$

Increase of magnetic field



no isochronism

$b = 0$



isochronism

$b > 0$

Decrease of magnetic field



no isochronism

Longitudinal motion only  
with second-order term of energy deviation but  
without coupling to betatron oscillation

Other conditions :  $b = \text{const.} \neq 0, E_r = 0$

$$v = 0, \omega_y = \omega_{\beta n}, \dot{\phi}_y = 0$$

$$\begin{cases} \dot{\phi} - 2 \left( \frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4} \right) \frac{\kappa c^4}{\gamma_n^4 \gamma_n^4 \dot{\theta}_n} \Gamma^2 = \frac{4\mathcal{E}}{m_0\gamma_n^3} b & \textcircled{1} \\ \dot{\Gamma} = \frac{2V_n}{2\pi m_0 c^2} (\cos\phi - \cos\phi_n) & \textcircled{2} \end{cases}$$

Useful relation

$$\dot{\phi}\dot{\Gamma} = \frac{2V_n}{2\pi m_0 c^2} (\dot{\phi}\cos\phi - \dot{\phi}\cos\phi_n)$$

$$= \frac{d}{dt} \left\{ \frac{2V_n}{2\pi m_0 c^2} (\sin\phi - \phi\cos\phi_n) \right\} \quad \textcircled{3}$$

$\otimes \times \dot{\Gamma}$

$$\dot{\phi}\dot{\Gamma} - 2 \left( \frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4} \right) \frac{\kappa c^4}{\gamma_n^4 \gamma_n^4 \dot{\theta}_n} \Gamma^2 \dot{\Gamma} = \frac{4\mathcal{E}}{m_0\gamma_n^3} b \dot{\Gamma}$$

Integration by time

$$\begin{aligned} & -\Gamma \left( \Gamma^2 + \frac{3}{2} \frac{1}{\frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4}} \frac{\gamma_n^4 \gamma_n^4 \dot{\theta}_n}{\kappa c^4} \cdot \frac{4\mathcal{E}}{m_0\gamma_n^3} b \right) \\ & = \frac{3}{2} \frac{1}{\frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4}} \frac{\gamma_n^4 \gamma_n^4 \dot{\theta}_n}{\kappa c^4} \frac{2V_n}{2\pi m_0 c^2} (\phi\cos\phi_n - \sin\phi - K) \end{aligned}$$

where  $K = \text{const.}$

Elimination of  $B_{z0}(r_n)$  by synchronous (isochronous)

$$\text{condition: } \delta B_{z0}(r_n) = \frac{m_0 \dot{r}_n \ddot{r}_n}{r_n \dot{\theta}_n} + \frac{m_0 \dot{\theta}_n \ddot{r}_n}{r_n \dot{\theta}_n} - m_0 \dot{\theta}_n \dot{\phi}_n$$

First-step approximation:  $\dot{r}_n \approx 0, \ddot{r}_n \approx 0$

$$\text{so that } \gamma_n \approx \frac{1}{\sqrt{1 - \left(\frac{r_n \dot{\theta}_n}{c}\right)^2}}$$

Further approximation:  $\dot{\theta}_n \approx 0$

Definition of transition -  $\alpha$ :  $\gamma_n^2 = 1 - \frac{v_n^2}{c^2} = 1 - \frac{v_n^2}{c^2}$

$$v_n \leq v_c$$

Three basic equations of motion

$$\dot{\phi} + \frac{kc^2}{\gamma_n^3 r_n^2} \Gamma - \left(\frac{3}{2} - \frac{1}{\gamma_n^2}\right) \frac{kc^2}{\gamma_n^4 r_n^3 \dot{\theta}_n} \Gamma^2 - \frac{kr_n \dot{\theta}_n}{r_n} x = \omega_y - \omega_{y,n} + \dot{\phi}_{y2}$$

$$\ddot{x} + \gamma_n^2 \dot{\theta}_n^2 x = \frac{c^2 \dot{\theta}_n}{\gamma_n r_n} \Gamma - 2 \left(\frac{3}{4} - \frac{1}{\gamma_n^2}\right) \frac{c^2}{\gamma_n^4 r_n^3} \Gamma^2 + \frac{8 r_n \dot{\theta}_n}{m_0 \gamma_n} b + \frac{8}{m_0 \gamma_n} E_r$$

$$\dot{r} = \frac{8V_n}{2\pi m_0 c^2} (\cos \phi - \cos \phi_n) + \frac{8V}{2\pi m_0 c^2} \frac{\cos \phi_n}{\sin \phi - \sin \phi_n}$$

Deduced equation of motion from upper two equations by elimination of  $x$

$$\dot{\phi} + \left(\frac{1}{\gamma_n^2} - \frac{1}{\gamma_c^2}\right) \frac{1}{\gamma_n r_n^2} \Gamma - 2 \left\{ \frac{3}{4} - \frac{1}{2\gamma_n^2} - \frac{1}{\gamma_c^2} \left(\frac{3}{4} - \frac{1}{\gamma_n^2}\right) \right\} \frac{kc^2}{\gamma_n^4 r_n^3 \dot{\theta}_n} \Gamma^2 = -\frac{k}{\gamma_n^2 r_n \dot{\theta}_n} \ddot{x} + \frac{8b}{m_0 \gamma_n^2 \dot{\theta}_n} + \frac{8E_r}{m_0 \gamma_n^2 \dot{\theta}_n r_n \dot{\theta}_n} E_r + \omega_y - \omega_{y,n} + \dot{\phi}_{y2}$$

Isochronous cyclotrons

$$\gamma_n = \gamma_c$$

General equations of motion

$$\ddot{x} + \gamma_n^2 \dot{\theta}_n^2 x = \frac{c^2 \dot{\theta}_n}{\gamma_n r_n} \Gamma - 2 \left(\frac{3}{4} - \frac{1}{\gamma_n^2}\right) \frac{c^2}{\gamma_n^4 r_n^3} \Gamma^2 + \frac{8 r_n \dot{\theta}_n}{m_0 \gamma_n} b + \frac{8}{m_0 \gamma_n} E_r$$

$$\dot{\phi} - 2 \left(\frac{3}{4} - \frac{5}{4} \frac{1}{\gamma_n^2} + \frac{1}{\gamma_n^4}\right) \frac{kc^2}{\gamma_n^4 r_n^3 \dot{\theta}_n} \Gamma^2 = -\frac{k}{\gamma_n^2 r_n \dot{\theta}_n} \ddot{x} + \frac{8b}{m_0 \gamma_n^2 \dot{\theta}_n} + \frac{8E_r}{m_0 \gamma_n^2 r_n \dot{\theta}_n} E_r + \omega_y - \omega_{y,n} + \dot{\phi}_{y2}$$

$$\dot{r} = \frac{8V_n}{2\pi m_0 c^2} (\cos \phi - \cos \phi_n) + \frac{8V}{2\pi m_0 c^2} \cos \phi_n$$

$$\dot{\phi} + \frac{kc^2}{\gamma_n^3 r_n^2} \Gamma - \left(\frac{3}{2} - \frac{1}{\gamma_n^2}\right) \frac{kc^2}{\gamma_n^4 r_n^3 \dot{\theta}_n} \Gamma^2 - \frac{kr_n \dot{\theta}_n}{r_n} x = \omega_y - \omega_{y,n} + \dot{\phi}_{y2}$$

Equations of motion of a special particle

See general equations of motion



# Longitudinal-Transverse Coupling Motion with Second-Order Term of Energy Deviation in Circular Accelerators

Konji Sato, RCNP, Osaka Univ., 13 June 2001

Cylindrical coordinates:  $(r, \theta, z)$

Equation of motion in a median plane

$$z = 0, \dot{z} = 0$$

$$\left\{ \begin{aligned} \gamma &= \frac{1}{\sqrt{1 - \left(\frac{\dot{r}}{c}\right)^2 - \left(\frac{r\dot{\theta}}{c}\right)^2}} \\ m_0 \gamma \ddot{r} + m_0 \dot{\gamma} \dot{r} - m_0 \gamma r \dot{\theta}^2 &= q r \dot{\theta} B_z(r) + q E_r \\ m_0 c^2 \dot{\gamma} &= \frac{qV}{2\pi} \dot{\theta} \frac{\cos \phi}{\sin \phi} \end{aligned} \right.$$

where  $\phi = \omega_y t + \phi_{y0} - k\theta$

$$q > 0; B_z(r) < 0 \text{ so that } \dot{\theta} > 0$$

Synchronous (Isochronous) conditions

$$\phi_n = \omega_{y,n} t - k\theta_n = \text{const.}$$

$$\dot{\phi}_n = 0, \omega_{y,n} = k\dot{\theta}_n$$

$$\left\{ \begin{aligned} \gamma_n &= \frac{1}{\sqrt{1 - \left(\frac{\dot{r}_n}{c}\right)^2 - \left(\frac{r_n \dot{\theta}_n}{c}\right)^2}} \\ m_0 \gamma_n \ddot{r}_n + m_0 \dot{\gamma}_n \dot{r}_n - m_0 \gamma_n r_n \dot{\theta}_n^2 &= q r_n \dot{\theta}_n B_{zn}(r_n) \\ m_0 c^2 \dot{\gamma}_n &= \frac{qV_n}{2\pi} \dot{\theta}_n \frac{\cos \phi_n}{\sin \phi_n} \end{aligned} \right.$$

Definitions of deviation

$$\phi - \phi_n = -k(\theta - \theta_n) + (\omega_y - \omega_{y,n})t + \phi_{y0}$$

$$\dot{\phi} - \dot{\phi}_n = -k(\dot{\theta} - \dot{\theta}_n) + \omega_y - \omega_{y,n} + \dot{\phi}_{y0}$$

$$x = r - r_n$$

$$\Gamma = \frac{r - r_n}{r_n}$$

Definitions of deviation

$$B_z(r) = B_{zn}(r_n) \left(1 - \pi \frac{r - r_n}{r_n}\right) + b$$

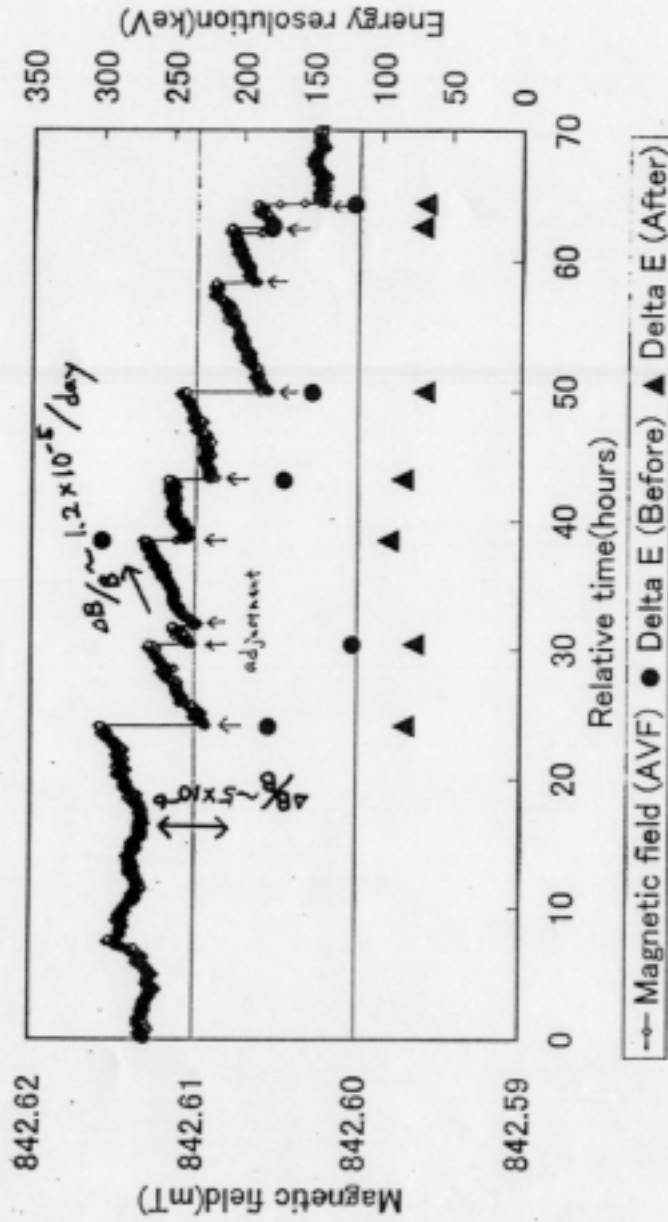
$$V = V_n + U$$

Equation of motion with second-order term of energy deviation

$$\frac{\dot{\theta} - \dot{\theta}_n}{\dot{\theta}_n} = -\frac{r - r_n}{r_n} - \frac{\left(\frac{\dot{r}_n}{c}\right)^2}{\left(\frac{r_n \dot{\theta}_n}{c}\right)^2} \frac{\dot{r}_n \dot{\theta}_n}{\dot{\theta}_n} + \frac{\frac{qV}{2\pi}}{\frac{qV_n}{2\pi}} \frac{\dot{\theta} - \dot{\theta}_n}{\dot{\theta}_n} - \frac{\frac{1}{2} \left\{ \frac{3}{2} - \frac{1}{2} - \frac{3}{2} \left(\frac{\dot{r}_n}{c}\right)^2 \right\}}{\left(\frac{r_n \dot{\theta}_n}{c}\right)^2} \left(\frac{\dot{\theta} - \dot{\theta}_n}{\dot{\theta}_n}\right)^2$$

$$\frac{\dot{\theta}^2 - \dot{\theta}_n^2}{\dot{\theta}_n^2} = -2 \frac{r - r_n}{r_n} - 2 \frac{\left(\frac{\dot{r}_n}{c}\right)^2}{\left(\frac{r_n \dot{\theta}_n}{c}\right)^2} \frac{\dot{r}_n \dot{\theta}_n}{\dot{\theta}_n} + 2 \frac{\frac{1}{2}}{\left(\frac{r_n \dot{\theta}_n}{c}\right)^2} \frac{\dot{\theta} - \dot{\theta}_n}{\dot{\theta}_n} - 3 \frac{\frac{1}{2}}{\left(\frac{r_n \dot{\theta}_n}{c}\right)^2} \left(\frac{\dot{\theta} - \dot{\theta}_n}{\dot{\theta}_n}\right)^2$$

392 MeV Proton



サイクロトロンが分かった！  
 - 誰も気付かなかった、サイクロトロンのお忘れ物 -

円形加速器の軌道理論において、エネルギーのずれの二乗の項を含むときの非線形運動が、磁場のずれに影響された運動

シンクロトロン、サイクロトロン、FFAGの中では、サイクロトロンが特殊な加速器  
 エネルギーのずれの一次の項を消すように装置を製作  
 エネルギーのずれの二乗が磁場のずれに比例  
 エネルギーのずれが磁場の正のずれの平方根に比例

AVFのフラット・トップ化に加えて、非線形運動を制御して、縦方向運動を整合させる装置の開発ができれば、これまでにRCNPが実現してきた世界記録をさらに更新できる

佐藤の経験では、エネルギー分解能が数倍良くなることが期待できる

サイクロで、やれば良いこと  
 新装置のアイデアと開発、AVFフラット・トップ化  
 予算、マン・パワー  
 物理の成果が出るのであれば、前段サイクロを更新

RCNPイノベーション  
 - ビームが良くなってもイノベーションとは呼ばない -

モチベーションは物理  
 ム・ム・ム？！

超々高品質ビーム核物理実験  
 緊急アピール！

知恵を出せ、頭を捻れ、総力をあげよ、結集せよ、  
 そして、RCNPとしての物理を研究せよ

# Ultra-Precise Beams 実現の小史

平成13年7月12日

大阪大学核物理研究センター 佐藤健次

核物理研究センターのサイクロトロン・カスケードでは、リング及びAVFの電磁石鉄芯温度の安定化を通して、磁場を長時間、高度に安定化することで、Ultra-Precise Beamsを実現できた。

過渡的渦電流による磁場の空間分布測定の必要性を主張

- ①コイル電流を調整すると過渡的渦電流が発生するはず
- ②それも、長い時定数で、六極状の磁場分布のはず

リング・サイクロトロンでの磁場分布の時間変化の測定

- ①大中小及びビームの外の異なる半径でのNMR測定
- ②トリムコイル冷却水の温度を変化させたときの測定で短い時定数の高速応答の磁場変化に遭遇
- ③コイル電流の変化に対する磁場分布の変化の測定データについては全く思い出せないのは、上のデータに驚いたためか？
- ④リング・サイクロトロン鉄芯温度測定開始

リング・サイクロトロンでの新しい磁場調整方法

- ①磁場が変化すると、コイルの電流ではなく、冷却水温度を調整

GRAND-RAIDENでの画期的測定結果

- ①0度方向での低バック・グラウンド測定が実現

AVFも行け行けで、NMRを開発して、磁場測定を実現

- ①絶対値の測定でなくとも、磁場の変化が読めれば良い
- ②磁場勾配を作れる補正コイル付きNMRの開発に成功

スローガン「磁場の安定化が全てである」を提唱

スローガン「磁場よりも温度」の実現に向けた改造

- ①機能付加：リング及びAVFの電磁石鉄芯やコイルの温度制御のため、コイル冷却水の入口温度を制御する方式を導入
- ②鉄芯温度を一定に保つには、入口と出口の平均温度を一定に保つよりは、出口温度を一定に保つ状態に近いことを経験的に学び、それを評価できるモデルを提案

冷却水系改造の成果

- ①磁場が長時間、高度に安定化した
- ②運転停止時も冷却系を連続運転することにした結果、磁場の強さの変更に必要な時間を短縮できた
- ③空調の影響や空気の流れの影響の制御の必要性を認識できるようになり、その温度管理と風向変更板を設置
- ④コイル電流を調整することが激減したので、過渡的渦電流の発生を無視できると考えて良さそうである
- ⑤加速電圧用高周波共振器の冷却水の温度制御の不具合を、周波数調整用トリマーの動きから発見し、修正

Ultra-Precise Beamsの実現

- ①サイクロトロン・ビーム単独で $\Delta E/E \sim 2 \times 10^{-4}$
- ②WS新ビーム・ラインと GRAND-RAIDENでの分散整合と角度分散整合で $\Delta E/E \sim 4 \times 10^{-5}$

サイクロトロンでのビーム軌道理論の新展開の根拠

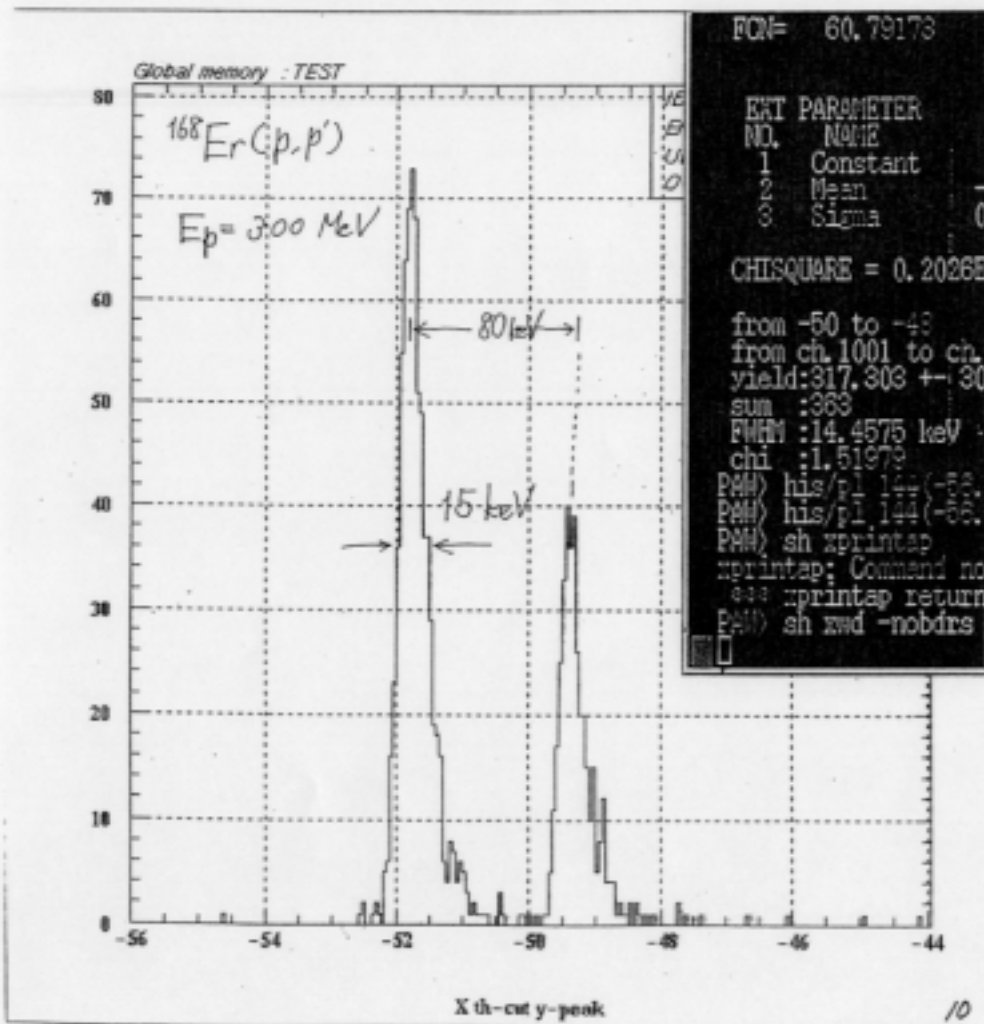
- ①磁場が猛烈に安定になると高品質ビームの実現が可能
- ②他方、磁場がわずかにずれると、品質が極端に悪化
- ③線形軌道理論では、こうした現象の説明が困難であると思われ、非線形効果を導入することを提案したい

非線形効果を矯正できる新種の装置の必要性を指摘

- ①エネルギーのずれと加速位相の間の関係において収差を矯正する装置が必要
- ②例えば、ビーム輸送系ならば六極電磁石のような装置

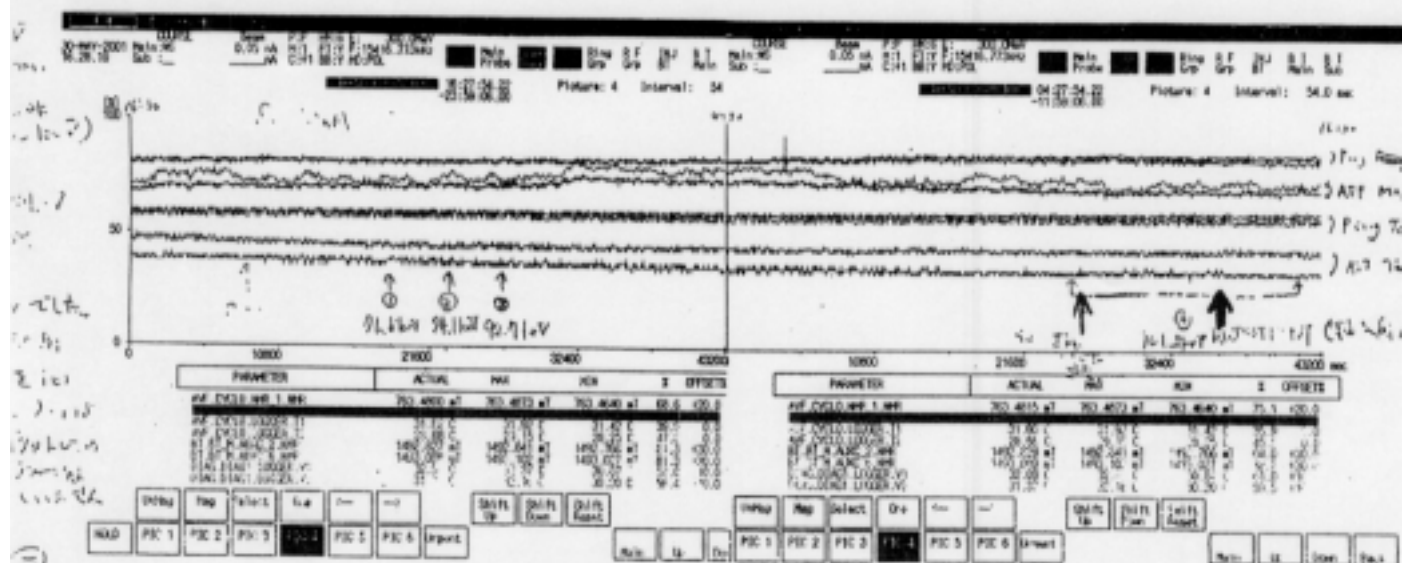
WS-course project

- E-resolution 15 keV at  $E_p=300\text{MeV}$
- $\frac{\Delta E}{E} \sim 5 \times 10^{-5}$  : World Record



Ions	E (MeV)	$\Delta E$ (keV)	$\Delta E/E$	Current	Year/Month
H	300	55	$1.8 \times 10^{-4}$	1 nA	00/12
	392	62	$1.6 \times 10^{-4}$	2 nA	00/6
$^3\text{He}$	450	170	$3.8 \times 10^{-4}$	15 nA	99/5
$^4\text{He}$	400	193	$4.8 \times 10^{-4}$	—	99/11
		245	$6.1 \times 10^{-4}$	1 nA	
		259	$6.5 \times 10^{-4}$	10 nA	
Dispersive mode					
H	300	12.8	$4.3 \times 10^{-5}$	1 nA	00/4
	392	16.7	$4.3 \times 10^{-5}$	1 nA	00/6

300 MeV protons  
 24時間中、サイクロの10<sup>9</sup>X-9調整は1回のみ  
 (FEL, ΔEのこれまでの最高は55keV)

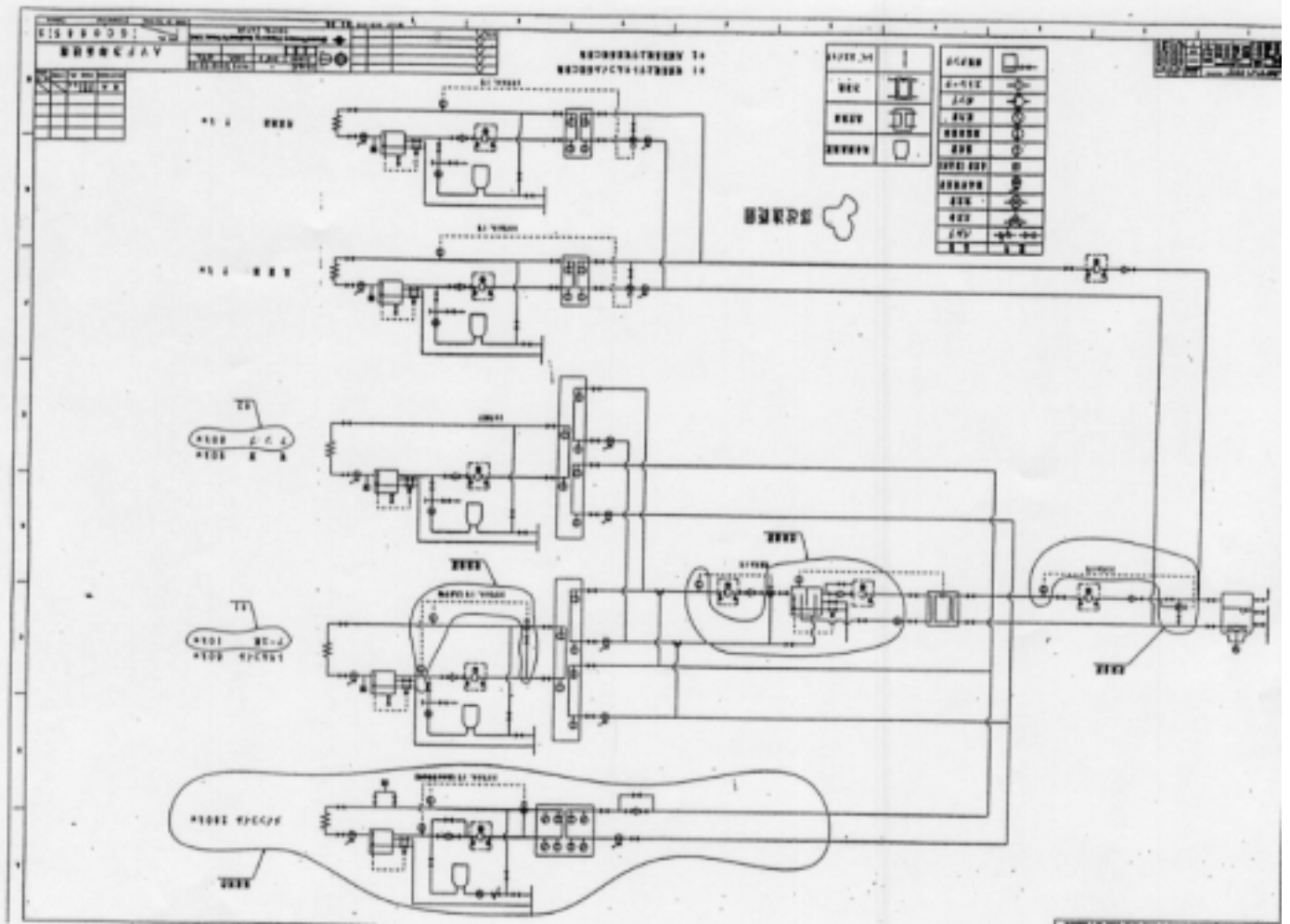


リングNo.2磁場 (9/9 4He 210 MeV 立ち上げ時)

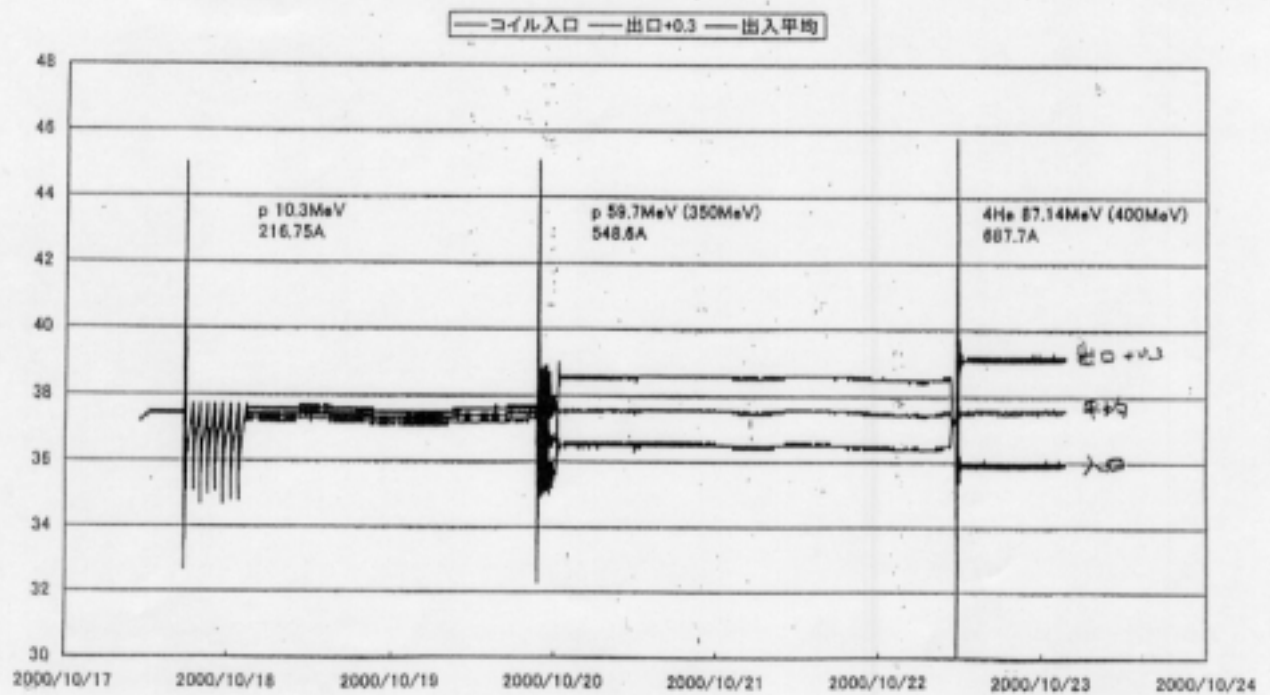


満電流のクリティカルポイントのつもりが、たまに、それに成功!





AVF主コイル出入口温度



## Remarkable Achievement

### High-Resolution Measurement of Magnetic Field of AVF Cyclotron by NMR Probes



Resolution of 1 mG:  
very helpful for tuning of AVF cyclotron

### 磁場よりも温度!

測定で気付かれた鉄芯の温度の問題

参った! 鉄芯の温度が変われば磁場の強さが変わり、磁場測定にならない。

鉄芯の温度が一定に出来る電磁石の開発が必要。

### 補足

リングサイクロトロンと  
AVFサイクロトロン

リングサイクロトロン

トリムコイルの冷却水の温度を制御すると、NMRで測定している磁場を一定にできる。

高品質ビーム、世界記録!

400 MeV陽子でエネルギー半値幅65 keV。

$1.7 \times 10^{-4}$

これをターゲットなしで、GARND RAIDENにて、分散整合と角度分散整合を同時に実現させると、16 keVに見えた。

$4 \times 10^{-5}$

1 ppm 程度の  
安定性が必要!

AVFサイクロトロン

この夏期運転休止期間中に、主コイルとトリムコイルの冷却水温度を独立して制御出来るように改造した。

しかし、そこで見えてきたことは、NMRを固定しているトリムコイル枠が温度で変形する効果であり、このため、NMRの読みを頼りに出来ないということである。

これを称して、対症療法と呼びたい。

### AGOR

AGORが発信して世界を駆け巡っていきそうな話:

サイクロトロンにおいて最後に残された問題は磁場の再現性で、その解決には、鉄芯温度の安定化と鉄芯温度の再現性を高める必要がある。

透磁率・温度依存性



磁場勾配のある

AVFサイクロトロン磁場の時間変動を

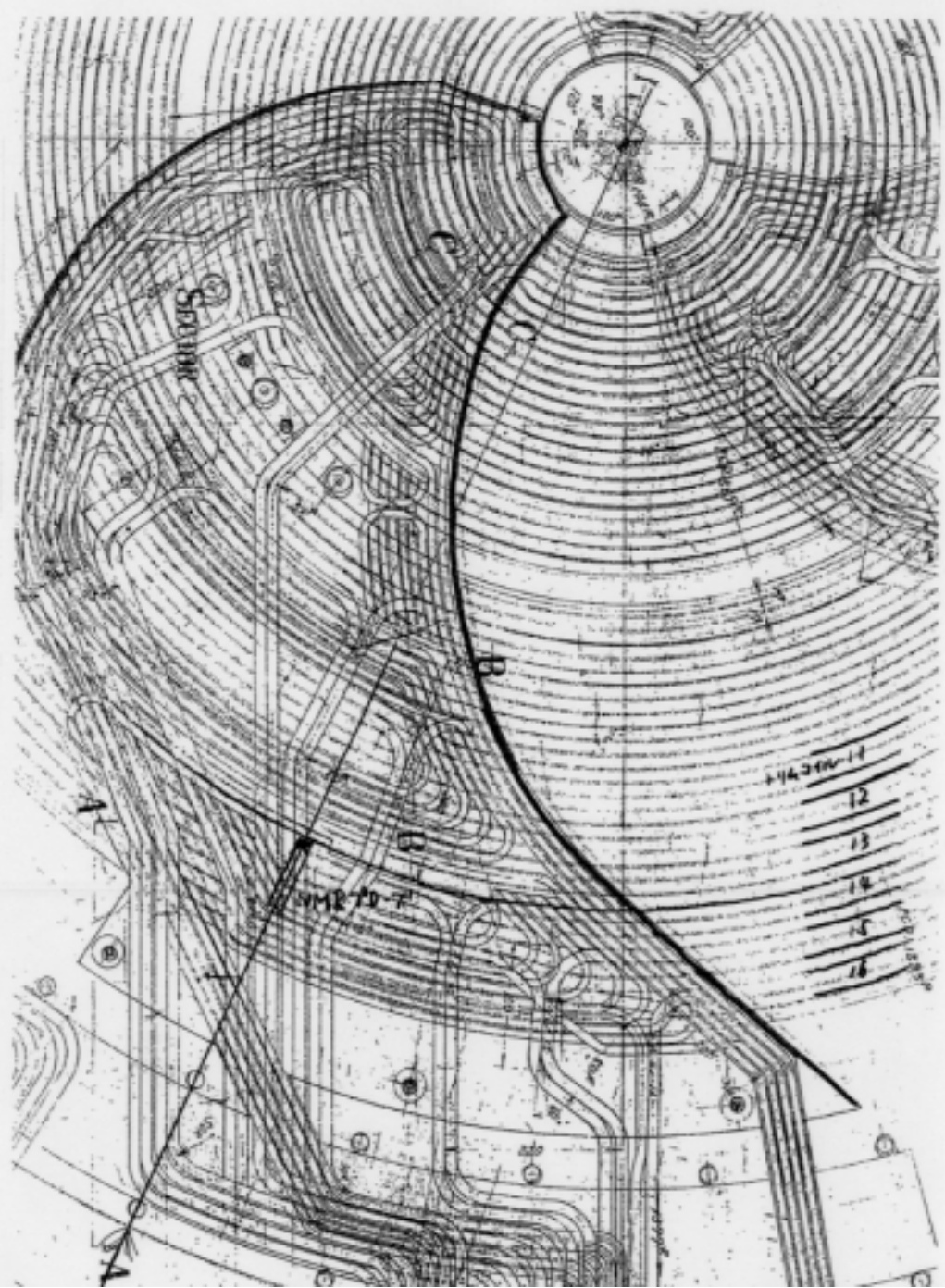
NMRプローブで測定したい。

磁場の絶対値を測定しなくとも良い。

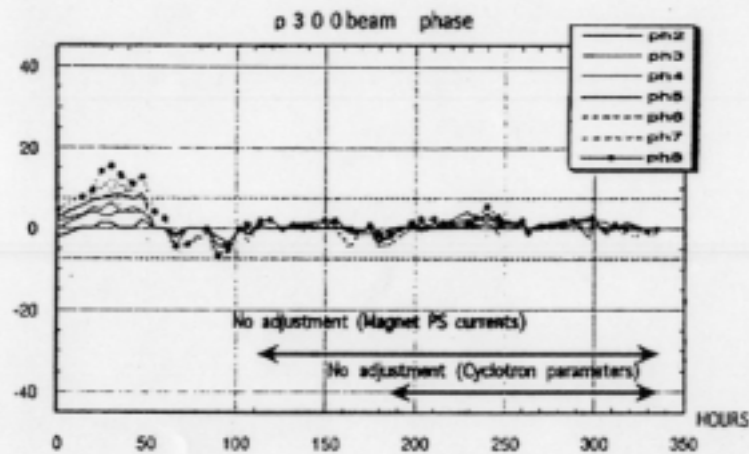
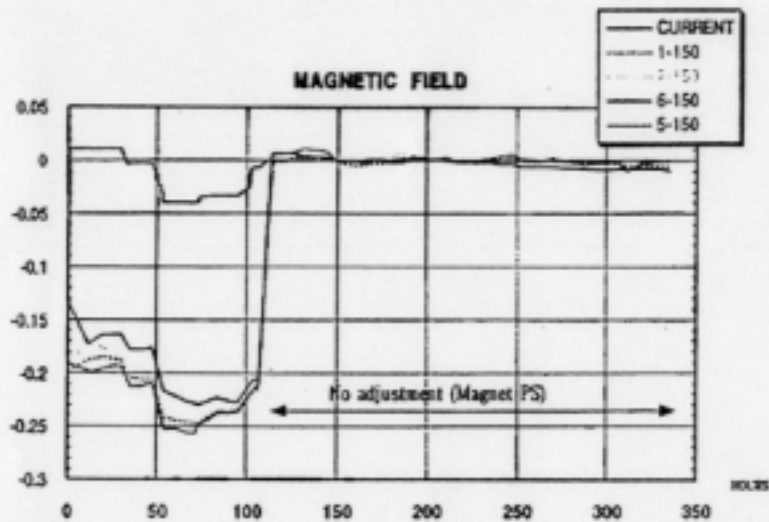
磁場勾配相殺用補正コイルを設けた

NMRプローブを完成。

その写真が見当たらない!!!



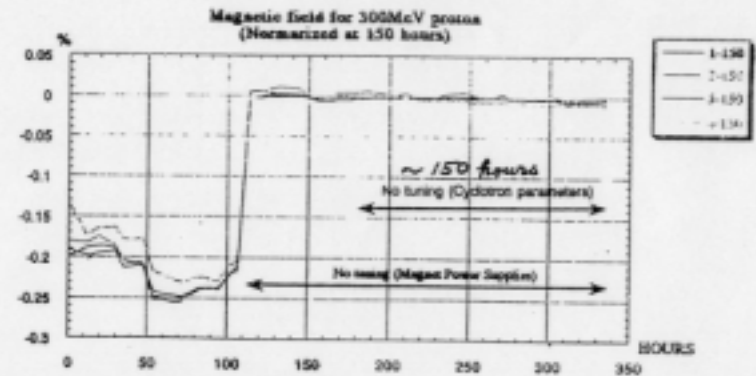
## Ring cyclotron Remarkable achievements in July, 1997



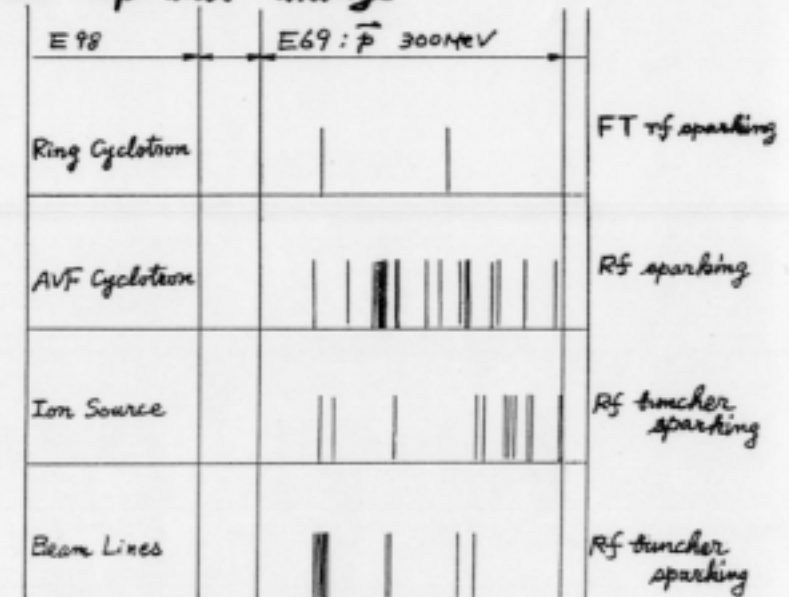
No adjustment of Ring cyclotron parameters:  
in a long term of about 10 days

## Upgrading of AVF-Ring Cyclotron Cascade

Reproducible remarkable achievement in July, 1997, of  
highly-stable long-term operation of Ring Cyclotron  
without any tuning of all Ring Cyclotron parameters



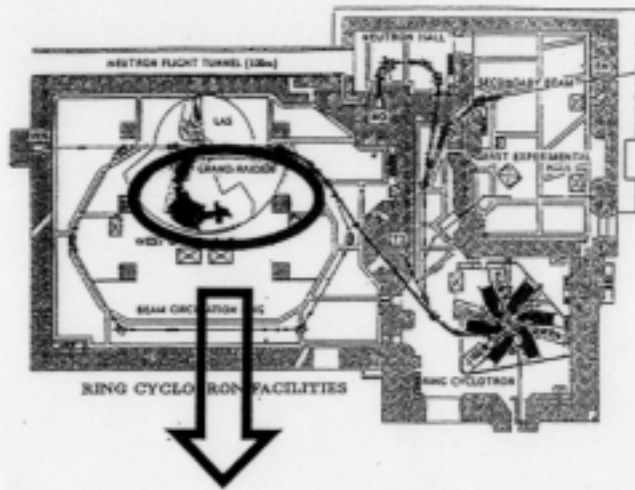
## Beam Trips and Tunings



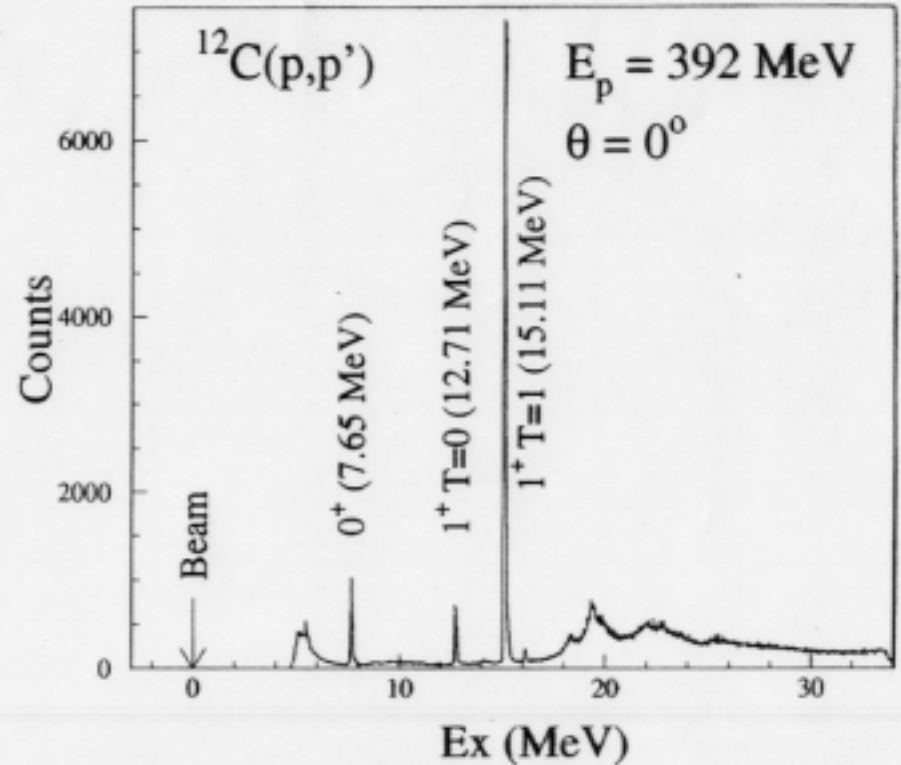
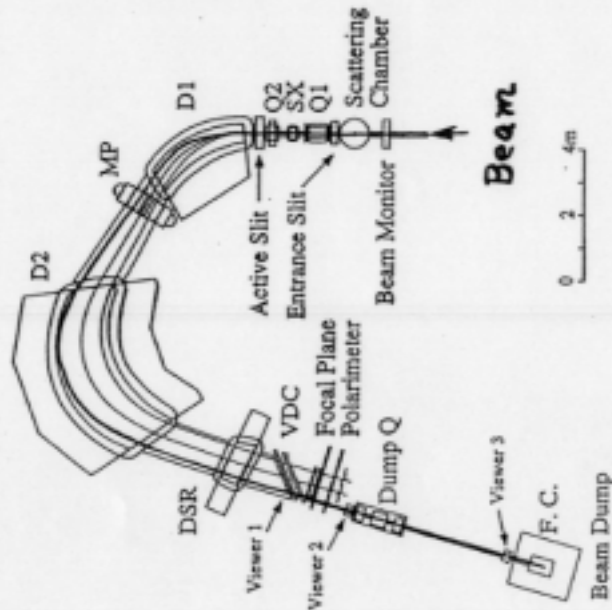


## Remarkable achievements in 1997

The world's nicest low background  $0^0$ -measurement at  $E_p = 392 \text{ MeV}$   
at GRAND RAIDEN



GRAND RAIDEN



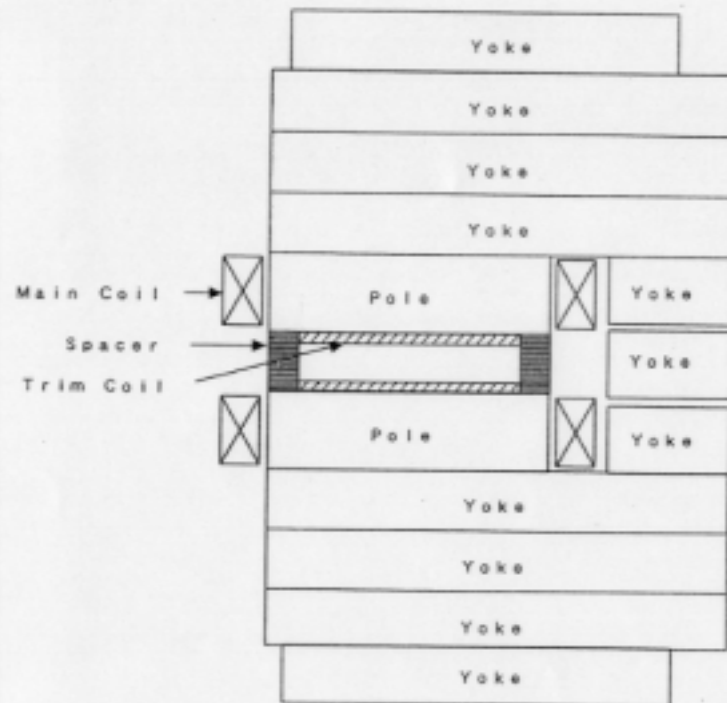
$$\Delta E_{\text{beam}} \cong 140 \text{ keV}$$

$$\left(\frac{\Delta E}{E}\right)_{\text{beam}} \cong 3.6 \times 10^{-4}$$

$$|\theta| \leq 0.5^\circ$$

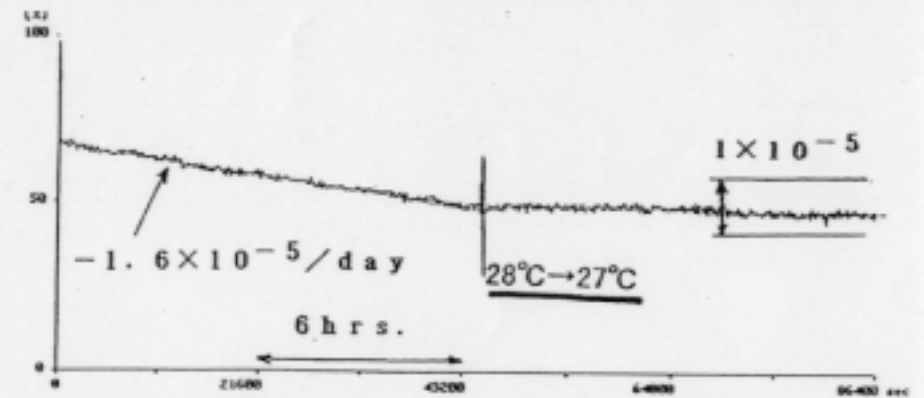
The best performance;  
attainable for a long term

## Ring cyclotron



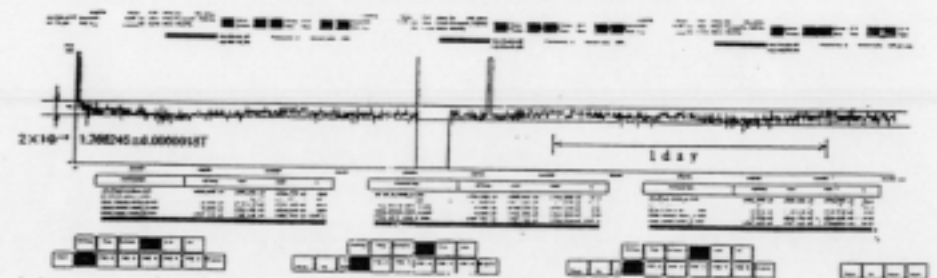
Structure of the Sector Magnet for the Ring Cyclotron

### Adjustment of temperature of cooling water of magnet coils



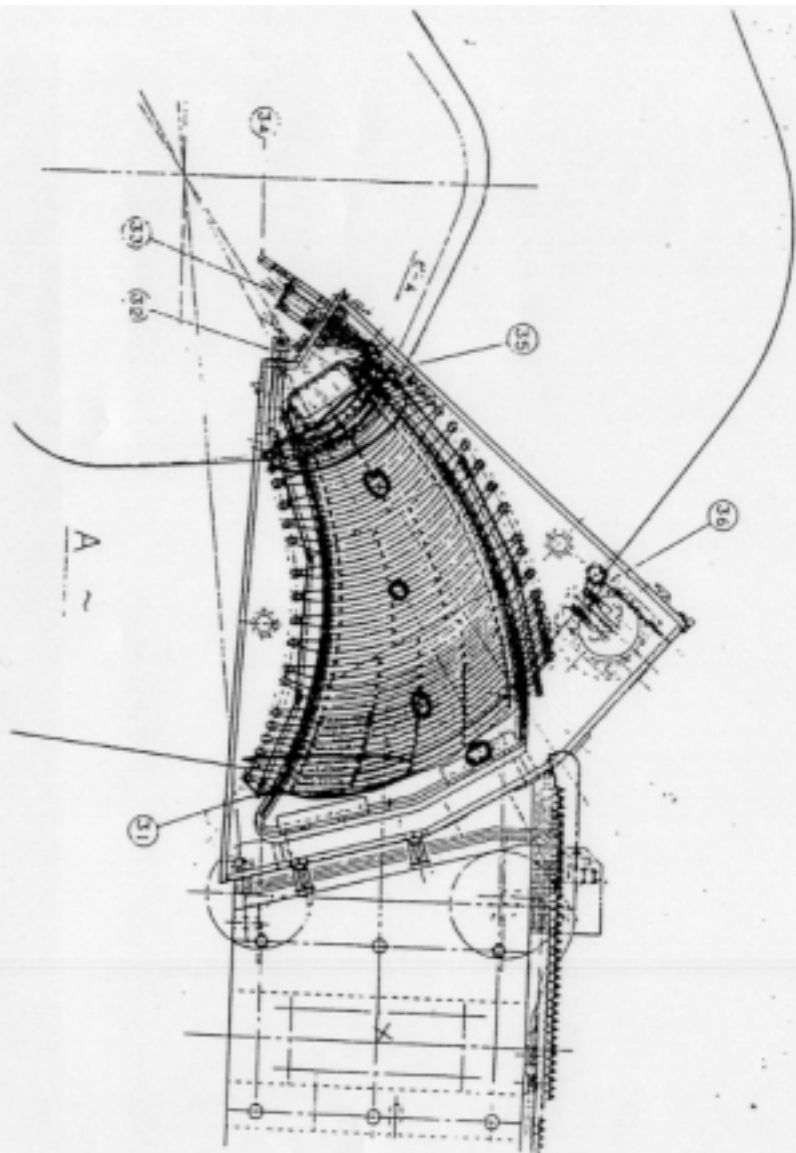
### Remarkable achievement in February, 1997

Trend data of the magnetic field for the Ring Cyclotron  
1997/02/08 1:55 - 02/11 1:55 (72hours)



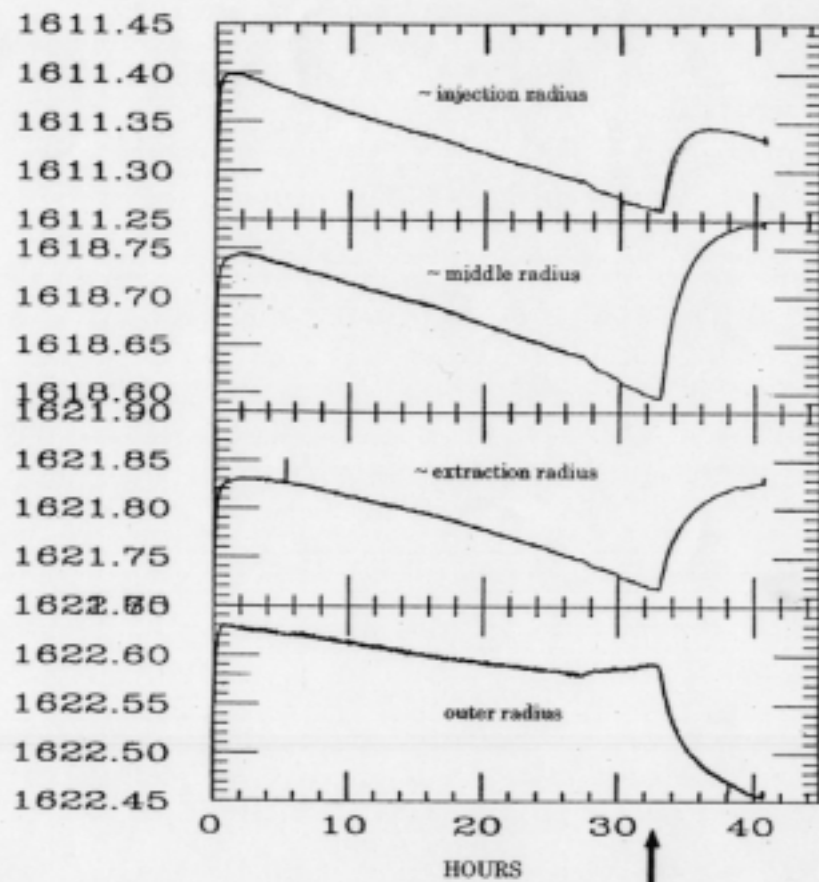
Relative field change:

less than  $2 \times 10^{-6}$  in a long term of 3 days



## Ring cyclotron

Field change with temperature of cooling water of magnet coils



temperature change of cooling water

Fast response

due to

a special structural feature of magnets

## 1.1 Unique experimental fact of a long time-constant

H.N. Jungwirth of NAC, South Africa

"Field Setting for a Cyclotron Sector Magnet"

Proc. of MT9, 1985, Zurich, pp.138-141.

Transition measured from 1.25 T to different field level

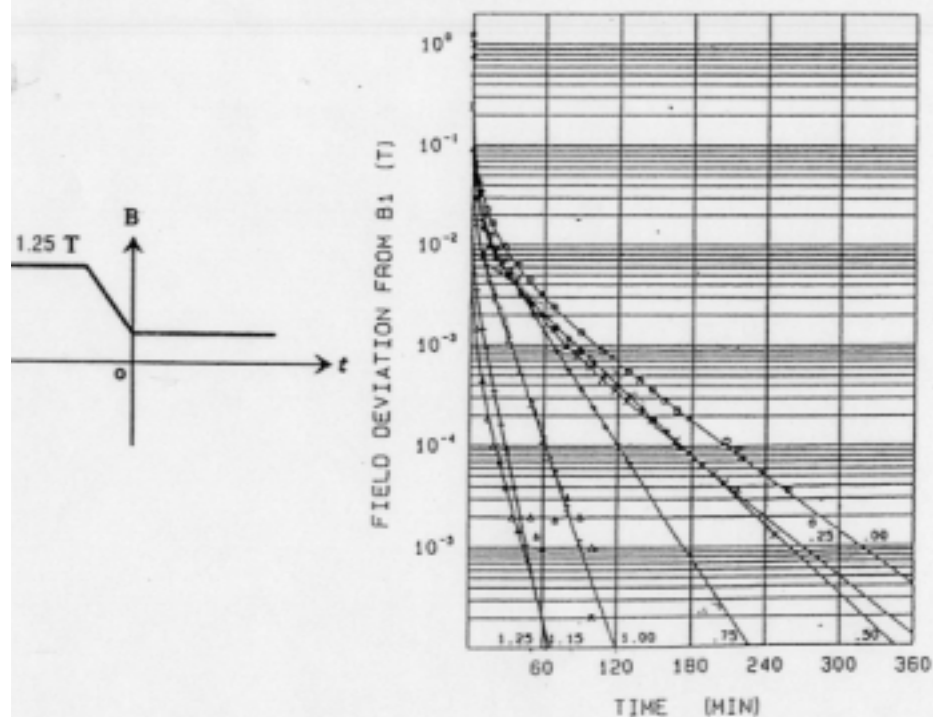


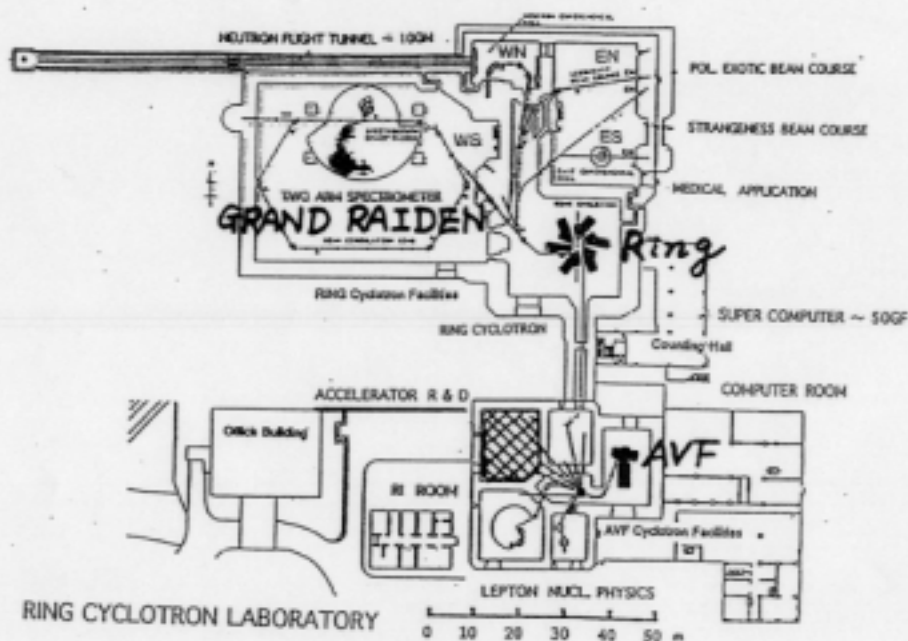
Fig. 3 - Field transition characteristics measured at different field levels  $B_1$  in the pole gap of a sector magnet.

### Reproducibility

Six standard excitations ranging from 0.25 T to 1.25 T have been set many times, both by hand and under computer control, using these procedures during the field measurements for the sector magnets. The mapping equipment made use of two Hall-probes as field sensors with a resolution of 20  $\mu\text{T}$ . The field was also monitored with NMR-probes to within 2  $\mu\text{T}$  at one or two positions in the pole gap, sometimes up to 12 hours after the field has become stable according to the setting procedure. No field drift was observed except for variations within the stability limit of the power supplies and at maximum excitation, where a remanent magnetization deficiency cannot be completely avoided.

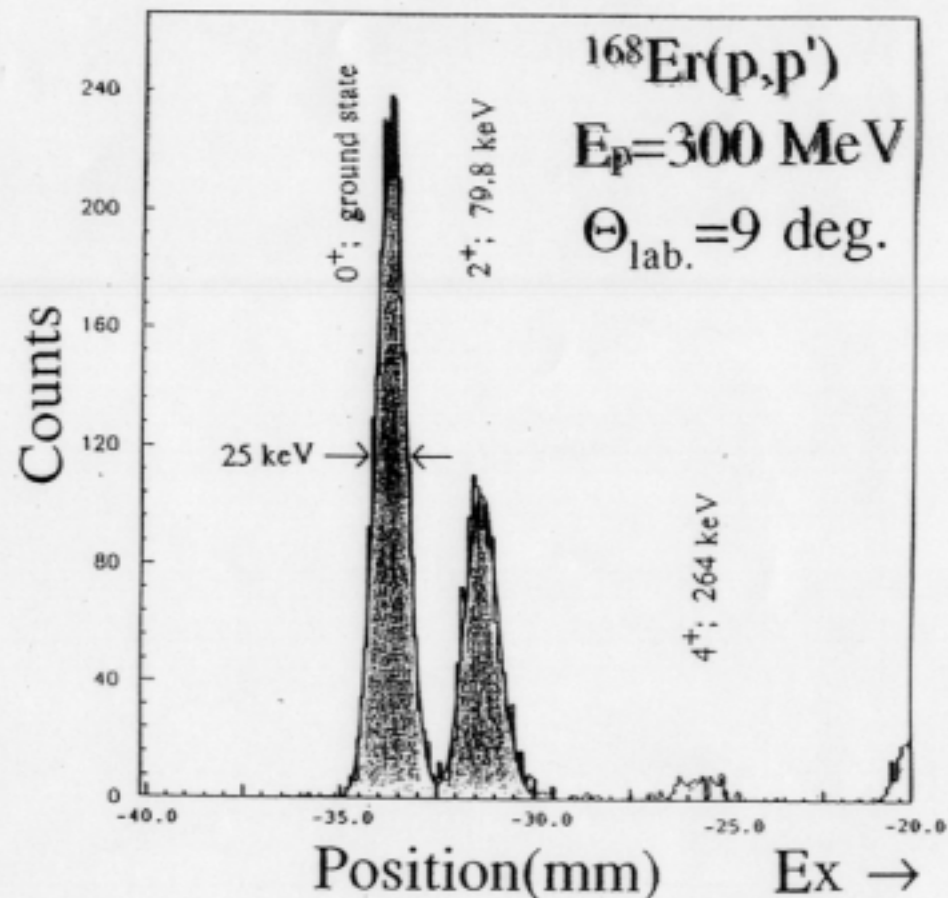
When making use of computer controlled field setting a field reproducibility of 0.2  $\mu\text{T}$  was measured at maximum excitation, improving gradually to about 50  $\mu\text{T}$  at lower field levels. Deviations by up to a factor of two larger were encountered when different operators were setting the power supplies manually, because of increased variations in the time structure of the procedure, including somewhat differently ramped currents. Most of the deviations for computer controlled setting have been observed after fields with high trim-coil excitation, but these should now be erased during the modified first stage of the procedure. When the magnets were fully excited for the first time after they had not been in operation for a few days or longer, the maximum field was up to 0.5  $\mu\text{T}$  lower than during continuous operation probably due to temperature effects.

# I. Overview of AVF-Ring Cyclotron Cascade



- 1971 RCNP established.
- Construction of the AVF cyclotron started.
- 1975 The 1st beam extracted from the AVF cyclotron.
- 1987 Construction of the Ring Cyclotron started.
- 1991 The 1st beam extracted from the Ring Cyclotron.
- 1997 Remarkable achievement of highly-stable long-term operation of the Ring Cyclotron without any tuning of all cyclotron parameters.

The measurement of a high resolution (p,p') spectrum at GRAND RAIDEN



$$\Delta E_{\text{beam}} \cong 100 \text{ keV}$$

$$\left(\frac{\Delta E}{E}\right)_{\text{beam}} \cong 3.3 \times 10^{-4}$$

The best performance;  
attainable for a short term only