Robinson Instability under Cavity Voltage Feed-back Observation at the SPring-8 Storage Ring

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1 Introduction

Robinson instability under cavity voltage feed-back loop was analized in [1]. We apply it to the SPring-8 storage ring to find the stable region of the parameters of the feed-back loops. We use the eigenvalue equation Eq.() and Eq.() in [1] to obtain the growth rate of the instability. At solving them, we have to care that C, R, G_0 and Z_f may have frequency dependence. We used an iteration scheme to solve it.

2 Application to the SPring-8 Storage Ring

We apply above analysis to the SPring-8 storage ring which has three RF stations(B,C,D). Each station has one klystron, eight single-cell RF cavities and two feed-back loops, one for amplitude and one for phase, to control the sum of voltages of cavities. The parameters of the SPring-8 storage ring are shown in Table 1.

Table 1
Parameters of the SPring-8 storage ring

Electron Energy	E_0	7.975	GeV
RF Frequency	f_{RF}	508.58	MHz
Harmonic Number	h	2436	
Radiation Loss/Turn	U_0	8.91	MV
Acceleration Voltage/Turn	V_c	12	MV
Max. Stored Current	I_0	100	mA
Synchrotron Frequency	f_s	~ 1.5	kHz
Number of RF stations	N_s	3	
Number of Cavities/Station		8	
Cavity Parameters			
Shunt Impedance	R_c	3	MΩ
Unloaded Q	Q_c	40000	
Coupling Factor	β	2	
Detuning Phase Angle	$\Delta\psi$	-5	deg.

2.1 Static Cavity Voltage

To apply the theory in [1], we have to know the impedance of the cavity and static value of the cavity voltage.

We assume that every cavity in each station has the same parameters and the sum of voltages of these cavities is controlled by feed-back loops. Thus, we treat these 8 cavities as one cavity whose impedance is 8 times of a actual single caivty.

The impedance of a cavity has a form

$$Z(\omega) \quad = \quad R_L \cos\psi e^{i\psi}, \quad \tan\psi = Q_L \left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega} \right) \quad (1)$$

where $Q_L = Q_c/(1+\beta)$ and $R_L = R_c/(1+\beta)$ are constant of a cavity and are shown in Table **??**, and ω_r is the resonant frequency which is dynamically controlled by a tuner control system.

To obtain ω_r which determine the impedance, we have to analyze the voltage in a cavity. The static part of phasers of cavity voltages are shown in Fig. 1 where $\tilde{i}_b = -2I_0$ is the static part of the beam current[2] , $\tilde{V}_b = Z(\omega_{RF})\tilde{i}_b = -2I_0 R_L \cos\psi e^{i\psi}$ is the beam induced voltage, and I_0 is the stored current.

We do not show the suffix 0 which was used to tag static part of parameters in [1] and we use ϕ instead of ϕ_c in [1].

The resonant frequency ω_r is controlled by the cavity tuner control system to control $\Delta\psi$ and $V_c = |\tilde{V}_c|$ is controlled by the amplitude feed-back loop and ϕ should fulfill the equation $V_c \cos\phi = U_0$ to compensate the energy loss of the beam. Now we have $\tilde{V}_b$, V_c, ϕ and $\Delta\psi$.

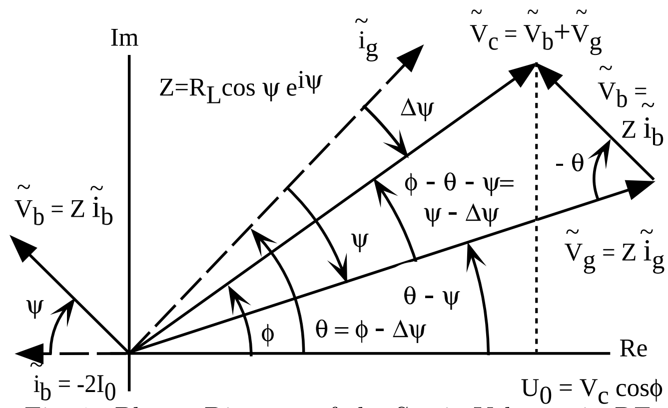

Fig. 1 Phaser Diagram of the Static Voltages in RF Cavity. The positive direction of the angles is counter-clockward.

Grancing Fig. 1, we have

$$|\tilde{V}_b| \sin\theta = |\tilde{V}_c| \sin(\psi - \Delta\psi), \quad \theta = \phi - \Delta\psi \qquad (2)$$

and

$$\tan\psi \quad = \quad \tan\Delta\psi - \frac{1}{\cos\Delta\psi} \frac{2I_0 R_L}{V_c} \sin(\phi - \Delta\psi) \; (3)$$

where we used $|\tilde{V}_b| = |Z\tilde{i}_b| = 2I_0 R_L cos\psi$. The resonant frequency ω_r is obtained through $\tan\psi$ and its definition Eq. (1).

2.2 Feed-Back Controller

Each feed-back loop has one feed-back controllers which consists of a filter circuit and an amplifier of gain G_0. The filter circuit is shown in Fig. 2 and the first stage of the circuit is a high-pass filter and the second is a low-pass filter. The matrix element of the filter for one feed-back loop is

$$Z_f = \frac{1}{i\omega C_L (Z_H + R) + 1}, \quad Z_H = \frac{R_H}{1 + i\omega C_H R_H} \qquad (4)$$

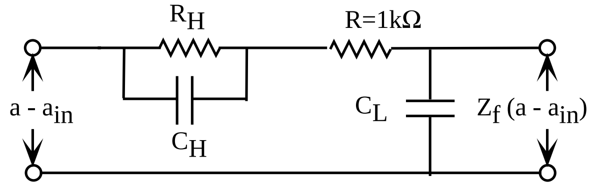

Fig. 2 The filter circuit Z_f in a feed-back controller of the SPring-8 storage ring. The values of R_H, C_H and C_L are listed in Table 2.

Table 2
Filter Parameters. The raw k is for amplitude feed-back loop and the raw ϕ is for phase.

Filter A $C_H = 0\mu F$ $C_L = 24.7\mu F$				
		$R_H/k\Omega$ at Station		
set		B	C	D
A-1	k	10	10	10
	ϕ	10	10	10
A-2	k	1.1	1.1	1.1
	ϕ	1.1	1.1	1.1
A-3	k	1.1	1.1	1.1
	ϕ	10	10	10
A-4	k	10	10	10
	ϕ	1.1	1.1	1.1

Filter B $C_H = 10nF$ $C_L = 4.7\mu F$				
		$R_H/k\Omega$ at Station		
set		B	C	D
B-1	k	10	10	10
	ϕ	10	10	10
B-2	k	3	10	10
	ϕ	10	10	10
B-3	k	3	3	10
	ϕ	10	10	10

where $R = 1k\Omega$ and the other parameters are shown in Table 2.

At the experiment, we used two set of C_H and C_L, Filter A and Filter B, and used several set of value of R_H for six feed-back loops; A-1,2,3,4 for Filter A and B-1,2,3 for Filter B, as shown in Table 2.

The frequency response of the feed-back loop at zero beam current is shown in Fig. 3 and Fig. 4 for Filter A and Filter B with several values of R_H.

Filter A has narrower frequency range compared with Filter B because of the large value of C_L.

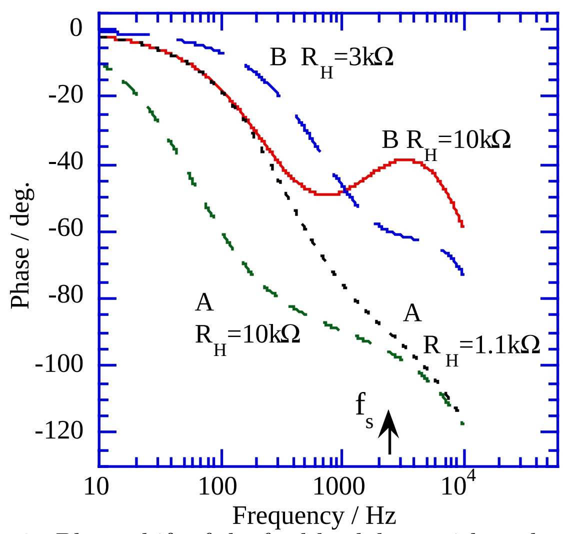

Fig. 3 Phase shift of the feed-back loop without beam current for parameters listed in Table 2.

2.3 Growth Rate and Frequency Shift

The calculated growth rate are shown in Fig. 5 and Fig. 6 for parameters in Table 2.

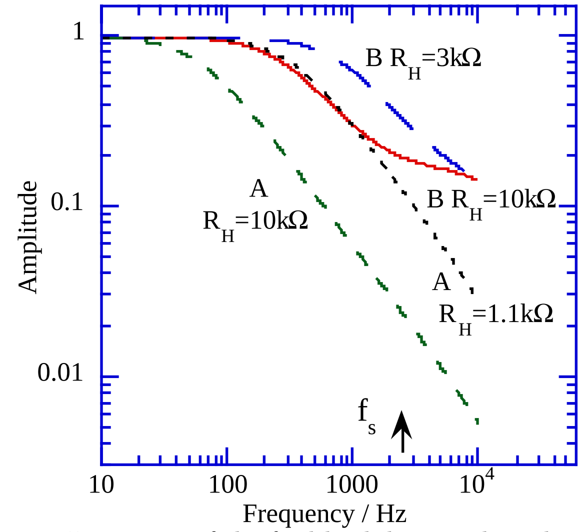

Fig. 4 Loop gain of the feed-back loop without beam current for the parameters listed in Table 2

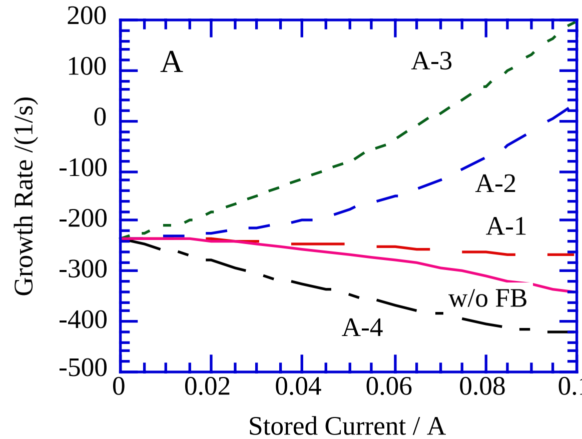

Fig. 5 Growth rate vs stored current for filter A. The growth rate includes radiation damping.

The growth rate is positive above 0.6 A with the parameter B-3.

With the parameter A-3, the gain of amplitude feed-back loop at the sychrotron frequency is larger then A-1 and growth rate is larger than A-1. On the other hand, with the parameter A-4, the gain of phase feed-back loop at the sychrotron frequency is larger then A-1 and growth rate is smaller than A-1.

3 Simple Model

We show a simple model to explain the effect of the feed-back loop on synchrotron motion of a beam.

We assume that Q value of a cavity is lower enough so that the cavity voltage respond immediately to the change of driving current such as i_b or i_g.

Assume the beam executes synchrotron motion. If the beam has a shift of the timing $\tau < 0$ as shown in Fig. 7, this delay produces shift of the beam current, $\Delta\tilde{i}_b$,which produce a shift of the cavity voltage, $\Delta\tilde{V}_b$. The feed-back loops try to compensate this shift; the amplitude feed-back loop produces $\Delta\tilde{V}_{gk}$ which reduce energy gain of beam and the phase feed-back loop produces $\Delta\tilde{V}_{g\phi}$ which increase energy gain of beam. If the phase delay of the feed-back loop is 0, the total energy

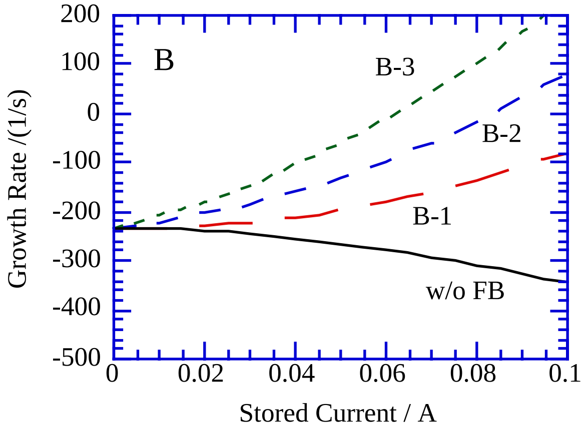

Fig. 6 Growth rate vs stored current for filter B. With B-3, Growth rate exceeds 0 at stored current more than $\sim 0.6A$.

gain of the beam during the energy shift $\delta < 0$ or during $\delta > 0$ is 0.

In actual case, the feed-back loops have a phase delay as shown in thus total energy gain of the beam by $\Delta\tilde{V}_{gk}$ during $\delta < 0$, is negative and the total energy gain during $\delta > 0$ is positive hence the amplitude feed-back loop excite the synchrotron osciilation.

On the other hand, $\Delta\tilde{V}_{g\phi}$ increase the erergy gain during $\delta < 0$ and decrease the erergy gain during $\delta > 0$, which means the phase feed-back loop damps the synchrotron oscillation.

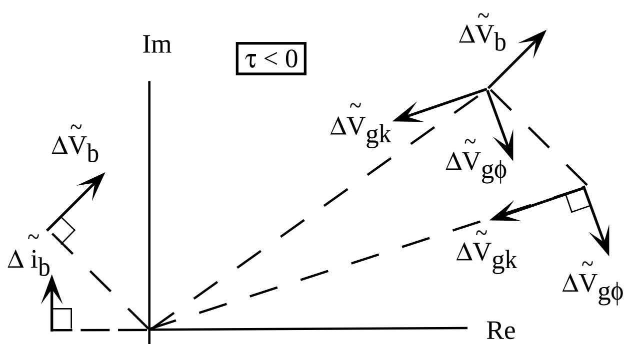

Fig. 7 Phaser diagram of the small shift of the cavity voltage. $\tilde{V}_b$ is produced by small timing delay of the beam for $\tau < 0$. $\tilde{V}_{gk}$ and $\tilde{V}_{g\phi}$ are voltage produced by the amplitude and the phase feed-back loop to $\tilde{V}_b$. The static part of voltages are the same as Fig. 1.

From Eq.() in [1], the amplitude of synchrotron oscillation of the beam, $|\breve{\varphi}|$, is $Q_s = \frac{1}{2}\tau_E\omega_S \sim 20$ times larger than the modulation of the cavity voltage $|\breve{\phi}|$ or $|\breve{k}|$ which drives $\breve{\varphi}$. The loop gain of the feed-back should be smaller than $1/Q_s$ at the worst case that phase delay is 90 deg. But in case of Filter B, the gain at f_s is still higher than $1/Q_s$ and may become unstable.

4 Observation

The experiment was performed with the paramter A-1 and B-3. The sychrotron motion of the beam was excited by a phase modulaton on the RF acceleration voltage and the phase oscillation of the beam current was observed by a phase detector.

The observed frequency response of the synchrotron motion is shown in Fig. 8. As the stored current increases, the peak height become higher and the resonance width become narrower which means damping time is longer because instability growth rate cancel the radiation damping rate.

Above stored current 0.6 A, the large amplitude of the synchrotron motion was observed without excitation and a large sideband peaks of RF frequency in a beam signal, was obserbed. Its peak height is proportional to the amplitude of the synchrotron motion of bunchs.

With A-1, which is the nominal parameter of the storage ring, the growth rate is far below zero and the synchrotron sideband at RF frequency Observed synchrotron oscillation at the experiment is shown in Fig. **??**.

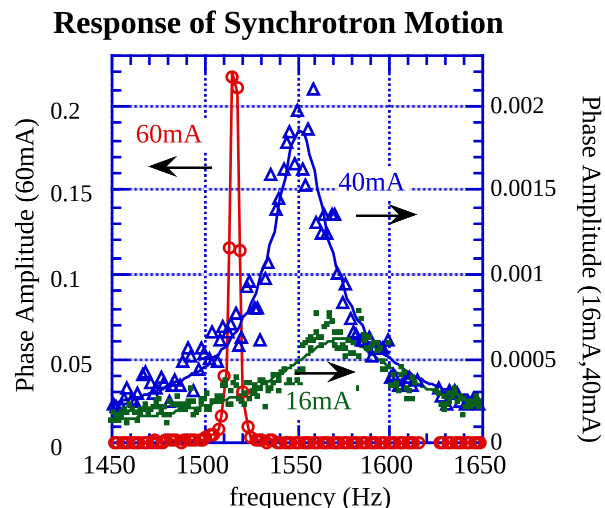

Fig. 8 Observed frequency response of the amplitude of phase osciilation of the beam with Filter B. The data for 60mA is 2 magnitude larger than the data at 16mA and 40mA. The damping time/rate of the synchrotron motion from the width of the data are $5.5ms/182s^{-1}$ for 16mA, $12ms/83s^{-1}$ for 40mA and are the expected value from the theory, shown in Fig. 6. the beam was unstable at 60mA. The peak height is also proportional to the damping time.

5 Conclusion

The effect of the cavity voltage feed-back loop on the robinson instability was analized and growth rate and synchrotron frequency shift is obtained. It shows that ,in actual machine which has slower synchrotron oscillation frequency and large beam loading like the SPring-8 storage ring, the frequency response of the feed-back loop must be slower enough to suppress gain at the synchrotron osicllation frequency to get stable operation at high current.

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References

[1] T. Nakamura, "Robinson Instability under Cavity Voltage Feed-back", This proceedings.
[2] P. Wilson, SLAC-PUB-2884(1991),SLAC.